On Ulam's Packing Conjecture

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Damasceno, Engel & Glotzer (unpublished)
Best packing shapes are trivial
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Worst packing (convex) shape?
Ulam's Conjecture

“Stanislaw Ulam told me in 1972 that he suspected the sphere was the worst case of dense packing of identical convex solids, but that this would be difficult to prove, ”

--- 1995 postscript to “Packing Spheres”. 
Ulam's “Last” Conjecture

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Naive motivation: sphere is the least free solid (three degrees of freedom vs. six for most solids).
In 2D disks are not worst packing
Why can we improve over circles?
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\[ f(\theta) \]

\[ \sum_{i=1}^{6} f(\theta_i + \phi) \]
Why can we improve over circles?

\[ f(\theta) \]

\[ \sum_{i=1}^{\phi} f(\theta_i + \phi) \]
Why can we improve over circles?

\[ f(\theta) \]

\[ \sum_{i=1} f(\theta_i + \phi) \]
Why can we improve over circles?

\[ f(\theta) \]

\[ \sum_{i=1}^{\infty} f(\theta_i + \phi) \]
Why can we improve over circles?

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\[ f(\theta) \]

\[ \sum_{i=1} \ f(\theta_i + \phi) \]
Why can we not improve over spheres?

\[
\sum_{i=1}^{12} f(R\hat{u}_i) = \text{const.} \quad \text{if and only if} \quad f(u) = \text{monopole} + \text{dipole}
\]
Why can we not improve over spheres?

Theorem (Kallus, Nazarov): Sphere is locally worst among convex, centrally-symmetric solids.

arXiv: 1212.2551
Higher dimensions, covering

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arXiv: 1212.2551, 1301.5895