Emergence is a formal theory about

SCALE (spatial scale; temporal scale)

and (more generally)

HIERARCHY (under which falls certain kinds of modularity)

- individual in a group
  (individual to group cognition)
  [e.g. of course, group vs. supergroup]

- economic systems
  - individual decision
    - within a context

- layers in a computational system
  (protocols for internet communication; multiple systems - the "emergence of networks")

SANTA FE INSTITUTE
* Introduction

* Symmetry
  Permutations, Shifts, and Finite Group Theory
  Invariances of Equations of Motion
  Continuous Symmetries
  Semigroups & “approximate” Symmetries

* Symmetry Breaking
  Essential vs. Spontaneous
  Navier-Stokes & Turbulent Symmetry Breaking
  Symmetry Restored?

* Phase Transitions
  Ising and XY Models
  Annealing vs. Domain Wall Formation
  Effective Theories for Defects

* Emergence Defined

* Signatures of Emergence in Animal Society
A General Definition of Emergence

From the lower-level theory or level of organization, is qualitatively different an effective theory of the system at some scale, a system has emergent properties when...
Signatures of Emergence in Animal Society

Emergence Defined

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Phase Transitions

Symmetry Restored?
Navier-Stokes & Turbulent Symmetry Breaking
Essential vs. Spontaneous

Symmetry Breaking

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Continuous Symmetries
Invariances of Equations of Motion
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Symmetry

Topics
Symmetry Group

\{C, B, A\} \\
\{A, C, B\} \\
\{B, A, C\} \\
S_3 = \{\text{shift right, shift right twice, swap first two}\} \\
\text{swap first and last} \\
\text{swap second two}
\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix} \sim (\varnothing, \varnothing) H
\]

\[
\begin{cases}
0 & \text{otherwise} \\
1 & \text{if } \varnothing = \varnothing
\end{cases}
= (\varnothing, \varnothing) H
\]

\{ \varnothing \in A', B', C \}

Symmetry Group
Essential Symmetry Breaking

$S_3 \rightarrow \mathbb{Z}_2$

$H(\sigma_i, \sigma_j) \sim \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$H(\sigma_i, \sigma_j) \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
Continuous Symmetries

O₃ \rightarrow O₂ \rightarrow Z_{2n}

all rotations in 3 dimensions
a.k.a. O₃

only rotations in 2 dimensions
a.k.a. O₂

discrete rotations in 2 dimensions
a.k.a. Z_{2n}

(where n is the number of longitude marks)
But the solution doesn't

Governing the system have a symmetry
(Equations (and boundary conditions)

Spontaneous Symmetry Breaking
