An Evolutionary Approach to Institutional Persistence and Change

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First Draft: November 2005
This Draft: July 31, 2011

Abstract

Some institutional transitions are implemented as the deliberate outcome of bargaining among a small number of groups, but many are more decentralized, with a large number of private actors informally adopting new practices that are later confirmed by changes in formal governance structures. For example, land tenure norms, changes in conventional crop shares, shifts in inheritance practices, and traditional property rights all are informal institutions, or conventions, that persist for long periods of time and sometimes experience rapid changes in the absence of government policies. To capture these informal and decentralized aspects of institutional persistence and change, we study transitions between conventional contracts among members of two classes. The driving mechanism in our model comes from intentional deviance from conventions by individuals, leading to some contracts being selected over others in the long-run. Transitions between contractual conventions occur when sufficiently many individuals deviate from (rather than conform to) the status quo convention. We identify conditions under which efficient and/or egalitarian contractual conventions are likely to be long-run stable equilibria under a stochastic evolutionary dynamic. We endogenize the population sizes of the two classes and obtain conditions under which barriers to intergenerational mobility increase the probability of unequal institutions. We also let the rate of deviation from the status quo convention vary with the degree of inequality and group network structure. Finally, we introduce a government motivated to support the long-term interest of one of the groups, and identify the conditions under which it will adopt redistributive strategies.

JEL codes: D02 (Institutions), D3 (Distribution), C73 (Stochastic and Dynamic Games; Evolutionary Games)
Keywords: institutional persistence, evolution, stochastic stability, collective action, class, income distribution

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1 Introduction

Economic institutions such as labor relations and land tenure often persist over centuries, while transitions among these institutions are sometimes abrupt. A large recent literature has shown that initial institutional differences persist, causing long term economic and social effects (for surveys see Nunn (2009) and Acemoglu, Johnson and Robinson (2005)). Banerjee and Iyer (2004), for example, show that the informal institutions surrounding the Zamindari land tenure system in India persisted long after the formal institution was abolished. Other research has found that informal institutions such as crop share or female labor supply norms exhibit substantial long-run hysteresis, even in the face of large changes in technology and agricultural fundamentals (Bardhan 1984, Young and Burke 2001, Alesina, Giuliano, and Nunn 2011).

While many institutional transitions are implemented as the deliberate outcome of bargaining among a small number of groups, some are more decentralized, with a large number of private actors informally adapting new practices that are later confirmed by changes in formal governance structures. In many countries and periods this decentralized and informal aspect of institutional transition is evident in, for example, labor contracts, changes in conventional crop shares, shifts in inheritance practices, and changes in economic relationships between men and women.

Recent contributions to the political economy and institutions literature have modeled institutional persistence and change as the outcome of bargaining between representative agents of a small number of economic groups. Acemoglu and Robinson (2008), for example, consider a model in which formal political institutions change while the informal economic institutions persist. In contrast to this and other political economy models, in our approach institutions are not directly chosen, but rather emerge as the largely unintended consequence of individual actions of large numbers of agents, none of whom is powerful enough to choose an institution for the entire society. The distinction between the two approaches is evident in two very different cases of the demise of European feudalism. Consistent with the political economy approach, the emancipation of Russia’s serfs by Tsar Alexander II in 1863 was a deliberate choice to implement a new set of institutions resulting from bargaining within Russia’s elite (Blum 1971). In contrast, the demise of English serfdom was not the result of explicit bargaining among social groups. The historian E.B. Fryde (Fryde 1996 pg 6) writes:
throughout the 1380s and long beyond them...the servile velleins refused with ever increasing persistence to accept the implications of serfdom, ... In this atmosphere of frequent local disorder and of continuous tension between lords and tenants, the direct exploitation of domanial estates would largely disappear from England in the fifty years after the [1381] Great Revolt.

Similarly, in France protracted agrarian conflict culminating in the 1789 peasant rebellions, forced local lords to abandon many of their feudal privileges. The abolition of seigniorial dues by the Estates General in 1789 confirmed the new order, it did not introduce it (Markoff 1996).

South Africa’s transition to democracy, which we will take up in some detail in the next section, provides a direct contrast between the evolutionary and political economy approaches. Acemoglu and Robinson (2006) write that “the basic structure of apartheid was unaltered” until “De Klerk concluded that the best hope for his people was to negotiate a settlement from a position of strength”(p13). For Acemoglu and Robinson, South Africa’s new institutions were introduced as the result of the formal constitutional negotiations beginning in 1990. Consistent with their view that economic institutions will change only after the political institutions change owing to commitment failures, they conclude that the change in economic institutions resulted from the introduction of a new political system. However, our reading of the historical evidence is that fundamental changes in economic practices and hence de facto economic institutions predate De Klerk’s rise to prominence in the National Party, and are more plausibly seen as the cause of the subsequent political transition, rather than its consequence.

Which of these approaches captures the essential dynamics of institutional changes will, as these cases suggest, vary from case to case. We share with Acemoglu and Robinson a perspective that emphasizes the intentional pursuit of group objectives and social conflict as key ingredients in a theory of institutional persistence and change. This differs from the evolutionary game theory approach to institutional innovation and change in which the observed institutions are the outcome of a process of random experimentation and adaptation.

Like both of these approaches, our model identifies conditions under which inefficient economic institutions persist in the long run. But in contrast to the models developed by Acemoglu and
Robinson (2006, 2008), commitment problems play no role in explaining inefficient institutions in our approach. Rather, an inefficient institution may persist due to persistent coordination failures, as in Axtell, Epstein and Young (2003), or Ellison (1993). But in contrast to Young (1998), in our model an inefficient institution may endure even when it implements highly unequal outcomes as long as intergenerational mobility into the upper class is sufficiently restricted.

By synthesizing the intentional behavior among conflicting groups stressed by political economy and the decentralized and stochastic process aspects emphasized in evolutionary approaches we hope to match several stylized facts about institutional transitions. These include the fact that economic institutions sometimes change before political institutions (the demise of feudalism in early modern Western Europe, e.g. Brenner 1976), the long term persistence of institutions that are both inefficient and unequal (Sokoloff and Engerman 2000), punctuated institutional equilibria (the end of Communist Party rule, e.g. Lohmann 1994), and the fragility of highly unequal economic institutions in modern industrial economies compared to their robust persistence in pre-modern times (Trigger 2003, Hobsbawm 1964).

For concreteness, we study the emergence and persistence of contracts that govern the size of the joint surplus and its distribution between two classes, and we identify conditions under which efficient and/or egalitarian contracts are likely to emerge and to persist. We represent these institutions as conventions between such discrete classes of economic actors as employers and workers or landlords and sharecroppers.

Are there common structural properties that account for the emergence and persistence of evolutionarily successful contracts? To answer this question we study transitions between contracts that result when a sufficiently large number of individuals play idiosyncratically rather than adopting a best response (Young 1993a, Kandori, Mailath, and Rob 1993). However, in contrast to these models, and as in Naidu, Bowles, and Hwang (2010) and Bowles (2004), we represent idiosyncratic play as intentional challenges to the status quo convention rather than random behavioral experimentation or errors.

Transitions occur when the number of individuals in one class who reject the terms given by the status quo contractual arrangement is sufficient to induce best-responding individuals in the other class to deviate from the status quo contract as well. We will show that the dynamic resulting from
intentional (rather than random) deviations from the status quo convention is more plausible than the existing evolutionary models: in our dynamic institutional transitions are induced only by the idiosyncratic play of those who will benefit if a transition should occur, and the distributional interests of a group is favored if it is smaller and if its rate of deviance is greater. The opposite is the case when idiosyncratic play is random. By specifying a historically plausible dynamic we can explore the effects on institutional persistence of such characteristics as the size and distribution of the joint surplus associated with each set of contracts, impediments to mobility between classes, and the information available to members of each class.

In the next section we examine a historical case that illustrates the main aspects of the institutional dynamic we wish to model. Then we introduce a contract game and study the contractual equilibrium selection process when idiosyncratic play is intentional in the sense that deviations from best responses are limited to those which would benefit the deviant individual, were sufficiently many others to do the same. We show that if class sizes and rates of idiosyncratic play are equal, this dynamic reproduces a result analogous to Young’s contract theorem (Young 1998), namely that the conventions selected by this dynamic are both efficient and egalitarian. We then let the sizes of the two classes differ. In contrast to existing unintentional idiosyncratic play models, our dynamic selects contracts that favor the less numerous class. If the poorer class is the more numerous (as is typically the case) the contractual equilibria selected need not risk dominant, and may be both unequal and inefficient.

We then study the evolution of class sizes resulting from inter-generational mobility across class boundaries. We model barriers to upward mobility(e.g. credit constraints) of the type studied by Galor and Zeira (1993), Banerjee and Newman (1993), Mookerjee and Ray (2006), Bowles and Gintis (2002), and Benabou (1998), among others, and show that for a given barrier to mobility there exists a unique equilibrium distribution of class membership and distribution of the joint surplus between the two classes. By limiting the size of the well off class, barriers to upward mobility support higher levels of inequality in equilibrium. This is true for two reasons: the selected contract is more unequal, and the endogenously determined class sizes allow the richer class to engage in contracts with a larger number of the poor.

We then explicitly model idiosyncratic play by taking account of the amount of information available to members of each class, distinguishing in this way between modern and pre-modern
class structures. We suggest that the less segmented intra-class information structure and height-
ened polarization of incomes in early capitalism may have provided conditions favorable to working
class challenges to the status quo, and partly as a result, to the emergence of a redistributive state.
We then introduce governmental policies of redistribution, and also let the rate of idiosyncratic
play vary with the degree of economic polarization at each state. The penultimate section suggests
extensions to address the influence of technical change, variations in inheritance systems, endoge-
nous barriers to upward mobility, collective action, demographic structure, governmental capacities
and the tension between bargaining power and political power.

2 Decentralized Transitions: South Africa

We combine the decentralized individual-based dynamic of evolutionary game theory with the
group distributional conflict approach common to political economy because we think that in many
historically important cases both aspects were important. Among these cases is the transitions to
democratic rule in South Africa.

The labor market aspects of South African apartheid were a convention regulating the patterns
of racial inequality that had existed throughout most of South Africa’s recorded history and had
been formalized in the early 20th century and strengthened in the aftermath of World War II. For
white business owners, the convention might be expressed: Offer only low wages for menial work to
blacks. For black workers the convention was: Offer one’s labor at low wages, do not demand access
to skilled employment. These actions represented mutual best responses: As long as (almost) all
white employers adhered to their side of the convention, the black workers’ best response was to
adhere to their aspect of the convention, and conversely.

The power of apartheid labor market conventions is suggested by the fact that real wages of
black gold miners did not rise between 1910 and 1970, despite periodic labor shortages on the
mines and a many-fold increase in productivity (Wilson 1972). But a series of strikes beginning
in the early 1970s and burgeoning after the mid-1980s with the organization of the Congress of
South African Trade Unions (COSATU) signaled a rejection of apartheid by increasing numbers
of black workers. The refusal of Soweto students to attend classes taught in Afrikaans and the
ensuing 1976 uprising returned civil disobedience to levels not experienced since the anti-pass law
demonstrations a decade and a half earlier, including the one at Sharpeville at which 69 protesters had been killed by police. The acceleration of urban protests loosely coordinated by the United Democratic Front (UDF), contributed to what whites came to call the “ungovernability” of the country and its businesses. Figure 1 depicts these trends.

Many business leaders concluded that adherence to the apartheid convention was no longer a best response, leading them independently to alter their labor relations, raising real wages and promoting black workers. An executive of the Anglo American Corporation, South Africa’s largest, commented: “...in the business community we were extremely concerned about the long-run ability to do business...” (Wood 2000:171) Starting in the mid 1980s, the Corporation developed new policies for ‘managing political uncertainty’ and to address worker grievances, even granting workers a half day off to celebrate the Soweto uprising. In September 1985, Anglo American’s Gavin Relly led several business leaders on a clandestine “trek” to Lusaka to seek common ground with African National Congress leaders in exile. In 1986 the Federated Chamber of Industries issued a business charter with this explanation: “the business community has accepted that far reaching political reforms have to [be] introduced to normalize the environment in which they do business.” FCI (1990). An official of the Chamber of Mines described the situation in 1987

The political situation in the country was really dismal and we knew that we were going to have one mother of a wage negotiation. And that the issue wasn’t what level of increases we negotiated; the issue was do we survive or not? Will there, after this negotiation, still be such a thing as managerial prerogative. Who controls the mines, really? That was what it would boil down to. (Wood 2000: 169)

In addition to conceding many of their black employees’ workplace demands, business-led pressure for political reforms mounted, joined by reform advocates from the government’s intelligence services, churches and others. Late in 1989, four years after the state of emergency had been declared in response to the strike wave and urban unrest, F. W. de Klerk replaced the intransigent P. W. Botha as State President. In 1990 he lifted the ban on the African National Congress, the South African Communist Party and other anti-apartheid organizations, and released Nelson Mandela from prison. Mandela was elected president in South Africa’s first democratic election in 1994.
Figure 1: Political and economic disturbances in South Africa, 1960-1994 (Sources: Strikers: Statistics South Africa; Detentions: Institute of Race Relations, Yearbooks; Political Instability: Federke, De Kadt, and Luiz 2001)

Note the following about this process. First, the concession of best-responding businesses to the idiosyncratically-playing black workers occurred well before and constituted one of the causes of the political transition. The redistribution of economic resources thus predated and contributed to the redistribution of political resources. Second, the process of transition was extremely abrupt, bringing to an end in less than a decade de facto class and race relations that had endured for a century. Third, while trade unions, ‘civics’ (community organizations), and other groups were involved in the rent strikes, student stay aways, and strikes against employers, the rejection of apartheid was highly decentralized and only loosely coordinated prior to the unbanning of the ANC in 1990. We now model an abstract transition process with these general features.
3 Institutional Equilibrium Selection

3.1 Contracts

Contracts differ both in the kinds of incentives that they provide and the distribution of the joint surplus that they implement. To illustrate the kinds of contracts among which decentralized selection may take place, suppose the B’s are land owners while A’s are those who farm the land. As we will see, contract $E$ is an equal but inefficient sharecropping contract and contract $U$ is an unequal but efficient fixed rental contract. The share is that which maximizes the landowner’s profits (subject to the farmers incentive compatibility constraint for the supply of labor), while the rent is determined by the exogenously given bargaining power of the two parties. Under both contracts, hours of labor, $L$, produce output, $q$, according to $q = f(L)$, where $f$ is a concave, increasing production function satisfying the Inada conditions. The farmer’s (A’s) utility varies with income $y$ and hours worked: $V(y, L) = y - h(L)$. The landowner’s (B’s) opportunity cost of holding the land is $k_c$. The farmer’s utility-maximizing labor supply under either contract is $L(s), L' > 0$ where $s$ is the share of the residual output retained by the farmer and is equal to 1 in the fixed rental contract and $s \in (0, 1)$ in the share contract. Under the rental contract the farmer (as residual claimant) works $L(1)$ hours, so total output is $f(L(1))$. Subtracting from this the disutility of the farmer’s labor $h(L(1))$ and the opportunity cost of the land, the joint surplus is $f(L(1)) - h(L(1)) - k_c$

Under the share contract the owner’s profits of $(1 - s)f(L(s))$ are maximized at a share $s^* < 1$, under which terms the farmer works $L(s^*)$ hours, yielding a total output of $f(L(s^*))$ and a joint surplus of $f(L(s^*)) - h(L(s^*)) - k_c$.

We consider a large population of agents of size $N = N_1 + N_2$, with $\gamma \equiv \frac{N_1}{N}$ the fraction of population that are of class A (we will rule out integer problems below). Each period, agents from class A are randomly matched with agents from class B and play the following contract game, with A as the row player illustrated in table 1. We term A the non-elite agents, or poor, and B the elite, or wealthy agents. Each agent in the pair proposes one of two contracts (termed $U$ or $E$) governing the distribution of the surplus (e.g. union recognition, crop-shares, or land tenure norms). If they fail to coordinate on a contract, both get 0, reflecting the fact that agents are bargaining over a discrete institution, agreement on which is necessary for the production of a surplus, rather than simply over a divisible surplus.
Returning to our illustrative contract above, define $k_c^*$ such that $(1 - s^*) f(L(s^*)) - k_c^* = s^* f(L(s^*)) - h(L(s^*))$, so that the share contract equally divides the surplus. As expected, the joint surplus under the fixed rental contract is larger, reflecting its superior incentives. To ensure that both contracts are Pareto optima, so that the interests of the classes are opposed, we assume the bargaining power of the landlords in the $U$ contract to be such that the rent, $R^*$ is fixed at $R^* > f(L(1)) - s^* f(L(s^*)) - h(L(1)) + h(L(s^*))$, so that farmers are worse off in the fixed rent contract.

We can also define $D = (1 - s^*) f(L(s^*)) - k_c^*$, and divide all the payoffs by $D$. Now by definition of $k_c^*$, the normalized joint surplus produced under sharecropping is $2$, and $\frac{1}{2}$ is the share that the tenant receives. Also define $\rho = \frac{f(L(1)) - h(L(1)) - k_c^*}{D} > 2$ as the joint surplus produced under the rental contract with $\sigma = \frac{f(L(1)) - h(L(1)) - R^*}{\rho} < .5$ being the share received by the tenant. This gives the normalized payoffs in the contract game below.

<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>E</th>
</tr>
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<tbody>
<tr>
<td>U</td>
<td>$\sigma \rho, (1 - \sigma) \rho$</td>
<td>0,0</td>
</tr>
<tr>
<td>E</td>
<td>0,0</td>
<td>1,1</td>
</tr>
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Table 1: Payoffs in the Contract Game

### 3.2 Dynamics

The dynamic governing contractual offers is a familiar myopic best-response dynamic with inertia. Each period, all players are matched with a member of the other class to play the contract game. Each time they are matched, agents play the strategy, $U$ or $E$, that they played last with probability $1 - \nu$ or revise their strategy with probability $\nu$. If they revise their strategy and do not play idiosyncratically, they play the best-response to last-period’s distribution of strategies.

We can represent this dynamic by a stochastic dynamical system, where the states represent the fraction of each population playing $U$, the unequal strategy. The state space is given by $X = \Delta_R \times \Delta_C$, where $\Delta_R = \{ \frac{i}{N_1} | i \in \{1, .., N_1\} \}$ and $\Delta_C = \{ \frac{j}{N_2} | j \in \{1, .., N_2\} \}$ where $N_1$ is the size of the row population and $N_2$ is the size of the column population, and each $i$ and $j$ is the
number of the row and column population, respectively, that is playing strategy $E$. Let $p \in \Delta_R$ and $q \in \Delta_C$ be vectors denoting the number of agents playing each strategy in the row and column population, respectively. We will denote a state as $\theta = (p, q) \in X$ and denote the best-response of a row (respectively column) agent as $BR_R(q)$ (respectively $BR_C(p)$), since each side chooses a strategy in response to the distribution of play in the opposing group.

The unique mixed strategy Nash equilibrium of the 2-strategy game defined above is defined as $(p^*, q^*)$ with $p^* \equiv \frac{1}{1+\sigma}$ and $q^* \equiv \frac{1}{1+(1-\sigma)p}$. For convenience, we suppose parameter values are such that $p^* N_1 \notin \Delta_R$ and $q^* N_2 \notin \Delta_C$, allowing us to rule out mixed strategy Nash equilibrium from the set of states.

This defines a random dynamical system as follows:

$$p_{t+1} = \alpha_R(t) BR_R(q_t) + (1 - \alpha_R(t)) p_t$$  \hspace{1cm} (1)

$$q_{t+1} = \alpha_C(t) BR_C(p_t) + (1 - \alpha_C(t)) q_t$$  \hspace{1cm} (2)

where $\alpha_R(t), \alpha_C(t) \sim Bernoulli(\nu)$ are i.i.d.

The dynamic can be represented as a Markov process: $P^\nu: X \to X$, defined by $P^\nu(\theta'|\theta) = \text{Prob}(\theta - (\frac{x_1}{N_1} p, \frac{x_2}{N_2} q) + (\frac{x_1}{N_1} BR_R(q), \frac{x_2}{N_2} BR_C(p)) = \theta')$ where $x_1 \sim Bin(N_1, \nu), x_2 \sim Bin(N_2, \nu)$ where $Bin(N_i, \nu)$ is a binomial distribution with $N_i$ draws with probability of success given by $\nu$.

**Proposition 3.1.** For generic contracting games and sufficiently large population sizes, the only recurrent classes of this Markov process are the strict pure Nash equilibria, where both players co-ordinate on the same contract.

**Proof.** In Appendix. \hfill $\square$

We now add a perturbation to this dynamic. Suppose that when agents revise their strategies, they play a non-best response with probability $\varepsilon$ if the status-quo contract is not their preferred contract, and 0 otherwise. When row players deviate they play contract $E$. When column players deviate, they choose contract $U$. This formulation of the perturbations is the key difference
between our model and the standard stochastic evolutionary game theory models that have \( \epsilon \) being the probability of playing a randomly chosen strategy. By contrast, our model has \( \epsilon \) as the probability of idiosyncratically playing the strategy that would be best for that sub-population were it to be played by both sub-populations in equilibrium. We describe the stochastic process more fully and apply it to a more general class of bargaining games in Naidu, Bowles, and Hwang (2010). The perturbations correspond to non-best-response behavior, which we term “deviant” or “idiosyncratic”. We have in mind rejections of the terms of the status quo contract by either side, such as lockouts, union decertification campaigns, private enclosures of common lands, strikes, slave revolts, and urban food riots.

This perturbed Markovian dynamic can be represented by another random dynamical system:

\[
\begin{align*}
    p_{t+1} &= \begin{cases} 
    \alpha_R(t)(1 - \beta_R(t))BR_R(q_t) + 1 - \alpha_R(t)(1 - \beta_R(t))p_t + \alpha_R(t)\beta_R(t) & \text{if } q_t < q^* \\
    \alpha_R(t)(1 - \beta_R(t))BR_R(q_t) + (1 - \alpha_R(t))(1 - \beta_R(t))p_t - \alpha_R(t)\beta_R(t) & \text{if } q_t > q^*
    \end{cases} \\
    q_{t+1} &= \begin{cases} 
    \alpha_C(t)(1 - \beta_C(t))BR_C(p_t) + (1 - \alpha_C(t))(1 - \beta_C(t))q_t - \alpha_C(t)\beta_C(t) & \text{if } p_t > p^* \\
    \alpha_C(t)(1 - \beta_C(t))BR_C(p_t) + 1 - \alpha_C(t)(1 - \beta_C(t))q_t - \alpha_C(t)\beta_C(t) & \text{if } p_t < p^*
    \end{cases}
\end{align*}
\]

where \( \alpha_R(t), \alpha_C(t) \sim Bernoulli(\nu), N_1\beta_R(t) \sim Bin\{N_1, \epsilon\}, \text{ and } N_2\beta_C(t) \sim Bin\{N_2, \epsilon\} \) are i.i.d.

This can be represented by a transition matrix given by \( P^{\nu,\epsilon}(\theta' | \theta) \). The long-run steady state of the dynamic is then given by the unique vector \( \mu(\nu, \epsilon) \in \mathbb{R}^{N_1 + N_2} \) that satisfies \( \sum_i \mu_i(\nu, \epsilon) = 1 \) and \( \mu(\nu, \epsilon)P^{\nu,\epsilon} = \mu(\nu, \epsilon) \). We are interested in the states that have positive weight in the distribution \( \mu^*(\nu) = \lim_{\epsilon \to 0} \mu(\nu, \epsilon) \). Following Foster and Young(1990) we call these stochastically stable states.

Recall that we are imposing \( \sigma \rho < 1 < (1 - \sigma) \rho \), reflecting the assumption that the poor agents do worse in the unequal contract. Notice that this game has two strict Nash equilibria \((U, U)\) and \((E, E)\). Agents are myopic, and play a best response to the distribution of play in the previous period. This will define a large state-space. If we suppose initially that the sizes of the two classes are fixed, then we can represent the state-space by \((p_t, q_t)\), where \( p_t \) and \( q_t \) denote the fraction of class A and class B playing 1 in period \( t \).

The implied Markov process induced by the best-response dynamic has two recurrent classes, which correspond to the strict Nash equilibria of the contract game, namely \((1, 1)\) (which corre-
sponds to \((E, E)\) and \((0, 0)\) which corresponds to \((U, U)\). It remains to show that the perturbed dynamic we have specified selects one of these Nash equilibria and does not cycle between them. The intuition is simple: idiosyncratic play can take the process from \((0, 0)\) to \((0, 1)\), and any state is accessible from \((0, 1)\). This is because from \((0, 1)\) the distribution of deviant play for both populations has full support (on the strategy space) given that each population is at the state that would be worse for it were it in equilibrium. Thus the process is ergodic. For small \(\epsilon\), the ergodic distribution will have almost all of its mass on the two pure Nash equilibria (the interior population equilibrium is ruled out by the finite population assumption and the fact that each agent plays a single strategy). The stochastically stable state will be one of these two pure Nash equilibria.

**Proposition 3.2.** As \(\epsilon\) goes to 0, the ergodic distribution of the perturbed Markov process will put mass 1 on one of the 2 recurrent classes of the unperturbed dynamic; there is no cycling.

*Proof.* Follows from Proposition 1 in Naidu, Hwang and Bowles (2010).

### 3.3 Institutional Selection

Suppose also that the status quo convention is \((U, U)\) namely the convention that favors the better off Bs. If sufficiently many As demand contract \(E\) rather than the status quo contract \(U\), best responding Bs will switch to offering contract \(E\). The minimum number of As deviating from the status quo to induce a switch from contract \(U\) to contract \(E\), \(R_{UE}\), is termed the resistance for a transition from \(U\) to \(E\) is given by (1). The corresponding resistance for a B-induced transition from the \(U\) contract to the \(E\) contract is given by (2). Without loss of generality, we normalize all the resistances by \(N\), so that these resistances refer to fractions of the two classes rather than number, approximated by:

\[
R_{UE} = \gamma \frac{(1 - \sigma)\rho}{1 + (1 - \sigma)\rho} \tag{4}
\]

\[
R_{EU} = (1 - \gamma) \frac{1}{1 + \sigma\rho} \tag{5}
\]

If the rates of idiosyncratic play do not differ between the classes, the population will spend most of the time at the convention whose displacement requires more deviations from the status quo. This is the stochastically stable state, given by \(i\) such that \(R_{ij} > R_{ji}\). In this case the expected waiting time before a transition out of \(i\) to \(j\) will exceed that of the reverse transition, so that the
population will spend more than half of the time near the convention given by $i$.

These resistances differ from those in the standard perturbed Markov process models in which the resistances that drive transitions are identified by letting $\epsilon$ go to zero so that transitions are induced by the idiosyncratic play of that group for which the least number are required to induce the best responders in the other group to switch strategies (Kandori, Mailath, and Rob 1993, Young 1993, Binmore, Samuelson and Young 2003). By contrast our resistances are the least number of idiosyncratic plays required to induce a transition by those who would benefit should a transition occur. In the contract game, it is always the case that the number of errors required to induce a transition is least for members of the sub-population that stands to lose from the transition, because inducing best responders in the opposing sub-population to switch to a contract that they prefer requires fewer idiosyncratic players than inducing a switch to a worse contract.

This is why in the standard model with random errors transitions are always induced by those who lose as a result. In our model transitions are induced by those who stand to gain, as agents to not 'experiment' with contracts under which they would be worse off. Thus the resistances that drive the two processes (intentional or random) are always different: resistances in the standard perturbed Markov process model are always less than one half, while ours are greater than one half.

4 Efficiency, Distribution and Persistence

We can now investigate how the level of equality and efficiency of a contract effects the persistence of the associated convention. Efficiency is measured by the level of the joint surplus, that is, $2$ in the equal contract and $\rho$ in the unequal contract, while the level of equality in the unequal contract is measured by the share of the surplus received by the least well off group, $\sigma$. Setting $R_{UE} = R_{EU}$ from (1) and (2) gives the characteristics of unequal contracts such that the population would spend approximately half of the time at the unequal and half at the egalitarian contract.

\[
\gamma = \frac{(1 - \sigma)\rho}{1 + (1 - \sigma)\rho} = (1 - \gamma)\frac{1}{1 + \sigma\rho} \iff \gamma = \frac{1 + (1 - \sigma)\rho}{1 + 2(1 - \sigma)\rho + \rho^2(1 - \sigma)\sigma}
\]
It is simple to check that if $\gamma = 1/2$, the stochastically stable equilibrium is risk-dominant. In the 2x2 contract game, this will be the contract that maximizes the product of the payoffs of the two classes, namely $\rho^2(1 - \sigma)\sigma$ for convention $U$ and 1 for convention $E$. Thus, if $\rho^2(1 - \sigma)\sigma > 1$ then $R_{UE} > R_{EU}$, and $U$ will be selected. The reverse inequality implies that $E$ is selected. We can generalize these results to the case where the class sizes differ. Our key result is that unequal and inefficient contracts that are not risk dominant will be selected if the class suffering the inequality is sufficiently large relative to the favored class.

**Proposition 4.1.** For the contract set above and the dynamic process with resistances $R_{UE}$ and $R_{EU}$, we have $\gamma^*(\sigma, \rho) = \frac{1 + (1 - \sigma)\rho}{1 + 2(1 - \sigma)\rho + \sigma(1 - \sigma)\sigma^2}$ such that if $\gamma > \gamma^*$ then $U$ is the stochastically stable state. Under the assumption that $(1 - \sigma)\rho > 1 > \rho\sigma$ we have both $\frac{d\gamma^*(\sigma, \rho)}{d\sigma} < 0$ and $\frac{d\gamma^*(\sigma, \rho)}{d\rho} < 0$. Further if $E$ is risk-dominant, i.e. $\rho^2(1 - \sigma)\sigma < 1$, then $\gamma^* > \frac{1}{2}$.

**Proof.** Differentiating with respect to $\rho$ and $\sigma$ yields:

$$\frac{d\gamma}{d\rho} = -\rho \left( -1 + \rho + \rho^2(\sigma - 1)^2 - 2\rho\sigma \right) \left( -1 + 2\rho(-1 + \sigma) + \rho^2(\sigma - 1)\sigma^2 \right) < 0$$

since the negative components of the numerator are no less than $-3$ (since $\sigma\rho < 1$), but the positive components are bounded below by 3 since $\rho(1 - \sigma) > 1$ and $\rho > 2$, so the net numerator is negative.

$$\frac{d\gamma}{d\sigma} = \frac{(1 - \sigma)(-1 - 2\rho\sigma - \rho^2(1 - \sigma)\sigma)}{(-1 + 2\rho(\sigma - 1) + \rho^2(\sigma - 1)\sigma)\sigma} < 0$$

since $\sigma < \frac{1}{2}$ implies $(1 - \sigma) > 0$.

Proposition 4.1 shows that for a given $\gamma$ there exists a locus of inequality and efficiency levels $\gamma^*(\sigma, \rho)$ that satisfies equation (6), so that if $\gamma > \gamma^*$ the unequal contract becomes stochastically stable. If the unequal contract is risk-dominant, then this can occur even if the A-population is smaller than the B-population. If contract $U$ is not risk-dominant, then stochastic stability requires that the A-population be larger than the B-population. As the total surplus of contract $U$ shrinks, it takes a larger and larger relative population of As to maintain the stochastic stability of contract $U$. This result is perhaps easiest to see by looking at the plot of $\gamma^*$ in Figure 2.

Similarly, as the inequality of contract $U$ increases, so that the As receive less and less of the
surplus, it takes a larger relative population size of the As for the unequal contract to be stochastically stable. If $\gamma < \frac{1+\rho}{1+2\rho}$ the resistance to moving from the equal to the unequal contract ($R_{EU}$) will be uniformly less than the resistance to moving from the unequal to the equal ($R_{UE}$), so the unequal contract will be selected even if contract $E$ offers nothing to the A class. This occurs because in a population all of whom are best responding by playing contract $U$ favored by the As if all of the Bs idiosyncratically select their preferred (unequal) contract, the expected payoff to the As of persisting with their preferred contract ($U$) is zero, so they will (weakly) best respond by conceding to the Bs and playing $E$. In order for the As to induce the Bs to concede to a switch from a contract in which they receive the entire surplus to the equal contract, it is not necessary for all poor to deviate, a fraction $\frac{\rho}{1+\rho}$ of them will be sufficient. But if $\gamma$ is sufficiently large, this required number of deviating As will exceed the critical number of deviating Bs to induce a shift in the opposite direction, namely $(1-\gamma)$, so the unequal contract will be selected. Thus the equilibrium selection process favors smaller classes.

The reason is not the incentive-based logic stressed by the political science literature on collective action inspired by Olson (1965); nor is it related to the fact that excess supply of a factor of production may disadvantage its ‘owners’ in markets. Rather the advantage of small size arises simply because smaller groups are more likely to experience realizations of idiosyncratic play large enough to induce a transition, as long as the rate of idiosyncratic play is less than the critical fraction
of idiosyncratic players required to induce a transition (which we assume throughout, given that the relevant resistances in our model are always greater than one-half). The standard evolutionary dynamic, however, will have the opposite prediction in this class of games, where a larger relative population size for the As favors the equal contract (Bowles 2004, Naidu, Bowles and Hwang 2010).

5 Intergenerational Mobility

The assumption that class sizes are given may now be relaxed. Suppose that becoming a member of the B class requires that one’s parents have joint income not less than some minimum amount, which, for simplicity, constitutes the next generations inheritance. This impediment to class mobility could arise because class membership requires that one undertake a project with a minimum size, for example owning capital goods sufficient to employ an economically viable team of workers. In this case, inheritance of the asset is required because members of the less well off class are credit constrained. Those who inherit less than this amount become members of the A class. In the resulting model, then, the stochastically stable contract and the relative sizes of the two classes will be jointly determined.

The state space of the dynamic with endogenous population shares is now given by: \( X' = \Delta_R \times \Delta_C \times \Gamma \), where \( \Gamma = \{ \frac{k}{N} : 0 < k < N \} \). Thus a state will now be a triplet, \((p, q, \gamma) \in X'\). Changes in the sizes of the two classes will occur when an offspring of a B parent has insufficient wealth to retain its parents upper class status, or when a child of an A has sufficient wealth to become a B. How often this occurs in general could depend on four things: the degree of class assortment in parenting, the inheritance rules in force (primogeniture or equal inheritance, for example), the minimal inheritance required for membership in the upper class, and the payoffs of the two parents. We assume equal inheritance to the two offspring of each couple and abstract from marital assortment as it will not affect the resulting equilibria in our model. We assume that when parents belong to the same class, the two offspring retain the parents’ class membership, the payoffs of two B’s always being sufficient for both offspring to become B’s and the payoffs to two A’s never being sufficient to allow their two offspring to become B’s.

The expected income of the cross-class couple is given by the expected income of the A parents, \( p_t q_t + (1 - p_t)(1 - q_t) \rho \sigma \), plus the expected income of the B parent. (who gains a mean payoff of
\(q_t p_t + (1 - q_t)(1 - p_t)\rho(1 - \sigma)\) in each of \(\frac{\gamma_t}{1 - \gamma_t}\) interactions with members of the A class). Thus:

\[
y_c(p_t, q_t, \gamma_t) := p_t q_t + (1 - p_t)(1 - q_t)\rho \sigma + (q_t p_t + (1 - q_t)(1 - p_t)\rho(1 - \sigma))\frac{\gamma_t}{1 - \gamma_t}
\]  

(9)

Note that because of restrictions on \(X'\), the maximum value of \(y_c\), which we denote \(y_{\text{max}}\) is equal to \(\rho \sigma + \rho(1 - \sigma)(N - 1)\) and \(y_{\text{min}} = 0\), when either \(p_t = 0\) and \(q_t = 1\) or \(p_t = 1\) and \(q_t = 0\).

The change in \(\gamma_t\) from one generation to the next is as follows. Letting \(\gamma_{t+1}\) represent the population fraction of As in the next generation, it will equal the fraction of As this generation plus the children of cross-class marriages that became As minus the children of cross-class marriages that became Bs. Each cross-class couple produces either 2 B children or 2 A children, subtracting or adding one member of the A class (since one of the parents is of each class). In the absence of class assortment in marriage, the number of cross-class couples is \(\gamma_t(1 - \gamma_t)\).

To capture the relationship between parental wealth and class mobility, we say that \(G\), the expected change in the size of the A class as a result of the cross class couple’s offspring, varies inversely with the ratio of the cross class couple’s income to the minimum inheritance to attain upper class membership so that \(G = G\left(\frac{y_c}{\overline{\pi}}\right)\), where \(y_c\) is expected cross-class income and \(\overline{\pi}\) is a measure of the barriers to mobility for the children of cross-class couples. Thus we can write:

\[
\gamma_{t+1} = \gamma_t + \gamma_t(1 - \gamma_t)G\left(\frac{y_c(p_t, q_t, \gamma_t)}{\overline{\pi}}\right)
\]

(10)

We impose the following conditions on \(G\)

- \(G\left(\frac{y_{\text{min}}}{\overline{\pi}}\right) = \frac{1}{N}\)
- \(G\left(\frac{y_{\text{max}}}{\overline{\pi}}\right) = -\frac{1}{N}\)
- \(\frac{dG(y)}{dy} < 0\) everywhere.

The assumptions on \(G\) are intuitive; when cross-class income is at its lowest, both children of a cross-class couple become As (the poor), which adds \(\frac{1}{N}\) to the fraction poor. When cross-class income is at its highest \(y_{\text{max}}\), then both children of a cross-class couple become Bs (the rich), which lowers the share of the population that is poor by \(\frac{1}{N}\). \(G\) is differentiable and decreasing in the average income of cross-class couples. Together, these assumptions imply that there exists a \(y^*\) such that \(G\left(\frac{y^*}{\overline{\pi}}\right) = 0\) and \(G'\left(\frac{y^*}{\overline{\pi}}\right) < 0\) and a set of possible steady states \((p, q, \gamma)\) such that
\[ y_c(p, q, \gamma) = y^*. \] At the mixed strategy Nash equilibrium \( p^*, q^* \), define \( \gamma^* \) by \( y_c(p^*, q^*, \gamma^*) = y^* \).

We look at the stable steady states of the unperturbed version of this dynamic, where \( \gamma_{t+1} = \gamma_t \), \( p_{t+1} = p_t \) and \( q_{t+1} = q_t \) at the same time.

By the assumptions on our state-space, interior \( p^* \) and \( q^* \) are not elements of \( \Delta C \times \Delta R \), so if \( y_c(p, q, \gamma^*) = y^* \) for some interior \( p, q \), the best-response dynamic will ensure that the process will transition to some other state \((BR_R(q), BR_C(p), \gamma^*)\). This rules out any mixed strategy equilibria as absorbing, and thus implies that we can restrict attention to the two pure strategy equilibria and the implied population shares. Thus the remaining stable steady-state will be characterized by either \( G(\frac{y_c(\gamma_E, 1, 1)}{\bar{\pi}}) = 0 \) or \( G(\frac{y_c(\gamma_U, 0, 0)}{\bar{\pi}}) = 0 \), where:

\[
\begin{align*}
\gamma_E^* &= \frac{\bar{\pi}y^* - \rho\sigma}{1 + \frac{\bar{\pi}y^* - \rho\sigma}{1 - \sigma}p} \\
\gamma_U^* &= \frac{\bar{\pi}y^* - 1}{\bar{\pi}y^*}
\end{align*}
\]

As expected, both are increasing in \( \bar{\pi} \). Increasing barriers to entry raises the share of workers in both steady states.

**Proposition 5.1.** *The only absorbing states of the dynamic \( p_t, q_t, \gamma_t \) are \((E, E, \gamma_E^*)\) and \((U, U, \gamma_U^*)\).*

*Proof.* See Appendix.

It is straightforward to compute the resistances for the transitions between these two recurrent states. Since the class sizes are different in each equilibrium, and the population that is playing idiosyncratically is different in each equilibrium, we modify the resistances in (1) and (2) by making the relative class fractions differ between the 2 contracts:

\[
\begin{align*}
R_{UE} &= \gamma_U^* \frac{(1 - \sigma)\rho}{1 + (1 - \sigma)\rho} \\
R_{EU} &= (1 - \gamma_E^*) \frac{1}{1 + \sigma\rho}
\end{align*}
\]

Thus we are able to explore the effects of exogenous changes in \( \bar{\pi}, \sigma, \) and \( \rho \) on the stochastically
stable contract, the equilibrium class sizes, and hence on the income inequality between members of the two classes. Intuitively we would expect that as the barrier to mobility increased (higher \( \pi \)) the poor class would be more numerous in equilibrium and that as a result the population would spend a larger fraction of the time at the unequal contract. The result of these two consequences of an increase in \( \pi \), one would expect, would be to increase the income difference between the two classes. Proposition 5.2 shows that these intuitions are correct.

**Proposition 5.2.** Given \( \rho, \sigma \), there exists a value of \( \pi^* = \pi^*(\sigma, \rho) \) such that, for all \( \pi > \pi^* \), we have \((U, U, \gamma_U^*)\) as the stochastically stable state.

**Proof.** Using 11 and 13, note that \( R_{UE}(\pi) \) is monotonically increasing and goes to \(-\infty\) as \( \pi \) approaches 0 and approaches \( \frac{(1-\sigma)\rho}{1+(1-\sigma)\rho} < 1 \) as \( \pi \) goes to \( \infty \). Also note that \( R_{EU} \) goes to \( \infty \) as \( \pi \) becomes large. Thus there exists a point \( \pi^* \) where \( R_{UE}(\pi^*) = R_{EU}(\pi^*) \), and above which \( R_{UE} > R_{EU} \), so that it is easier to escape from the equal state than the unequal state.

An implication of Proposition 5.2 is that the risk dominant contract will not be selected if the cost of vertical class mobility is sufficiently high.

An increase in \( \rho \) (from 10) lowers \( \gamma_U^* \) as it increases the income of the cross-class couple, thereby facilitating mobility out of the A-class, reducing the equilibrium size of that class and hence favoring them. In contrast to the exogenous population size model, however, the effect on equilibrium selection is ambiguous, as an increase in the productivity of the unequal contract (with no change in the equal contract) will increase the fraction of idiosyncratically playing As necessary to induce the best responding Bs to abandon the unequal contract and at the same time reduce the number of idiosyncratically playing B’s required to induce a shift from 1 to 0. However, a proportional increase in the productivity of both contracts, for example, scaling up the payoff matrix in Table 1 by some \( \omega > 1 \), does not affect the fraction of each class whose idiosyncratic play is sufficient to induce a transition. In this case the only effect of an increase in \( \rho \) is, assuming \( \pi \) fixed as above, to reduce the equilibrium size of the A-class in both contracts, favoring the As and unambiguously increasing the fraction of time spent at the more equal convention.
This section shows that the results from the previous section about which contracts are persistent are robust to endogenizing the class sizes. High barriers to mobility create asymmetric class sizes, which increases the returns to being rich, as there are now many workers to interact with. It is also harder for the poor to generate enough idiosyncratic deviance to tip the equilibrium to one that is favorable to them, and so a high barrier to mobility will favor an unequal contract. Even if one starts from equal population sizes and at the egalitarian contract, the intergenerational transmission dynamic will eventually produce few elites and many poor, and this will make it easier for the rich to obtain their preferred convention.

6 Network Structure and Inequality

Historical institutions that have implemented unequal outcomes have differed dramatically in ways not captured thus far by our evolutionary contract game. As a result we might expect that contracts with identical values of $\sigma$ and $\rho$ would experience quite different dynamics. The distributional consequences of an economic institution are thus necessary but not sufficient to characterize its historical trajectory. The structure of social interactions also matters.

Early industrial capitalism, for example, agglomerated workers in large establishments facilitating collective action. By contrast, earlier class systems, according to Ernest Gellner (1983), were characterized by “laterally separated petty communities of the lay members of society” speaking different dialects or even languages, presided over by a culturally and linguistically homogeneous class. Economic relations in such societies often took the form of patron-client relationships that endured over generations with little mobility of the clients among the patrons (Fafchamps 1992, Platteau 1995, Blau 1964).

The patron client relationship will support a very different dynamic from the relationship of employee to employer in the modern labor market. The reason is that these two institutions affect the information available to agents when they adopt best responses. Suppose that when adopting a best response the members of the two classes do not know the entire distribution of play in the previous period. Instead, players are organized into a bipartite network, and only know the distribution of play in a subset of the other population. While we could in principle investigate heterogeneous sets of opposing play known by each agent, we simplify dramatically and focus only
on the case where each agent knows the distribution of play of a fraction of the opposing class. A’s know the play of a fraction of B’s given by $k_A$ and B’s know the distribution of play in a fraction $k_B$ of As. Pre-capitalist agrarian institutions, in Gellner’s view, entailed $k_A < k_B$, for the upper class communicated readily amongst themselves and therefore had information about the recent play of a large segment of the less well-off class. The geographical, cultural and linguistic isolation of the As, by contrast, militated against information sharing beyond ones local community.

The advantage enjoyed by the B’s is not that a given B-patron may engage the A-clients of other B’s. Rather, by drawing information from a larger sample of As, the Bs less noisy signal of the distribution of play reduces the likelihood that their myopic best response will overreact to the chance occurrence of a high level of idiosyncratic play among their particular A-clients.

Assuming for simplicity that $\gamma_E^*, \gamma_U^*$ are given, the resistances $R_{EU}$ and $R_{UE}$ are:

\[
R_{EU} = k_A(1 - \gamma_E^*) \frac{1}{1 + \sigma \rho} \quad (15)
\]
\[
R_{UE} = k_B \gamma_U^* \frac{(1 - \sigma) \rho}{1 + (1 - \sigma) \rho} \quad (16)
\]

The two ‘scope of vision’ parameters in the resistances $(k_A, k_B)$ mean that more idiosyncratic players are required to induce a concession by the best responding members of the population that has more information. If $k_A$ is small, then it takes only a few idiosyncratic plays by Bs to convince the best responding As to concede to the unequal contract. As is evident from (14) and (15) an increase in $k_A$ is equivalent to an increase in the size of the B population and conversely for an increase in $k_B$.

**Proposition 6.1.** For given $\gamma_E^*, \gamma_U^*, \rho, \sigma < \frac{1}{2}$, there is a $\frac{k_A^*}{k_B^*} > 0$, such that for all $\frac{k_A}{k_B} < \frac{k_A^*}{k_B^*}$, the unequal contract is stochastically stable.

**Proof.** Follows immediately from comparing the resistances.

\[\square\]

The model thus suggests another possible reason for the trend in many countries over the past 2 centuries towards a reduction in the relative incomes of the well off (Piketty 2005). The geographic, industrial, and occupational mobility characteristic of modern labor markets (coupled
with the spread of literacy and greater ease of communication) made workers less responsive to the
demands of a small number of local employers, as they knew about the offers of employers outside
their local area. The effect would be to raise $k_A$ and thus to destabilize highly unequal contracts.

While we have considered only the simplest network structure, this framework could potentially
be extended to incorporate results from the literature on stochastic games on graphs to relate more
complex network properties to the stochastically stable equilibrium (Blume 1995, Ellison 2002,
Hojman and Szeidl 2006)

7 Inequality and the Rate of Idiosyncratic Play

Economic inequality may enhance the frequency of deviant play by the less well off group by providing
additional motives and opportunities to challenge the status quo contract (Scott 1976, Moore
1978, Wood 2003). To capture this insight we make the rate of idiosyncratic play state-dependent,
and study the response of a far-sighted government that on behalf of the myopic Bs seeks to deter
a transition to the egalitarian state.

Bergin and Lipman (1996) show that, if one allows $\epsilon$ to vary arbitrarily as a function of the
state, $\theta$, then one can choose a function that selects any recurrent class of the unperturbed process
as the stochastically stable state. But what error functions are empirically plausible? We would like
to capture the idea that idiosyncratic play by the poor will be greater in highly unequal societies.
To do this simply, we modify our preceding model, letting a state-dependent idiosyncratic play rate
be given by:

$$\epsilon(\theta) = \frac{1}{1 + \lambda \left( \pi^B(\theta) - \pi^A(\theta) \right)}$$

where $\lambda > 0$ captures the extent to which inequality increases idiosyncratic play, and $\pi^B(\theta), \pi^A(\theta)$
are the payoffs to the members of the two classes in state $\theta$. Sociological conditions favoring re-
jection of unequal contracts as well as religious or other cultural influences that make economic
inequality illegitimate will increase $\lambda$. In the equal contract, the B class idiosyncratically plays at
rate $\epsilon$, since $\pi^B(1) = \pi^A(1) = 1$, and in the unequal contract, the A class plays at a rate $\epsilon^{\phi(\lambda)}$,
where $\phi(\lambda) := \frac{1}{1 + \lambda (1 - \sigma \rho - \sigma \rho)}$, which is clearly increasing in $\sigma$. This implies the next Proposition.
Proposition 7.1. Consider any $\rho$, $\gamma$, and $\sigma$ such that $\gamma > \gamma^*(\sigma, \rho)$, so that contract $0$ would be stochastically stable if $\lambda = 0$, but $\epsilon(\theta)$ is as above. Then there exists some $\lambda$ such that for any $\lambda > \lambda$ the equal contract is selected.

Proof. Note that the resistances are now given by:

$$R_{UE} \times \phi(\lambda) = \gamma \left(1 - \sigma\right) \rho \times \frac{1}{1 + \lambda \left((1 - \sigma) \rho - \sigma \rho\right)}$$

$$R_{EU} = (1 - \gamma) \left(1 + \sigma \rho\right)$$

As $\lambda$ gets large, the resistance between $U$ and $E$ falls relative to $R_{EU}$, so $E$ becomes stochastically stable as long as $\lambda > \lambda$, where $\lambda$ is given implicitly by:

$$\gamma \frac{(1 - \sigma) \rho}{1 + (1 - \sigma) \rho} \times \frac{1}{1 + \lambda \left((1 - \sigma) \rho - \sigma \rho\right)} = (1 - \gamma) \frac{1}{1 + \sigma \rho}$$

We can solve explicitly for $\lambda$ to see that:

$$\lambda = \frac{\gamma \left((1 - \sigma) \rho^2 + (1 - \sigma) \rho\right)}{1 - \gamma} - 1$$

This result is simple, but it allows us to incorporate a key insight regarding the provision of government redistribution. We now endogenize a politically chosen level of redistribution at the unequal contract. To do so we introduce a tax $\tau$ that transfers utility from the B population to the A population in the unequal state.

Each B pays a tax on the surplus it receives from each worker. Each worker receives an equal share of the total taxes collected. Thus, in each transaction, B members receive $(1 - \tau)(1 - \sigma) \rho$, while A members receive $\sigma \rho + \tau(1 - \sigma) \rho \frac{(1 - \gamma) \rho}{1 - \gamma}$, since total taxes collected are $\tau(1 - \sigma) \rho (1 - \gamma) \frac{\gamma \rho}{1 - \gamma}$, which are shared among $\gamma$ As. So, for a given tax rate, the class difference in income is $(1 - 2\sigma) \rho - 2\tau(1 - \sigma) \rho$, and thus the rate of idiosyncratic play in the unequal contract is $\epsilon^{1/(1 + \mu(1 - 2\sigma) \rho - 2\tau(1 - \sigma) \rho)}$. As the tax rate rises, the rate of idiosyncratic play by the A class falls in the unequal state, as intended.
This now results in the following resistances:

\[ R_{UE}(\tau) \times \phi(\tau, \lambda) = \gamma \frac{(1 - \tau)(1 - \sigma)\rho}{1 + (1 - \tau)(1 - \sigma)\rho} \times \frac{1}{1 + \lambda((1 - \tau)(1 - \sigma)\rho - \sigma\rho - \tau(1 - \sigma)\rho)} \]
\[ R_{EU}(\tau) = (1 - \gamma) \frac{1}{1 + \sigma\rho + \tau(1 - \sigma)\rho} \]

(22)

(23)

What effect does this tax have on the expected duration of the unequal contractual regime. We can write the probability of exit, taking into account the effect of inequality on idiosyncratic play as:

\[ \mu_U(\tau, \lambda) = \sum_{j \geq \lceil R_{UE}\rceil} \left( \frac{\lceil \gamma N \rceil}{\lceil \gamma N \rceil - j} \right) e^{\phi(\tau; \lambda)} (1 - e^{\phi(\tau; \lambda)}) \]  

(24)

which, for \( \epsilon \) small, implies that we only have to concern ourselves with the minimum number of agents necessary to induce a transition. Thus (17) is on the order of \( \epsilon^{R_{UE}(\tau; \lambda) + \lambda((1 - 2\sigma)\rho - 2\tau(1 - \sigma)\rho)} \). Note that \( \mu_E(\tau) \) is the same as expression (4), save for the fact that \( R_{EU} \) is now a function of \( \tau \), since there is no change in the rate of idiosyncratic play in the equal contract.

The tax has three effects on \( \mu_U(\tau, \lambda) \) and they do not all have the same sign. As intended, it reduces the rate of idiosyncratic play by the As, making a transition out of the unequal contract less likely. But it has two counteracting consequences. By reducing the difference in the B’s payoffs in the two contracts it reduces \( R_{UE} \), the number of idiosyncratically responding As required to induce a transition out of the unequal contract. For analogous reasons, it also increases \( R_{EU} \), the number of idiosyncratically responding Bs required to induce a transition out of the equal contract. However, we can show that if the effect of polarization on idiosyncratic play of the poor is sufficiently great, redistributive taxation prolongs the unequal state.

**Lemma 7.2.** There exists a \( \bar{\lambda} \) such that for all \( \lambda > \bar{\lambda} \), we have \( \frac{d\mu_U(\tau)}{d\tau} < 0 \).

**Proof.** see Appendix.

25
8 Redistributive Politics

We now introduce a forward looking government that may seek to stabilize the status quo contract. One way that this could be done is to punish deviants, who in the current setup forgo one-period payoffs at the status quo contract but are not otherwise penalized. This could readily be modeled here by assigning negative values (rather than zero) to the off diagonal payoffs assigned to those who play idiosyncratically. Instead we focus on government redistribution of income under the unequal contract, as a device to reduce idiosyncratic play in the unequal state, thus prolonging the contract preferred by the upper class.

Suppose, now, that the government is a non-democracy, and implements the policy preferences of the $p$ percentile of the income distribution, choosing taxes and transfers to maximize the expected infinite-horizon income of the members of the class to which this percentile belongs. We first restrict attention to the case when $p > \gamma$; the state acts as the custodian of the long-term interest of the Bs, as in a non-democratic political system (Acemoglu and Robinson 2006). The state must weigh the costs of the tax on the per period income of the Bs against the effect of reduced income polarization on the probability of a transition out of the unequal to the equal state $\mu_U$. Thus the time spent in the unequal state is $\frac{1}{\mu_U}$ and the time spent in the equal state is $\frac{1}{\mu_E}$. Thus the expected (undiscounted) after-tax income of the B class is given by:

$$W^B(\tau) = \frac{(1 - \tau) \rho (1 - \sigma)}{\mu_U(\tau, \lambda)} + \frac{1}{\mu_E(\tau)}$$  \hspace{1cm} (25)

Proposition 8.1. If $\sigma \rho^2 > 2$ (which implies risk-dominance of $(U, U)$), then for sufficiently large $\lambda$ is a fixed $\epsilon > 0$ such that the government chooses a positive tax rate $\tau^* \in [0, \overline{\tau}]$ where $\overline{\tau} = 1 - \frac{1}{2(1 - \sigma)}$ is the tax rate that implements equal division of $\rho$ in the $(U, U)$ contract.

Proof. We prove this by showing that $W^B_0(\tau) > W^B_0(0)$. Since $W^B_0(\tau)$ is discontinuous at only a finite number of points, this implies that there exist a range of $\tau \in [0, \overline{\tau}]$ that satisfy the inequality, and thus an optimal $\tau^*$ exists in that interval. See Appendix for derivation and simulations. \qed

In the appendix we provide simulation results showing that there are interior tax rates that improve the long-run welfare of the rich for some sets of parameters. This proposition shows that if the unequal contract is efficient enough, in the sense of high $\rho$, then the elite will choose a positive
tax rate.

Thus as ideological conditions become more hostile to inequality one would expect far-sighted governments acting on behalf of the long term interests of the B class to subject the well to do to redistributive taxation. This proposition thus lies in the spirit of Acemoglu and Robinson (2006), where far-sighted elites choose partial reform in order to prevent (or delay) the transition to an equilibrium that is even more egalitarian.

9 Extensions

Our model of institutional equilibrium selection captures some key aspects of historically observed class dynamics. Among these are the ways that smaller group size may enhance bargaining power, the relationship between barriers to class mobility and the long run degree of income inequality, the importance of class differences in network structure and information, and the effects of state-sponsored ameliorative redistribution on the evolution of economic institutions. In each of these cases our model provides a novel micro-economic foundation for a widely observed regularity. The model is readily extended to take account of other aspects of institutional change.

Collective action. The only corporate actor we have considered thus far is the state; but individual members of each class may choose to act in unison, whether best responding or playing idiosyncratically. The members of a trade union may decide to work under the current contact, or to refuse to do so. Where members of such organizations may commit themselves to acting in unison, dynamics are affected in two ways. First the effective size of the class is reduced to the number of autonomously acting entities. The effect is to increase the fraction of time spent governed by the contract favored by the affected class. The second effect is to alter the rate of idiosyncratic play, the sign of the effect depending on the structure of the social interactions among the subgroups with each class. Models of social networks in which one’s behavior is influenced by (but not identical to) one’s neighbors provide a framework to take account of the fact that corporate bodies such as trade unions and business associations are rarely able to perfectly enforce action in unison (Young 2002, Durlauf 2001).

By embedding an explicit model of collective action as public goods game in the above dynamic
and assuming that some agents are other regarding. Other approaches provide more adequate behavioral foundations for deviant play. The result is that non best response play is correlated, with deviance from the status quo contract being largely absent when the number of potentially deviant players is insufficient to induce a transition (Kuran 1991, Bowles 2004).

**Endogenous barriers to class mobility.** Suppose that $\pi$ is the cost of capital necessary to hire the average number of As employed by a B, or $\kappa \gamma / (1 - \gamma)$ here $\kappa$ is the capital required to hire a single worker. Now as $\gamma$ increases, the cross-class couple, as before becomes richer, but the minimal amount required for their two children to become Bs also rises. So the cost of upward mobility will vary with the size of the working class. As a result $2\pi(\gamma) - y_c(\gamma)$ need not be monotonically declining in $\gamma$, so more than one change in $sgn(2\pi(\gamma) - y_c(\gamma))$ may occur over the interval $\gamma \in (\frac{1}{2}, 1)$. Thus, there may be multiple endogenously determined class sizes for a given contract, the same $\sigma$ and $\rho$ supporting a highly unequal society in which each of a small number of Bs profits from interacting with many As and a more egalitarian one in which more Bs interact with fewer As.

The cost of upward class mobility may also be endogenous due to the actions of the state. We have modeled state redistribution in the interest of perpetuating the status quo (unequal) contract by attenuating the associated income differences. Similar policies have been widely adopted to reduce $\pi$, the (private) cost of upward class mobility. Included are public education, meritocratic promotion rules and policies to relax the credit constraints facing the less well off. Notice, however, that these policies may have unintended effects. They could, as intended, succeed in raising $\tau_U$, the expected time spent in the unequal contract, if they reduced the income differences associated with the unequal contract, lowering $\lambda$ and hence reducing the responsiveness of the As idiosyncratic play to class disparities in income. But by reducing the equilibrium size of the A-class and hence the payoffs to the Bs in the unequal contract, these policies reduce $R_{UE}$ thereby facilitating transitions out of the unequal state, attenuating or reversing the intended effect.

**The pace of institutional change.** For reasonable updating processes, group sizes, and rates of idiosyncratic play, the waiting times for transitions from one basin of attraction to another are extraordinarily long, certainly surpassing historically relevant time spans. However, the above and other extensions can dramatically accelerate the dynamic process, yielding transitions over historically relevant time scales. First, most populations (nations, ethno-linguistic units) are composed of smaller groups of frequently interacting members. Migration among groups or emulation across
groups can induce even more rapid transition times for the population as a whole. Because groups are of quite variable size, the process may be considerably accelerated because the transition times will depend not on the mean group size but on the size of the smallest groups. Second, chance events affect the payoff structures as well as the behaviors of the members of the population. Variations in environmental effects on payoffs will thus shift the boundary of the basins of attraction, occasionally greatly reducing the size of the basin of attraction of the status quo convention. These effects in conjunction with non best-response play will accelerate the process of transition. Third, there are generally far more than two feasible conventions, and some of them may be adjacent (that is, the resistances among them are small.) A population may traverse a large portion of the state space by means of a series of transitions among adjacent conventions. Fourth conformism and collective action will reduce the effective numbers of players and tend to bunch deviant play, resulting in more frequent transitions.

10 Conclusion

We model institutional transitions in an evolutionary bargaining framework driven by intentional deviance from existing conventions. Our reformulation of evolutionary models of equilibrium selection highlights the fact that institutional change is often conflictual and initially propelled by the intentional yet substantially uncoordinated actions of a large number of actors. By contrast to these “bottom up” processes, some institutional transitions are the result of highly centralized bargaining among a small number of political parties or other corporate actors, better captured by standard political economy models. Other transitions may result from random experimentation by large numbers of actors (rather than intentional deviance from the status quo) and are best represented in existing stochastic evolutionary models. Because we do not have models that integrate both the bottom up and the top down aspects of institutional change with the unintentional and intentional sources of deviation from the status quo, research on institutional dynamics necessarily relies on a set of models, each designed to illuminate particular aspects that may be more or less relevant in given historical cases. Integrating the two classes of models seems like a promising area for research.

While we therefore do not claim generality for our model, we think that it nonetheless contributes new insights not available in existing models. For example, it provides a historically plausible dynamic that under some conditions results in efficient institutions being selected. But it
also shows that highly unequal and inefficient institutions may outlast (in an evolutionary sense) more egalitarian and efficient institutions if the barriers to upward class mobility are sufficiently great, as suggested by the work of Sokoloff and Engerman (2000) and Banerjee and Iyer (2006).

We think that this model may illuminate the dynamic of highly decentralized popular unrest and elite response during the French Revolution (Markoff 1996, Soboul 1964, and Rude 1959) and the U.S. civil rights movement (McAdam 1986). The model may also illuminate the emergence of some early welfare states (e.g. Bismarkian Germany, Mares (2004)) and systems of public education (e.g. United States, Katz 1968), and in addition to the case of the end of apartheid, the legalization and recognition of trade unions in the U.S. (Freeman 1998) and the end of Communist rule in Poland (Ekiert and Kubik 1999). But the historical work required to say if we are right remains to be done.
11 Appendix

11.1 Proof of Proposition 3.1

Suppose a state \(p_t, q_t\) is a strict Nash equilibrium, e.g. \((0, 0)\). Then \(p_{t+1} = 0\) and \(q_{t+1} = 0\), so \((0, 0)\) is an absorbing state, and an identical argument holds for \((1, 1)\). Now suppose \(p_t, q_t\) is not a strict Nash equilibrium, then there is a positive probability of moving to \((BR_R(q_t), BR_C(p_t))\), which could be either \((1, 0)\), \((0, 1)\) or \((0, 0)\) or \((1, 1)\). If it is either of the first two, then there is a positive probability of moving from \((1, 0)\) to \((1, BR_C(1))\) or \((BR_R(0), 0)\), as well as a positive probability of moving from \((0, 1)\) to \((0, BR_C(0))\) or \((BR_R(1), 1)\), all of which result in an absorbing strict Nash equilibrium. While we ruled out mixed strategy Nash equilibrium by assumption, we could easily incorporate it, by supposing that agents randomize equally across strategies when indifferent, thereby exiting any mixed population equilibrium.

11.2 Proof of Proposition 5.1

Intuition behind proof: a) first show \((0, 0, \gamma_U)\) and \((1, 1, \gamma_E)\) are absorbing states. Then show that from any state that is not one of these, a positive probability of transitioning exists.

Suppose a state \((p_t, q_t, \gamma_t)\) is a strict Nash equilibrium with corresponding population, e.g. \((0, 0, \gamma_U)\). Then \(p_{t+1} = 0\), \(q_{t+1} = 0\), and \(\gamma_{t+1} = \gamma_U\) so \((0, 0, \gamma_U)\) is an absorbing state, and an identical argument holds for \((1, 1, \gamma_E)\). Now suppose \((p_t, q_t, \gamma_t)\) is not a strict Nash equilibrium with corresponding population shares, then there is a positive probability (because of the inertia in the population dynamic) of moving to \((BR_R(q_t), BR_C(p_t), \gamma_t)\), which could be either \((1, 0, \gamma_t)\), \((0, 1, \gamma_t)\) or \((0, 0, \gamma_t)\) or \((1, 1, \gamma_t)\). If it is either of the first two, then there is a positive probability of moving from \((1, 0, \gamma_t)\) to \((1, BR_C(1), \gamma_t)\) or \((BR_R(0), 0, \gamma_t)\), as well as a positive probability of moving from \((0, 1, \gamma_t)\) to \((0, BR_C(0), \gamma_t)\) or \((BR_R(1), 1, \gamma_t)\), all of which result in an strict Nash equilibrium. Then \(\gamma_{t+1}\) will be either \(\gamma_E\) or \(\gamma_U\), resulting in an absorbing state.

11.3 Proof of Lemma 7.2

The derivative of the \(R_{UE}(\tau) \times \phi(\tau, \lambda)\) with respect to \(\tau\) is given by:

\[
\frac{\gamma}{(1 + (1 - \tau)(1 - \sigma)\rho)^2} \phi(\tau, \lambda) + \gamma \frac{(1 - \tau)(1 - \sigma)\rho}{(1 + (1 - \tau)(1 - \sigma)\rho)} \frac{d\phi(\tau, \lambda)}{d\tau}
\]
Expanding and factoring gives us:

\[
\gamma \frac{-(1 - \sigma)\rho}{(1 + (1 - \tau)(1 - \sigma)\rho)^2} \frac{1}{1 + \lambda((1 - 2\tau)(1 - \sigma)\rho - \sigma\rho)} - (1 - \tau) (1 + (1 - \tau)(1 - \sigma)\rho) 2\lambda(1 - \sigma)\rho,
\]

Factoring further:

\[
\gamma \frac{(1 - \sigma)\rho}{(1 + (1 - \tau)(1 - \sigma)\rho)^2} \frac{1}{1 + \lambda((1 - 2\tau)(1 - \sigma)\rho - \sigma\rho)} (-1 + \frac{(1 - \tau)(1 + (1 - \tau)(1 - \sigma)\rho) 2\lambda(1 - \sigma)\rho}{(1 + \lambda((1 - 2\tau)(1 - \sigma)\rho - \sigma\rho))})
\]

This will have the same sign as:

\[-1 + \frac{(1 - \tau)(1 + (1 - \tau)(1 - \sigma)\rho) 2\lambda(1 - \sigma)\rho}{(1 + \lambda((1 - 2\tau)(1 - \sigma)\rho - \sigma\rho))}
\]

Which is positive when:

\[(1 - \tau)(1 + (1 - \tau)(1 - \sigma)\rho) 2\lambda(1 - \sigma)\rho > (1 + \lambda((1 - 2\tau)(1 - \sigma)\rho - \sigma\rho))\]

Or equivalently:

\[
\lambda 2((1 - \tau)(1 + (1 - \tau)(1 - \sigma)\rho)(1 - \sigma)\rho - ((1 - 2\tau)(1 - \sigma)\rho - \sigma\rho)) > 1
\]

so we can define \( \bar{\lambda} \) as:

\[
\lambda > \bar{\lambda} \equiv \frac{1}{(1 - \tau)(1 + (1 - \tau)(1 - \sigma)\rho)(1 - \sigma)\rho + ((1 - 2\tau)(1 - \sigma)\rho - \sigma\rho))} > 0
\]

Which proves the Proposition.

### 11.4 Proof of Proposition 8.1 and Simulation

\[
W^B(\tau) = \frac{(1 - \tau)\rho(1 - \sigma)}{\mu_U(\tau, \lambda)} + \frac{1}{\mu_E}
\]

We can evaluate this at \( \tau = 0 \) and \( \tau = \bar{\tau} \) and check for conditions under which \( W^B(\bar{\tau}) > W^B(0) \):

\[
\frac{(1 - \bar{\tau})\rho(1 - \sigma)}{\mu_U(\bar{\tau}, \lambda)} + \frac{1}{\mu_E(\bar{\tau})} > \frac{\rho(1 - \sigma)}{\mu_U(0, \lambda)} + \frac{1}{\mu_E(0)}
\]
Rearranging and using the definition of $\bar{\tau}$:

$$\frac{\rho}{2\mu_U(\bar{\tau}, \lambda)} - \frac{\rho(1 - \sigma)}{\mu_U(0, \lambda)} > \frac{1}{\mu_E(0)} - \frac{1}{\mu_E(\bar{\tau})}$$

Using order notation, we can write this as:

$$\rho(O(\epsilon^{R_{UE}(\bar{\tau})\phi(\tau, \lambda)}) - O(\epsilon^{R_{UE}(0)\phi(0, \lambda)})) > O(\epsilon^{R_{EU}(0)}) - O(\epsilon^{R_{EU}(\bar{\tau})})$$

Note that if $\phi(\bar{\tau}, \lambda) = 1$, for all $\lambda$, as there is no inequality at the maximal tax, while $\phi(0, \lambda)$ approaches 0 as $\lambda$ gets large. Thus the left-hand side of the above expression is positive and of the order $O(\epsilon^{R_{UE}(\bar{\tau})}) = O(\epsilon^{-\frac{1}{1+\sigma\rho}})$. Also we have $R_{EU}(0) = \frac{1}{1+\sigma\rho} > \frac{1}{1+\rho} = R_{EU}(\bar{\tau})$, which means the right-hand side is positive and of order $O(\epsilon^{-R_{EU}(0)}) = O(\epsilon^{-\frac{1}{1+\sigma\rho}})$. Thus the inequality is satisfied at small $\epsilon$ and large $\lambda$ if:

$$\frac{\rho}{1 + \frac{\rho}{2}} > \frac{1}{1 + \sigma\rho}$$

Manipulation of this yields $\sigma\rho^2 > 2$, which proves the Proposition.

The simulation below shows how $W^B(\tau)$ looks for $\lambda = 0$ and $\lambda = 1$, for $N = 10, \epsilon = .3, \rho = 10, \sigma = .1$.

![Simulation Figure](image)

**Figure 3:** How $W^B(\tau)$ looks for $\lambda = 0$ (left) and $\lambda = 1$ (right), for $N = 10, \gamma = .5, \epsilon = .3, \rho = 10, \sigma = .01$. The horizontal line indicates the payoff $W^B(0)$ at each.
References


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