Strategic Achilles: Is Prior Information About Your Opponent’s Strategy Valuable?∗

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Abstract

Is having prior information about your opponent’s strategy helpful? We investigate this question using an adaptive framework where agents must implement their strategies as finite automata. Some agents are informed of the strategic complexity of their opponents and condition their strategies on that information. Intuitively, one might think that such strategic information should be advantageous, or at least should never hurt since it can be ignored. Notwithstanding this intuition, we find that informed agents do slightly worse than uninformed ones across a wide class of games. In fact, there is a class of games where they do significantly worse. This latter result is tied to the need of the informed agents to condition their strategies on the complexity of their opponents, leaving them vulnerable to manipulation after periods when certain levels of complexity do not arise in the uninformed population. Hence, any benefits accrued to the informed agents by their enhanced abilities come at a potential cost, just as the mythical Achilles could only become immortal by being vulnerable at the heel.

Keywords: Strategic Choice, Pre-Play Information, Neutral Evolution, Automata.

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1 Introduction

In many strategic situations it may be possible to acquire some prior information about your opponent’s strategy. Here, we pursue the question of how valuable is such knowledge. More specifically, we study a setting in which some players receive information about the degree of sophistication (complexity) of their opponent’s strategy prior to play. Intuitively, one would imagine that such information should be useful in the design of one’s own strategy and, at the very least, leave one no worse off as the information can always be ignored. For example, suppose you know that your opponent is going to use an extremely simple strategy in a repeated Prisoner’s Dilemma game. Then, you might infer that she will always cooperate or always defect, regardless of your play. Such prior knowledge might be useful when it comes to designing your own strategy. Notwithstanding the above intuition, we find that the value of such prior information is rather meager and, surprisingly, it may even prove to be disadvantageous at times.

The intuition that prior information about your opponent’s strategy should be of great value fails on many levels. First, it may be the case that, in practice, such information has only limited value. Good strategies always need to discover, and exploit, information about their opponent’s strategy as play proceeds, and having to do so while playing rather than prior to play may come at only a slight cost. Thus, in the case of the repeated Prisoner’s Dilemma introduced above, a strategy like Tit For Tat is a robust choice that can quickly—during the course of play—acquire needed information about, and effectively adapt to, the strategies of a variety of opponents. Second, agents are not passive entities in these environments, and if information is being transmitted about their strategies prior to play, they may be able to alter this information so as to confound its value, eliminating any potential strategic advantage.

While these two arguments are sensible explanations for the ineffectiveness of prior information (at least post hoc), they do not provide an adequate explanation for why having such information can actually be disadvantageous in some situations (especially since the argument that the information can always be ignored is so compelling). We leave the full explanation for this latter phenomenon for the penultimate section of the paper, and simply remark here that the need of the informed agents to condition their strategies on the complexity of their opponents leaves them vulnerable to manipulation during periods when certain levels of complexity do not arise in the uninformed population.

While counterintuitive, the above results have some basis in standard game theory. Game theory suggests that agents attempt to optimize their strategies against one another, so even if we alter the information conditions facing the agents, we would expect an effective response to be reflected in the resulting strategies. Moreover, if one agent can precommit to a strategy in, say, a Battle-of-the-Sexes game, the other agent, given knowledge of that commitment, can be forced to play for its inferior equilibrium.¹ The system we explore here explicitly incorporates both adaptation and strategic structure. We consider a world in which there is a remarkably rich set of possible strategies and where agents adapt their behavior over

¹Interestingly, this is not an explanation of the phenomena we later observe. As we discuss later on, the interplay between adaptation and strategic structure is crucial for our results.
time, rather than optimize. As will become apparent, such assumptions do not just bring the system closer to reality but also introduce some new opportunities for dynamic behavior.

To explore the above system we analyze two-player games where strategies are embodied as simple computer programs: finite automata. Agents can adapt their strategies using an evolutionary algorithm which is capable of creating effective strategies given the environment (Miller, 1988, 1996). We use automata theory (see Hopcroft and Ullman, 1979) to derive a simple measure of the strategic sophistication (or complexity) of an agent’s strategy, and allow the informed agents to condition their strategies on the complexity of their uninformed opponents.\(^2\) We investigate an entire class of 2 × 2 games as defined by Rapoport and Guyer (1978), and compare the performance of informed and uninformed agents both when playing against common populations of opponents and when playing against each other.

2 Model

We consider a system with two populations, each having \(N\) agents. Agents from one population take the role of row players, while those from the other act as column players. In every generation, each of the \(N\) agents from a given population plays a repeated 2 × 2 game with each of the \(N\) agents from the other population for \(n_R\) rounds (keeping the game fixed across all rounds and generations). Agents accumulate payoff from every game that they play in a given generation. At the conclusion of each generation, the strategies of the players in each population are modified using an evolutionary algorithm that allows the agents to “learn” better strategies over time.

2.1 Strategies and Complexity

An agent’s strategy is modeled as a finite automaton, specifically, as a Moore machine (Miller, 1988, 1996). As depicted in Figure 1, a finite automaton consists of a set of states, one of which is designated as the initial state. Each state has an associated action and transition function. The associated action gives the action that the machine will play in the current round of the game. The transition function determines the state that the machine will enter at the next round, and it is conditioned on the observed play of the opponent in the current round. In a 2 × 2 game, the transition function contains two possible transitions, one for each action of the opponent. Note that in an automaton of, say, twelve states, not all of the states may be “accessible,” that is, given the initial state there may be no possible set of transitions (opponent’s actions) that can lead to the inaccessible state. Below, we will fix the maximum number of automaton states to \(n_S\), and then allow the evolutionary algorithm (see below) to determine the actions and transitions of each state. This approach allows the system to endogenously select the number of accessible states (up to the maximum) that will be used by the automaton.

\(^2\)We also did a limited investigation of other types of prior information, such as behavioral tendencies, and found similar results. Note that we only investigated situations where partial information about the opponent’s strategy was revealed. Though, we suspect that even at the extreme—that is, the case where the entire strategy is made known to the opponent—many of our core insights would still hold.
Figure 1: Three sample automata. States are given by the large circles, transitions by the labeled arcs, and actions are shown by the labels in the interior of each state. Assuming each machine starts in its left-most state, the machine in (a) always plays $D$, regardless of its opponent’s action. The machine in (b) plays Tit For Tat, beginning by playing $C$ and then mimicking its opponent’s behavior. The machine in (c) plays a Grim-Trigger strategy, starting out and continuing to play $C$ until the opponent plays $D$, at which time it switches to playing $D$ thereafter.

Agents can either be uninformed or informed. An uninformed agent does not have any information about its opponent, and always begins in the same initial state (here, assumed to be state one) at the start of each multi-round game with a new opponent. An informed agent is given some pre-play information about its opponent’s strategic complexity. The informed agent conditions its initial state on this information, and plays different strategies conditional on the information that it receives. Here, each informed agent maintains a cutoff parameter, and if the incoming information is at or below this cutoff, the agent begins the game in state one, otherwise it begins in state two.

Informed agents condition their strategy on the strategic complexity of their uninformed opponents. We measure strategic complexity by the minimum number of automaton states needed to fully describe the strategy (thus, the strategic complexity of an agent is between 1 and $n_s$). We note at the outset that this is an imperfect measure of strategic complexity. While it does capture the notion that a strategy that always takes the same action in each round (having a minimized size of one) is simpler than one that conditions its move on the previous action of the opponent (with a minimized size of two), it can produce some

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3Note that two automata can have different sizes, yet yield the same strategic behavior. For example, a single state automaton with action $C$ and transitions back to that state is equivalent to a multi-state automaton, with each state having action $C$ and transitions that allow the entire automaton to be traversed. It can be shown (see Hopcroft and Ullman, 1979) that any automaton has a minimized-state isomorphic automaton, which we find using an algorithm discussed in Harrison (1966, p. 311).

4We never have informed agents playing against one another. Doing this would require a new definition of strategic complexity, as informed agents are essentially implementing two strategies within a single automaton. Alternative measures could include, say, the maximum of the two minimized sizes or the minimum size of an automaton that could realize both strategies.
anomalies. For example, we might consider a strategy that takes a random action in each round or one that plays action $C$ for one hundred rounds and thereafter plays $D$ both rather “simple,” yet such strategies require a large number of states to implement as a deterministic Moore machines. Notwithstanding these issues, this complexity measure is commonly used in the literature (see for example, Abreu and Rubinstein, 1988).

2.2 Evolving Strategies

We implement a simple evolutionary algorithm to allow strategies to adapt to one another in the environment. Recall that in every generation each agent from a given population plays a fixed-length, repeated game with every agent from the other population, accumulating payoffs. At the end of each generation we first select strategies for reproduction. Specifically, within each population we select two agents uniformly at random (with replacement), and place a copy of the strategy corresponding to the agent with the highest accumulated payoff (with ties being broken randomly) into the next generation. This process is repeated $N$ times, thus maintaining a constant population size across generations. Note that this process biases the new generation toward the better performing strategies within the population, though there is no guarantee that the current best strategy is maintained or that the worst one is eliminated. Also, notice that it is the accumulated scores against the players in the other population that determine the relative standing—and the ultimate reproductive success—of a given strategy within its own population.

After selection, we modify some of the strategies to introduce new strategic forms into the population. With probability $p_M$, a given strategy is selected for “mutation” and we randomly choose a state to be mutated. With probability 0.5, the action associated with that state is altered, otherwise one of the two transition states (chosen at random) is altered to a uniformly selected random state across all possible $n_S$ states. Finally, if we are mutating an informed agent, with probability $p_C$ its cutoff is changed to a number selected uniformly at random from 1, . . . , $n_S$.

2.3 Experiments

For each experiment conducted below, we performed multiple trials of the system, with $n_G$ generations in each trial. In the first generation, we generate the strategies at random, whereby for each possible state we choose both an action and the two transition states uniformly at random. For informed agents, we initially choose a cutoff uniformly at random from 1, . . . , $n_S$. The parameters used in our simulations are summarized in Table 1.

3 Results

To investigate the phenomena associated with our model, we conduct a series of numeric experiments. In Section 3.1 we consider the case in which an evolving population of either informed or uninformed agents plays a Prisoner’s Dilemma game against a fixed population of uninformed agents. This allows us to see if the evolutionary system is capable of discovering
Table 1: Parameter settings for the numeric experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population Size</td>
<td>N</td>
</tr>
<tr>
<td>Maximum States</td>
<td>(n_S)</td>
</tr>
<tr>
<td>Mutation Probability</td>
<td>(p_M)</td>
</tr>
<tr>
<td>Cutoff Mutation Probability</td>
<td>(p_C)</td>
</tr>
<tr>
<td>Rounds per Game</td>
<td>(n_R)</td>
</tr>
<tr>
<td>Number of Generations</td>
<td>(n_G)</td>
</tr>
</tbody>
</table>

Figure 2: The payoff matrix of the Prisoner’s Dilemma game. The two players (Row and Column), can each choose between two actions, \(C\) (cooperate) and \(D\) (defect).

productive strategies in a simple, static environment. In Section 3.2 we consider the case where agents from both populations evolve their strategies. We analyze this coevolving system across a canonical set of \(2 \times 2\) games. Based on the results of these experiments, in Section 3.3 we focus on a class of games where we find that informed agents do worse than uninformed agents.

### 3.1 Evolution Against a Fixed Population

We first study the relative performance of evolving informed and uninformed agents against a fixed population of uninformed agents. This allows us to test whether our evolutionary mechanism is able to successfully discover productive strategies in a simplified environment, as well as provides some initial hints about the potential value of pre-play strategic information.

We focus on a Prisoner’s Dilemma game, with the payoff matrix given in Figure 2. We have a fixed population consisting of uninformed agents who play a set of pre-defined strategies across all generations. The other players consist of an evolving population of either informed or uninformed agents, who adapt their strategies as described in Section 2.2, with the exception that for these experiments we had populations of size 24 versus 25.\(^5\)

For the fixed populations we considered a number of different combinations of strategies. The strategies we considered included Always Defect (AllD, Figure 1(a)), Tit For Tat (TFT, Figure 1(b)), Grim Trigger (GT, Figure 1(c), which cooperates as long as the opponent never defects, but if that happens it defects thereafter), Always Cooperate (AllC), and Random (Rand, where each agent has a fixed, randomly created strategy created using the same algorithm that we use for initializing strategies in the evolving populations discussed above).

\(^5\)This allows us to evenly divide a population into two halves.
Table 2: Average payoffs for the informed ($\bar{u}_{inf}$) and uninformed ($\bar{u}_{uninf}$) populations against the different fixed populations described in the text. The $p$-values (to test for equal population means) are calculated using a two-sample $t$-test assuming unequal variances (two-tailed).

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$\bar{u}_{inf}$</th>
<th>$\bar{u}_{uninf}$</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AllD+TFT</td>
<td>1.43</td>
<td>1.42</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>AllC+GT</td>
<td>2.33</td>
<td>2.08</td>
<td>&lt; 0.0001</td>
</tr>
<tr>
<td>Rand</td>
<td>2.05</td>
<td>2.05</td>
<td>0.94</td>
</tr>
</tbody>
</table>

The combinations of strategies we considered were **AllD+TFT** and **AllC+GT**, where there were twelve each of the two strategy types given in the label, as well as a population of Random strategies, **Rand**. For each of these three experimental conditions, we ran fifty trials of 500 generations each.

The results of these experiments are summarized in Table 2, where the averages are taken over all agents across all generations. We find that there are two cases, **AllD+TFT** and **AllC+GT**, where the average payoff of an evolving population of informed agents over uninformed agents is statistically significant, though the practical advantage is slight in the first case.

In the following discussion, we focus on the benchmark of optimal play by the evolving agents—in general, the observed play approaches such an ideal. In the first three experiments, the two strategies composing the fixed population differ in the number of minimized states (either one or more than one), so the prior information does allow the informed agents to clearly distinguish the strategy about to be employed by the opponent if their cutoffs are set appropriately. In the **AllD+TFT** case, this prior information allows an informed player to always defect or always cooperate depending on the information about the opponent, and receive a total payoff of 10 from the AllD and 20 from the TFT during the ten round game. Without this information, an uninformed player could still play Tit for Tat, insuring mutual cooperation when playing a TFT (and a total payoff of 20), at the cost of being defected on for one round by an AllD (and a total payoff of 9 across the ten rounds). Thus, the uninformed agent can receive a payoff of 29 versus 30 for the informed (which imply average payoffs per round of 1.45 versus 1.50 respectively). In our experiment, the payoffs approach these values, though the informed agents are farther from this ideal than the uninformed—as we explore below, this may due to slower learning by the informed agents.

The **AllC+GT** environment was designed to give the informed players a clear advantage, and that is reflected in the statistically and practically significant advantage of these players over the uninformed. In this environment, the informed players can “know” in advance whether to always defect (against an AllC) or always cooperate (against a GT), which if they do perfectly implies an average payoff of 2.5 per round. The average payoff of informed agents in our experiments is below this ideal payoff, but significantly higher than the payoff of 2.1 that they would receive if they would not discriminate among agents with different

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6Similarly, defecting in round one will insure mutual defection with an AllD (total payoff of 10), but at the cost of a defection by TFT in the next period (resulting in a total payoff here of $3 + 0 + 8 \times 2 = 19$). In either case, the resulting payoff for the uninformed is 29.
strategic complexity. Uninformed players are in a tough position in this environment, as the only way they can differentiate between the two types of opponents is to defect, but such a defection will result in defection by the opponent forever after. If the uninformed agent simply chooses cooperate, regardless of its opponent’s strategy, it will receive an average payoff of 2 (and its opponents will receive an average payoff of 2 as well). If instead the uninformed agent always defects, it will receive an average payoff of 2.1 (and its opponents will average .45). Our data indicate that the uninformed agents followed this latter approach, and thus received the slightly higher payoff.

Finally, in the Rand condition, the information value from knowing the opponent’s complexity is essentially neutralized given the random design of the strategies, and in such environments Always Defecting is the dominant strategy.

Our explorations of the environments above were conducted for two main reasons. First, we wanted to insure that our evolving agents were able to develop effective strategies in a relatively constrained, fixed environment. Indeed, we found that the algorithm was able to evolve novel and effective strategies, approaching the theoretical best strategies. Second, we wanted to acquire a useful benchmark for the potential value of prior information about your opponent’s strategy. A priori, we felt that these types of fixed-opponent environments should favor the informed agents, though experimentally we found that such an advantage did not materialize in any significant way in most environments.

While there are some circumstances under which the informed will prosper over the uninformed agents, such as in AllC+GT, these tend to be fairly contrived. We suspect that, in general, prior information about your opponent’s strategic choices—perhaps short of knowing its entire strategy—may not provide that much of an advantage even when the opponents are not allowed to adapt. The main reason for this lack of a substantial advantage is that uninformed agents, though lacking access to pre-play information, often are able to generate, and react to, similar information generated during the course of play. Thus, we find that, at least in some environments, careful strategic choice may be a sufficient substitute for the potential advantage brought about by prior information.

3.2 Coevolving Populations

In the experiments in the prior section, our evolving agents played against a fixed set of opponents. In more realistic settings, opponents are not passive entities, but actively alter their strategic choices in an attempt to neutralize any advantages of their opposition. To explore this scenario, we now analyze a system in which an evolving population of either informed or uninformed agents plays against an evolving uninformed population. Since both populations are simultaneously evolving their strategies, we have a coevolving system where strategic adaptations by players on one side of the game may lead to new strategic innovations on the other side, and so on.

We consider all 78 canonical games of Rapoport and Guyer (1978) along with a few additional games not fully captured by this set. The 78 canonical games of Rapoport and Guyer (1978) represent all $2 \times 2$ games in which players have strict ordinal preferences over outcomes. Since ordinal preferences do not sufficiently specify payoffs for repeated games, we
directly transform the ordinal preferences into their associated cardinal values (with payoffs constrained to \(\{1, 2, 3, 4\}\)). Twelve of the 78 games in the canonical set are symmetric. Thus, there are 66 asymmetric games, implying a total of 132 possible roles (as both a row and column player) for the players in these games. We also supplement the twelve symmetric games with three additional ones: Battle of the Sexes, Stag Hunt, and Pure Coordination.\(^7\)

First consider the 132 possible roles in the asymmetric games.\(^8\) In each game, we observe an evolving population of either informed or uninformed players matched against an evolving population of uninformed opponents, and compare the difference between the average payoffs received by either the informed or uninformed agents playing the same role in the game against uninformed opponents. We find that there is a statistical difference in performance between the two types of players in 62 of the 132 games and, of these, 55 (89\%) had the uninformed player earning more than its informed counterpart, with the remaining 7 games favoring the informed over the uninformed. While statistically significant differences were observed, the practical significance of the advantage was slight in most of these games. If we use a threshold of 0.1 as a practically significant difference in expected payoffs, then only three of the asymmetric games showed both statistically and practically significant differences, and all three of these games favored the uninformed counterpart.\(^9\)

Next, consider the 15 symmetric games. Only 4 of the 15 games showed a statistically significant payoff difference between informed and uninformed agents facing an uninformed opponent. In all four of these games, the uninformed agents did better than the informed ones, though only in one of these games was that difference practically significant (using, again, a 0.1 threshold).\(^{10}\)

Thus, we find that, in general there is no payoff advantage to being an informed versus an uninformed player in contests against an uninformed player. Indeed, there is some evidence that uninformed players do slightly better than informed players in such situations, though rarely is this performance difference of practical importance.

\(^7\)These three additional games are not captured by the canonical set because some of the action pairs lead to identical payoffs versus a strict order. The payoffs are \((3, 1, 0, 0; 0, 0, 1, 3)\) (Battle of the Sexes), \((3, 3, 0, 2; 2, 0, 2, 2)\) (Stag Hunt), and \((3, 3, 1, 1; 1, 1, 3, 3)\) (Pure Coordination) for the three games, where the first four numbers in each set are the payoffs for the first row, and the last four provide the payoffs for the second row. For instance, \((a, b, c, d; e, f, g, h)\) corresponds to the payoff matrix

\[
\begin{pmatrix}
  a & b \\
  e & f \\
  c & d \\
  g & h
\end{pmatrix}
\]

\(^8\)For the analysis below we consider the average payoffs received across the 500 generations and 100 trials. We first perform an \(F\)-test for each game to test for equal variances, and then use a two-sample \(t\)-test at the 5\% level (two-tailed), assuming either equal or unequal variances, depending on the outcome of the \(F\)-test, to check for differences in these means.

\(^9\)The three games were (RG55) \((2, 4, 4, 3; 1, 1, 3, 2)\), (RG57) \((2, 3, 4, 2; 1, 1, 3, 4)\), and (RG65) \((2, 4, 3, 1; 1, 2, 4, 3)\), where RG\(x\) refers to game \(x\) in Rapoport and Guyer (1978), with the focal agents in the role of row players. The first two of these games have a single Nash equilibrium that is also a dominant strategy for the row player, but the row player gets a low payoff, and thus these were classified as single, unstable equilibria games by Rapoport and Guyer (1978). The third game has two Nash equilibria, that differentially favor the players, much like a Battle-of-the-Sexes game. In the penultimate section of the paper we address the dynamics of such games.

\(^{10}\)This game was (RG66) \((3, 3, 2, 4; 4, 2, 1, 1)\) commonly known as Chicken.
In the above analysis, we considered only the performance of the two types of agents against an uninformed opponent. Of obvious interest is whether one type of agent has an advantage when they face each other directly. To explore this question, we can consider the performance of the agents when they are embroiled in a symmetric game. Given the symmetry of the payoffs, such an environment provides an easy way to directly compare the relative effectiveness of informed versus uninformed strategies.

Of the 15 symmetric games, four\(^{11}\) of them showed a statistically significant difference between the payoffs of the informed versus uninformed agents. In all four of these cases, the uninformed agents did better than their informed opponents, though only in two of the games (Chicken and Battle of the Sexes) was this difference of practical importance (0.32 and 0.14, respectively).

The observation that informed agents have no advantage over uninformed agents in coevolving environments is not too surprising in light of the outcomes observed in Section 3.1. Even in the static environment studied there, we saw that during the initial course of play uninformed agents often were able to generate, and act upon, information that was akin to that given to the informed agents prior to the game.

Moreover, in a coevolving environment, opponents no longer need to sit passively by, but rather can adapt their strategies so as to confound the information being revealed to the informed players. Uninformed agents could exploit this ability in a variety of ways. One possibility is that they evolve essentially random strategies, which as we saw before, would make prior information of little use. Of course, such evolution would also, in most situations, lead to reduced payoffs for the uninformed agents, and we observe that the evolving opponents do better than random. Another possibility is that all of the opponents evolve strategies with identical levels of complexity, and thereby eliminate the potential value of prior information. This hypothesis also finds little support in our data, as we often observe heterogeneity across strategic complexity. A third way that evolving opponents could confound informed agents would be by evolving faster than the informed agents. While all of the agents must evolve their state actions and transitions, the informed agents must do this across two strategies (that may intermingle), as well as evolve the cutoff value that determines the starting state. It may be that this additional evolutionary burden might hamper the speed at which informed agents can react to the evolution of the uninformed agents. To test this possibility, populations of informed and uninformed agents were allowed to evolve against a fixed population of randomly created, uninformed strategies. We found that the number of generations it took for the payoffs to reach a steady state was slightly longer for the informed versus uninformed agents, but not significantly so.

Another explanation for the inability of informed agents to exploit their presumed advantage is that uninformed agents might adjust their strategic complexity to exploit the conditional nature of the strategies used by the informed agents. The strategic complexity of a given strategy can often be altered through the addition or subtraction of some strategic elements.\(^{12}\) To test this proposed explanation, we considered a system where we allowed the

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\(^{11}\)These games were (RG7) \((3,3,4;2,4,1,1)\), (RG9) \((3,3,4,1;1,4,2,2)\), (RG66/Chicken) \((3,3,2,4;4,2,1,1)\), and (Battle of the Sexes) \((4,2,1,1;1,1,2,4)\).

\(^{12}\)Recall that our measure of strategic complexity relies on the size of an isomorphic, minimized machine,
Figure 3: An example of a strategy with strategic complexity greater than 1 that plays defect (D) in most rounds. This strategy cooperates in the first round and then defects thereafter.

populations to have alternating periods of evolution. For example, in a Prisoner’s Dilemma game we created a population of uninformed agents half of which played Always Defect and half of which played Tit For Tat. We then allowed a population of randomly created informed agents to evolve against this population and found that they quickly set their cutoff values to one, above which they always cooperate and below which they always defect. We then fixed the informed population and allowed the uninformed population to evolve, and found that they evolved strategies with complexities greater than one that for all practical purposes always defected (see Figure 3 for an example), and thereby exploited the willingness of the informed agents to cooperate with agents that have lengths greater than one (such agents, at least in the initial epoch, always played Tit For Tat). This simple experiment shows that opponents do have the ability to adapt their “strategic structure,” as well as their strategies, so as to counter the prior advantage given to the informed agents.

Thus, we find that the presumed strategic advantage of the informed agents can be quickly neutralized by the reactions of the uninformed agents. Uninformed agents are able to change their strategic structure, and by so doing, manipulate the pre-play message that is sent to the informed agents. Indeed, at the extreme, in environments where the informed agents place too much “confidence” in these messages, uninformed agents can easily exploit the informed ones, making mimicry an effective strategy. In less extreme circumstances, we still find that the uninformed strategies are able to alter their strategic structure in ways that eliminate the information value of the pre-play messages.

Earlier, we showed how in static environments the advantage of pre-play strategic information may not be substantial, as substitutes for such information can often be generated during the initial course of play. In this section we have explored why, in coevolving systems, any potential advantage of the informed agents may be further eroded, as strategic

and thus the only way that this measure can be altered is via a real change in the strategy—that is, a necessary condition for changing the strategic complexity is that there is some set of possible circumstances (plays by the opponent) where the strategy takes a different action. Thus, if we have, say, an Always Cooperate strategy (minimized size of one), adding some additional accessible states that also cooperate will not change its complexity. However, one could manipulate this strategy’s minimized size in a way that, for all practical purposes, results in the same strategy, for example, suppose we create a strategy that always cooperates unless the opponent is observed to follow some arbitrary, predefined sequence of consecutive actions for ten plays (for example, CDCDCDCDCD or CCCCDDDDDCD), in which case the strategy will defect once and continue anew. Such a strategy needs at least eleven states to be implemented, even though, in actual play, its observed behavior is likely to be identical to Always Cooperate.
modifications by the uninformed opponents can reshape the information that is transmitted in ways that confound its value. There is one key result above that we have heretofore ignored, namely that there is a class of games where informed agents are at a disadvantage over uninformed agents. We felt that this observation was worthy of a separate discussion, to which we now turn.

3.3 The Disadvantages of Information

In the coevolutionary system discussed above we found that, in certain games, informed agents receive significantly lower average payoffs than uninformed ones. This is rather surprising, given that a basic tenet of having additional information is that, at the very least, it should do no harm as it can always be ignored.13

In fact, across the class of $2 \times 2$ games that we explored, we found that informed agents do worse than uninformed agents in coevolving systems in certain games. These games, Chicken and Battle of the Sexes, are coordination games that have two pure-strategy equilibria, where the players differ as to which equilibrium each prefers. To simplify the discussion, we will focus on the Battle of the Sexes, though parallel arguments can be made in the case of Chicken.

3.3.1 Coevolving Agents in the Battle of the Sexes

We consider evolving populations of informed versus uninformed agents playing a repeated Battle of the Sexes game. We use the payoffs $\{3, 1, 0, 0; 0, 0, 1, 3\}$, and for rhetorical convenience designate the actions so that if both players take action $U$ the outcome is the preferred equilibrium for the uninformed players in the one-shot game, $U^*$, and if they both play $I$ the preferred equilibrium of the informed agents, $I^*$, is attained.

The dynamics of this coevolving system are complex as illustrated in Figure 4. In general, the system tends to reach metastable states, where we define a metastable state as a sequence of generations in which a large fraction of the games produce stable action pairs. We find that the two metastable states observed in the system—which turn out to be most action pairs attaining either $I^*$ or $U^*$—are punctuated by rapid transitions between them. As can be seen in Figure 4, the agents coordinate on $U^*$ in the majority of rounds, but there are occasions where these epochs of $U^*$ are disrupted by rapid transitions to other states, most often $I^*$. Given this pattern, we would expect that the uninformed agents would tend to earn higher payoffs than the informed agents. Indeed, the average payoffs of the uninformed agents (1.86) are significantly higher than those of the informed agents (1.72). We found very similar results across other parameter values, such as reducing the maximum number of states allowed per machine, in the experiment.

13 In one of the latter experiments discussed in Section 3.2 we also found that informed agents did worse than uninformed ones, but for an obvious reason. There, the informed strategies were first evolved against fixed opponents where the information they received easily separated the two types of opponents they were facing, allowing them to take the appropriate actions with each. We then fixed the informed strategies—implying that the information could no longer be ignored—and allowed the previous opponents to evolve, which they readily did by mimicking the messages, but not the behavior, of the earlier strategies.
To understand the dynamics of this system, we initially consider the transition probabilities across the various metastable states. In particular, we examine the entry and exit probabilities from the two states. Define a $U^*$-metastable state as one in which action pairs give $U^*$ at least 90% of the time. Similarly, an $I^*$-metastable state occurs when 90% of the action pairs lead to $I^*$. To investigate the system, we looked at three trials of 10,000 generations each (the large number of generations in each trial minimizes the impact of end-of-trial truncation). We found that during any given trial, there were on average 30 $U^*$-metastable states and 15 $I^*$-metastable states, with the $U^*$-metastable states averaging 176 consecutive generations in length and the $I^*$-metastable states lasting only 99 generations. Thus, the $U^*$-metastable states were twice as likely to arise and, once they occurred, were harder to escape. Both of these effects would lead the system to favor $U^*$, benefitting the uninformed agents.

The analysis of the transition probabilities above suggest that there is something inherent in the dynamics that promotes, and maintains, the $U^*$ over the $I^*$ outcome. To gain further insight into this phenomenon we consider a simplified variant of our system, where we limit our agents to extremely simple machines and have them play a one-shot game. We find similar results in our experimental system with $n_S = 2$ and $n_R = 1$.

The strategy space in this simplified system is greatly reduced, with each population having four possible strategies.\footnote{Here we ignore the fact that identical strategies can be represented in different ways within, say, two-state machines. This distinction is similar to the one biologists make between the same phenotype being represented by multiple genotypes. While this could influence the dynamics, we avoid such details here to simplify the discussion. That being said, the more general observation that identically behaving strategies may be represented in different ways, is important to our argument.} Uninformed agents can play action $U$ and have either a low strategic complexity (we will designate such strategies using a lower case $u$) or a large one (designated by $U$) or they can play action $I$ and have low ($i$) or high complexity ($I$). Informed agents can play $U$ regardless of their opponent’s strategic complexity ($uU$, where we adopt the convention of using lower-case (upper-case) letters to indicate the action when confronting a low-complexity (high-complexity) opponent), play $U$ when their opponent has low complexity and $I$ otherwise ($uI$), play $I$ when the opponent has low complexity and $U$ otherwise ($iU$), or play $I$ regardless of their opponent’s complexity ($iI$). Thus, uninformed
agents can choose one of two actions linked with one of two possible signals, while informed players can condition their single action to the two possible signals of either low or high complexity. The payoff space for all of these possible strategies is given in Figure 5.

Consider the implications of Figure 5.\textsuperscript{15} For the moment, assume that we have only one agent on each side, and that these agents seek to best respond to the current action of their opponent. First, note that if the two players find themselves not coordinating (leading to one of the eight possible (0,0) payoffs), the selective pressures will quickly place them in one of the coordinated equilibria (either $U^*$ or $I^*$) with equal probability.\textsuperscript{16} Second, if we consider the best responses for the eight pure-strategy equilibria in Figure 5, we find an intriguing pattern. For the four equilibria that favor the uninformed players (that is, $U^*$), the best-response transitions are

$$(u, uI) \leftrightarrow (u, uU) \leftrightarrow (U, uU) \leftrightarrow (U, iU),$$

where the directions of the arrows indicate a possible best-response transition between the various strategies. Note that all of these transitions keep the system in the equilibrium that favors the uninformed players. Thus, once the system finds itself at $U^*$, we would expect it to continue in this equilibrium, with perhaps some drift among the underlying strategies.

The situation is quite different for the four equilibria that favor the informed players, $I^*$. In this case, the transitions are:

$$(u, uI) \leftrightarrow (I, uI) \leftrightarrow (I, iI) \leftrightarrow (i, iI) \leftrightarrow (i, iU) \rightarrow (U, iU),$$

where the four strategies in the interior keep the system in the equilibrium favoring the informed players, $I^*$, while the two profiles on either end result in the equilibrium favoring

\textsuperscript{15}The argument presented here is easily formalized, and is similar to the results obtained by Kandori et al. (1993) and Kandori and Rob (1995) on the stochastic stability of population states under the perturbed best-response dynamic. While this dynamic gives qualitatively the same results as our mutation and selection process, it appears difficult to obtain analytic results for the perturbed best-response dynamic in the current setting.

\textsuperscript{16}Depending on which of the uncoordinated outcomes they start out in, there could be a bias. For example, if they are playing $(i, uI)$ three of the best responses put them in $I^*$ and the other one leads to $U^*$. Four of the eight uncoordinated outcomes have such asymmetries, with two of them favoring $I^*$ and two favoring $U^*$. 

\begin{table}
\centering
\begin{tabular}{c|cccc}
 & $uU$ & $uI$ & $iU$ & $iI$ \\
\hline
$u$ & 3,1 & 3,1 & 0,0 & 0,0 \\
$U$ & 3,1 & 0,0 & 3,1 & 0,0 \\
$i$ & 0,0 & 0,0 & 1,3 & 1,3 \\
$I$ & 0,0 & 1,3 & 0,0 & 1,3 \\
\end{tabular}
\caption{The payoff matrix for the stage game associated with the Battle of the Sexes game with the simplified strategies discussed in the text, where lower-case (upper-case) symbols are tied to low (high) complexity.}
\end{table}
the uninformed players, \( U^* \). Thus, if the players find themselves at \( I^* \), the system can easily drift away from this equilibrium favoring to the one favoring the uninformed. This is quite different from the dynamics controlling \( U^* \), which keep the system in that state.

Given the above, we can start to intuit the dynamics. If the system is at an uncoordinated outcome, then the best response forces will push it to one of the coordinated outcomes. Once the system is in one of the coordinated equilibria, the dynamic will allow it to drift across the various strategies. In the case of the coordinated equilibrium favoring the informed players, this drift will eventually result in the system moving to the equilibrium favoring the uninformed players. Once the system enters the equilibrium favoring the uninformed players, the dynamics will keep the system there. This intuition is consistent with what we observed in the full model, and provides an explanation for why the uninformed agents will do better than the informed agents in such a game.\(^{17}\)

As seen above, it is the asymmetric flow between the two coordinated equilibria that results in the uninformed players doing better than the informed ones. How does this asymmetry arise? As previously discussed, there are two unidirectional flows out of \( I^* \) and into \( U^* \). Suppose the two players are using strategies \((I, iI)\). In this case, the uninformed player has high complexity and takes action \( I \) and the informed player responds with action \( I \), leading to the outcome that favors the informed player. Now, suppose that the informed player’s strategy drifts to \( uI \), that is, the informed player decides to play action \( U \) if it meets a low-complexity agent. This change in strategy has no immediate impact on the game, as the uninformed player is of high complexity, so there is no change in either player’s action or the resulting payoffs. Also note that, because the informed player only sees high-complexity opponents, there is no chance to test the low-complexity part of its strategy. Thus, this change in the informed player’s strategy sets the stage for a major transformation, namely, if the uninformed player switches its strategy to \( u \), that is, lowers its strategic complexity and changes its action, it can invoke the previously unused part of the informed player’s strategy, and cause the outcome to switch to \( U^* \), resulting in the equilibrium of the one-shot game that favors the uninformed player. Hence, the ability of an informed agent to play an effective strategy conditional on its opponent’s strategy hampers it in developing an effective unconditional strategy. By contrast, since an uninformed agent cannot “specialize,” its strategy will remain effective. A similar argument can be made for the other asymmetric transitions (in this case, once the system enters \((i, iU)\), the uninformed player can alter its strategy to high complexity and play action \( U \), forcing the system into its favored equilibrium).\(^{18}\)

\(^{17}\)Obviously, the story is a bit more complicated than this when populations of agents with different strategies interact with one another. In this more complicated world, it is possible for mixes of strategies and stochastic events to lead to a variety of outcomes, for example, knocking the system out of one of the equilibria and throwing it into a condition with general uncoordination. However, even in this more complicated system, where populations employ all possible strategies, it can be shown that a similar bias exists. For example, in such systems there is a positive probability of acquiring strategies that involve the informed agents playing action \( U \) in some circumstances, even if most pairs of agents play \( I \) initially.

\(^{18}\)In the above discussion, we ignore the fact that the informed agents’ cutoff parameters also evolve. This only adds to the mechanism described above. For example, suppose we start with strategy profile \((I, iI)\) and the informed agent’s strategy drifts to \( uI \), as described above. If the informed agent raises its cutoff sufficiently, it will see its opponent as having a low complexity and will therefore play action \( U \), which
Notwithstanding our initial intuitions that informed agents could never be worse off than uninformed agents—since, in the worst case scenario, they could always ignore the information and act as uninformed agents—we find that the need to react to information is detrimental to these agents. While the informed agents do attempt to “ignore” the information and seek to push the system to, and maintain it in, their favored equilibrium, the structure of the informed strategies is such that there is a critical vulnerability. When the uninformed agents all begin to send the same signal, the behavior of the informed agents when they experience the unused signal is no longer subject to any selective pressure, and can consequently drift. Then, by switching to sending the previously unused signal, the uninformed agents can break out of the unfavorable equilibrium and force the system into the favorable one.

Moreover, once this drift is exploited by the uninformed agents, it is in the individual interest of each informed player to acquiesce. Even though acquiescence benefits the individual player, it is detrimental to the informed population as a whole, as the short-term gain is offset by the long-term loss when the informed population is knocked out of its preferred equilibrium.

The mechanisms above result from both the adaptive dynamics that allow the agents to “learn” their strategies and the underlying structures that represent these strategies. This mechanism is quite different than, say, one that relies on the uninformed agents pre-committing to action \( U \) and having that information conveyed to, and acquiesced by, the informed agents. Instead, our focus is on the interaction of two interesting phenomena. First, we consider issues induced by learning, in this case, the difficulty of adapting to events that don’t occur. Second, we focus on how the actual structural implementation of a strategy can matter—a topic that has not been widely investigated (see, however, papers by Abreu and Rubinstein (1988) and Binmore and Samuelson (1992)).

4 Discussion

Throughout our investigations we not only find that having pre-play information about your opponent’s strategy is of little value in adaptive environments, but that it can actually be harmful. These results are rather counter intuitive, as it would seem \textit{a priori} that such information should be of value, and at the very least, it should do no harm.

To be clear, we did find a narrow set of circumstances under which pre-play information about an opponent could be of limited value. In these worlds, agents faced opponents that did not evolve, and the information about strategic complexity revealed the type of strategy and allowed the informed agent to implement an appropriate response prior to play. However, if uninformed agents were allowed to adapt their own strategies in such an environment, any potential advantage of the informed agents was rapidly dissipated, as the uninformed agents developed strategies that could quickly acquire the needed information during the initial course of play, and thereafter respond appropriately to the opponent.

induces its opponent to choose \( U \). This provides another mechanism by which the system might drift to the equilibrium preferred by the uninformed agents.
While using early moves to probe your opponent’s strategy could potentially be costly, for example, compromising some early payoff opportunities to gather information or causing your opponent to do something drastic (like setting off a grim trigger), we find that in general uninformed strategies may not be at a dramatic disadvantage vis-à-vis informed strategies in such worlds.

We also found that when agents are allowed to adapt to one another, the uninformed opponents can alter their strategies in environments where explicit information is transmitted so as to confound the value of the information that is received prior to play. In our model, the agents were able to alter the structure of their strategies so that the information that was revealed to their informed opponents held little value. The fact that similar strategic ends can often be embodied in very different structures allows such mimicry to happen. In the system we studied, this typically required the opponents to add structural elements that increased the complexity of their strategies. These transformations, while perhaps requiring some slight compromises in terms of the implemented strategy, were sufficient to overcome any advantages the pre-play signal might have conveyed to the informed agents.

Finally, we observed that there is a class of games in which the informed agents are actually at a disadvantage to the uninformed agents. This is a surprising outcome, as one might think that informed agents can never do any worse than uninformed agents, since they could always ignore the information they were given. The class of games in which informed agents are at a disadvantage includes those games with multiple equilibria that differentially favor the players, such as Battle of the Sexes and Chicken. We find that uninformed agents can take advantage of the informed player’s need to process the pre-play information. This aspect of the informed agent’s strategic structure, interacting with the overall adaptive system, provides a convenient purchase from which the uninformed agents can gain an advantage. The need to respond to pre-play information allows parts of the structure of the informed agent’s strategy to drift randomly when left unused. While such drift is inconsequential in the short-term, it sets the stage for more radical implications in the long-term when the previously unprobed, and currently maladaptive, piece of strategic structure is invoked in response to a new signal sent by the uninformed agents.\textsuperscript{19}

Thus, notwithstanding strong intuitions that pre-play information about your opponent’s strategy would be quite useful, we find that in general such information is of little benefit. Even in static situations where such information should be of the most value, uninformed agents are able to generate similar information during the initial play of the game at low cost. Moreover, in less static situations, the information that gets transmitted becomes an endogenous part of the strategy, and thereby its potential value is compromised. Perhaps most surprisingly, we find that even the capacity to respond to information becomes problematic, and may even be disadvantageous.

\textsuperscript{19}This is related to the notion of neutral mutations in biology, where changes to the underlying form of an organism (that is, the genotype), do not alter the physical form (the phenotype) of the organism. Since such changes do not alter the physical form of the organism, the fitness—and consequently the selective forces impinging on the organism—remains unchanged, and it is in this sense that such mutations are considered neutral. That being said, neutral mutations are not always innocuous, as they can lead the organism into a state where subsequent small changes, that normally would have no impact, become consequential.
One obvious question is whether the results we observe above are an artifact tied to the choice of information—here, the strategic complexity—that is being transmitted. At many levels, strategic complexity is a fundamental concept, and one that has substantially face validity in terms of what could be realistically transmitted in such games. Thus, the fact that it can be exploited is instructive, regardless. Nonetheless, we did consider some alternatives, for example, the degree of reciprocity, and found similar results. We do view as fair game the quest for finding other types of information about the strategy of one’s opponent that could be realistically divined prior to play and not succumb to the mechanisms we find above.

While the investigations above were motivated by an exploration of the importance of pre-play strategic information, they also illuminate a much deeper issue, namely the importance of strategic structure and adaptation on the dynamic behavior of a system. Our model above makes an explicit attempt to define strategic structure and recognize how such a structure can be adapted over time. With such notions, we can begin to explore a new realm of social system behavior. We saw above how a strategic structure that can accommodate pre-play strategic information can unintentionally interact with the adaptive system in a very nonlinear way, leading to rather counterintuitive results. Thus, the way in which a strategy is “deployed” in the world, may make a difference. In our model, parts of the strategic structure that were designed to react to the information about one’s opponent, can drift when they go unprobed by the actions of those opponents. While these drifts convey no short-term disadvantage to the informed agent—that is, they are neutral—in the longer term they set the stage for rapid transformations of the system into disadvantageous equilibria, caused by the invocation of the previously untested structure.

In Greek mythology, Thetis held her son Achilles by his heel as she dipped him into the river Styx to make him immortal. Alas, by holding him so, she inadvertently prevented the waters from touching the heel beneath her hand. Similarly, in the above adaptive system, it is the desire of an informed agent to be able to respond to information about his opponent that inadvertently creates the vulnerable heel of this strategic Achilles.

References


