Emergence

Symmetry Breaking, Topological Defects and Effective Theories

Lecture 1
9 AM
27 June 2011
A General Definition of Emergence

A system has emergent properties when an effective theory of the system at some scale, or level of organization, is qualitatively different from the lower-level theory.
A General Definition of Emergence

A system has emergent properties when an effective theory of the system at some scale, or level of organization, is qualitatively different from the lower-level theory.
A system is emergent when the *symmetries* of the lower-level theory are *violated* under aggregation.
Topics

Symmetry
Permutations, Shifts, and Finite Group Theory
Invariances of Equations of Motion
Continuous Symmetries
Semigroups & “approximate” Symmetries

Symmetry Breaking
Essential vs. Spontaneous
Navier-Stokes & Turbulent Symmetry Breaking
Symmetry Restored?

Phase Transitions
Ising and XY Models
Annealing vs. Domain Wall Formation
Effective Theories for Defects

Emergence Defined

Signatures of Emergence in Animal Society
Essential Symmetry Breaking

\[ H(\sigma_i, \sigma_j) \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ H(\sigma_i, \sigma_j) \sim \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ S_3 \rightarrow \mathbb{Z}_2 \]
Continuous Symmetries

- All rotations in 3 dimensions, a.k.a., $O_3$
- Only rotations in 2 dimensions, a.k.a., $O_2$
- Discrete rotations in 2 dimensions, a.k.a., $Z_n$ (where $n$ is the number of longitude marks.)

$O_3 \rightarrow O_2 \rightarrow Z_2$

Thursday, June 30, 2011
Traffic Jam without Bottleneck

Experimental evidence for the physical mechanism of forming a jam

Yuki Sugiyama, Minoru Fukui, Macoto Kikuchi, Katsuya Hasebe, Akihiro Nakayama, Katsuhiro Nishinari, Shin-Ichi Tadaki and Satoshi Yukawa

Movie 1

The Mathematical Society of Traffic Flow
Navier-Stokes Equation

\[ \frac{\partial v_i}{\partial t} + \sum_j v_j \frac{\partial v_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 v_i}{\partial x^2_j} \]

Symmetry: flip $x$ position, $y$ velocity

Uriel Frisch, Turbulence
Navier-Stokes Equation

\[ \frac{\partial v_i}{\partial t} + \sum_j v_j \frac{\partial}{\partial x_j} v_i = -\frac{\partial p}{\partial x_i} + \nu \sum_j \frac{\partial^2}{\partial x_j^2} v_i \]

Symmetry: flip x position, y velocity

Uriel Frisch, Turbulence
Navier-Stokes Equation

\[
\frac{\partial v_i}{\partial t} + \sum_j v_j \frac{\partial}{\partial x_j} v_i = -\frac{\partial p}{\partial x_i} + \nu \sum_j \frac{\partial^2}{\partial x_j^2} v_i
\]

Symmetry: flip x position, y velocity

...upon turning on non-linear term
Navier-Stokes Equation

\[ \frac{\partial v_i}{\partial t} + \sum_j v_j \frac{\partial}{\partial x_j} v_i = -\frac{\partial p}{\partial x_i} + \nu \sum_j \frac{\partial^2}{\partial x_j^2} v_i \]

y symmetry still exists...
Navier-Stokes Equation

\[ \frac{\partial v_i}{\partial t} + \sum_j v_j \frac{\partial}{\partial x_j} v_i = -\frac{\partial p}{\partial x_i} + \nu \sum_j \frac{\partial^2}{\partial x_j^2} v_i \]

y symmetry still exists... suddenly broken!
Control Parameter

Reynold's Number

\[ \text{Re} = \frac{L V}{\nu} \]

Characteristic Velocity and Length Scales

System Dissipation

Pablo Navarrete Michelini, University of Chile

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Reynold’s Number

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Characteristic Velocity and Length Scales

System Dissipation

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What is your symmetry?

continuous? A real number? A vector?
[a fundamental mechanism question]

What is your interaction?

pair wise? (neighbours on a graph, cars in a line?) Mean field? (Well mixed population?)
Higher-order? (Cooperative effects, spatial structures...)
[a fundamental mechanism question]

What is your control parameter?

Noise? Dissipation? Penalty for failure?
Strength of interaction? Of pairwise:triplet interaction ratio?
[when the system shows emergence]
Broken Symmetries & Topological Defects
The Cost Function of the Ising Model

\[ H[\{\sigma_i\}] = - \sum_{i,j} J_{ij} \sigma_i \sigma_j \]

\(Z_2\) symmetry (flip all +1 to -1, and vice-versa)
Erdős-Renyi Random Graph
Pigtail Macaques at Emory
Lattice
What is your symmetry?
Each node can be “up” or “down”, and the cost is invariant if you flip everyone at once (Z\textsubscript{2} symmetry)

What is your interaction?
pairwise interactions between neighbours on a lattice, that promote sameness (nodes want to be in the same state)

What is your control parameter?
noise (“temperature”) — higher noise means influence of neighbours is small
[Ising Simulation]
Topological Defects

domain walls

Cambridge / DAMTP
(see E.P. Shellard et al.)
More Elaborate Symmetries

Vortex Images: Käser, Maier & Rautenkranz (2007)
Annealing

many defects: (quenched) strong, but brittle — hard to cleave

few defects: (annealed) ductile, flexible
What is your symmetry?

continuous? A real number? A vector?
[a fundamental mechanism question]

What is your interaction?

pair wise? (neighbours on a graph, cars in a line?) Mean field? (Well mixed population?)
Higher-order? (Cooperative effects, spatial structures...)
[a fundamental mechanism question]

What is your control parameter?

Noise? Dissipation? Penalty for failure?
Strength of interaction? Of pairwise:triplet interaction ratio?
[when the system shows emergence]

updated worksheet: check wiki, or http://santafe.edu/~simon/practical.pdf
the Spin Glass
Memory & the Rugged Landscape

![Graph showing high temperature over perturbation steps](image-url)
Memory & the Rugged Landscape

Near Phase Transition

Perturbation Step

Average On/Off
Memory & the Rugged Landscape
Memory & the Rugged Landscape
Memory & the Rugged Landscape

[one of many] ground states
Sickness & Health

high-temperature
Salamanders!

Ising models for networks of real neurons

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(Dated: February 4, 2008)

Ising models with pairwise interactions are the least structured, or maximum–entropy, probability distributions that exactly reproduce measured pairwise correlations between spins. Here we use this equivalence to construct Ising models that describe the correlated spiking activity of populations of 40 neurons in the retina, and show that pairwise interactions account for observed higher–order correlations. By first finding a representative ensemble for observed networks we can create synthetic networks of 120 neurons, and find that with increasing size the networks operate closer to a critical point and start exhibiting collective behaviors reminiscent of spin glasses.

PACS numbers: 87.18.Sn, 87.19.Dd, 89.70.+c

Physicists have long explored analogies between the statistical mechanics of Ising models and the functional dynamics of neural networks [1, 2]. Recently it has been suggested that this analogy can be turned into a precise mapping [3]: In small windows of time, a single neuron either does ($\sigma_i = +1$) or does not ($\sigma_i = -1$) generate an action potential or “spike” [4]; if we measure the measurements of Refs [3, 7]. Under these conditions cells within $\sim 200 \mu m$ of each other have drawn from a homogeneous distribution; the decline at larger distance [8]. This correlated contains $N \sim 200$ neurons, of which we record from 9; experiments typically run for $\sim 1$ hr [10].

The central problem is to find the magneti
Critical Opalescence

\[ e^{-n_{\text{steps}}(\beta - \alpha)} \]

Balance! (Power law)

\[ \alpha, \beta \text{ not balancing above critical temperature} \]
Critical Opalescence
Critical Opalescence
Critical Opalescence