Abstract

This paper derives a Cobb-Douglas matching function as the aggregate of the number of matches made by each firm in a large economy using results from the theory of heavy tail distributions. The model is then calibrated using the empirical distribution of firm sizes and stylized facts about social networks. The implied elasticity of the matching function is found to be close to those reported by Shimer (2006). The residuals from a regression of V/U on the job finding rate are found to be graphically distributed with heavy tails, as implied by the model.

1 Introduction

Leading explanations for employment dynamics in macroeconomic theory are found in search and matching models (Diamond 1982, Mortensen 1982, Pissarides 1984). These models take frictional unemployment seriously, and posit the existence of a matching function that pairs unemployed workers with vacant jobs. A functional form for this matching function that is both analytically convenient as well as empirically verified is the Cobb Douglas form, where the number of matches $M = U^\alpha V^{1-\alpha}$, where $U$ is the number of unemployed, and $V$ is the number of vacant jobs.

There have been many attempts to model the microfoundations of the matching function (see Petrongolo and Pissarides 2001 for a survey). This paper constructs perhaps the simplest such model. The approach taken in this paper is closest to Dreze and Bean (1990), who assumed that the labor market was segmented into a large number of local labor markets, each of which cleared. They derived a CES matching function under the assumption that the number of unemployed workers and job vacancies were both log normally distributed. This paper differs from their work in two ways. Firstly, it looks at matching across firms instead of across local labor markets, allowing for empirical calibration, as well as avoiding the problems inherent in defining a “local labor market”. Secondly, it uses results from proba-
ability theory and Pareto distributions to generate a Cobb Douglas matching function.

A very recent model and calibration is given in Shimer (2006). The Shimer model also posits the existence of a large set of its segregated labor markets, each of which can exist in temporary disequilibrium. In the aggregate, this delivers a simulated Cobb Douglas matching function. However this formulation requires somewhat ad-hoc assumptions about how workers and jobs move between local labor markets. For example, it assumes that employers have no control over where a new job is created. Also, unlike this paper, it does not provide a closed-form expression for the implied matching function.

This paper attempts to provide extremely simple, but plausible microfoundations for the Cobb Douglas matching function based on results from the theory of heavy-tailed distributions. This is extremely close to the strategy employed by Jones (2005) to generate Cobb Douglas aggregate production functions. However, Jones uses results involving the limit of the maximum of a set of heavy tail distributions, while this paper uses results involving the sum of heavy tail distributions, or the weak central limit theorem. Also related to this paper is a recent paper by Gabaix (2006) that attempts to explain aggregate GDP volatility as the outcome of individual firm level volatility under the condition that the firms have a Pareto distribution of sizes. Gabaix also appeals to the weak central limit theorem to establish his results.

2 A Simple Model

First, I extend and modify a model due to Jackson and Rogers (2006) to generate a distribution of job-seekers across firms. Then I use the observed distribution of firm sizes, as well as an assumption of vacancies being proportion to firm size, to generate a Cobb-Douglas matching function of unemployed and vacancies.

Let $i$ index a finite set $I$, of size $n$ of firms. Let $U$ and $V$ be the aggregate levels of unemployment and vacancies. Suppose that latent firm sizes $b_i$ are Pareto distributed with parameter $\beta$:

$$Pr(b_i < B) = \frac{\gamma_1}{B^\beta}$$

Where $\gamma_1$ is a constant. Also suppose that each firm has access to a fraction of the unemployed of size $a_i$, and that this is also Pareto distributed with parameter $\alpha$.

$$Pr(a_i < A) = \frac{\gamma_2}{A^\alpha}$$

Where $\gamma_2$ is another constant. This distribution can be motivated in two ways. One is to suppose that
firms encounter unemployed workers through social networks. It is a stylized fact that social networks exhibit this type of “scale free” distribution of vertex degrees (Jackson 2006). However, a richer model would derive the pareto distribution of firm access to unemployment as an equilibrium outcome. Jackson and Rogers 2007, for example, develop a model of network formation that generates a power-law tail in the degree distribution.

Another motivation is if firms encounter unemployed workers through geographical location, so that firms are located in cities of heterogeneous sizes. It is another stylized fact that the distribution of city-sizes also follows a Zipf’s law distribution (Gabaix 1999, Rossi and Wright 2005). The assumption that firm sizes are independent of access to the unemployed is clearly unrealistic. However, as shown in the next section, even with this assumption we can matched some stylized facts about the data.

Suppose that each firm has a constant fraction of its size in vacancies, so that fractions of vacancies are also distributed with a Pareto distribution of parameter $\beta$.

Note that the number of matches made by each firm is $M_i = \min(a_i U, b_i V)$, so that each firm matches all its vacancies if there is excess unemployment in its network, and all unemployed workers in a firms network are matched if there are excess vacancies. Thus $M_n = \sum_{i=1}^{n} M_i$. Note also that simple algebra shows that each $M_i$ is also Pareto distributed with parameter $\alpha + \beta$.

$$Pr(M_i < m) = Pr(a_i < \frac{m}{U})Pr(b_i < \frac{m}{V}) = \frac{\gamma_2 \gamma_1}{U^\alpha V^\beta}$$

Without loss of generality, we can assume that $\gamma_1$ and $\gamma_2$ are equal to one. Let $z = (U^\alpha V^\beta)^\frac{1}{\alpha + \beta}$. Therefore, using the weak Central Limit Theorem for Pareto distributions we get

$$\lim_{n \to \infty} \frac{M_n}{z^n n^{\frac{\beta}{\alpha + \beta}}} =^d L(\alpha + \beta)$$

Where $L$ is a symmetric Levy-Stable Distribution with parameter $\alpha + \beta$ (Galambos 1976). See Gabaix (2006) for another use of this result of Levy stable distributions. Thus, if we let $M(U, V) = \lim_{n \to \infty} M_n$, we get:

$$M(U, V) = U^{\frac{\alpha}{\alpha + \beta}} V^{\frac{\beta}{\alpha + \beta}} n^{\frac{1}{\alpha + \beta}} l$$

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where \( l \) is a \( L(\alpha + \beta) \) random variable. Note that as the number of firms gets large, the number of matches also gets large, for the simple reason that there are more draws from the vacancies and unemployment share distributions.

3 Calibration

Empirical fits of Zipf’s law for firms implies \( \beta = 1.059 \) (Axtell 2001). If we assume that the exposure to unemployment is a result of the social networks of unemployed workers firms have access to, then we can use results from network theory. We can assume that these are generic social network graph, which often exhibit power-law tails (Jackson 2006). The models and evidence about the index of the tail vary highly, however. Barabasi and Albert use a preferential attachment model to get an index of \( \alpha = 2 \). As there is little work done on estimating this parameter for job-search networks, we take \( \alpha = 2 \) as given. If instead we assume that exposure to unemployment is a result of local labor markets, then we can appeal to Zipf’s law for cities, and set \( \alpha = 1 \).

The job finding rate is given by \( \frac{M}{U} = f = (\frac{V}{U})^{\frac{\beta}{\alpha + \beta}} n^{\frac{1}{\alpha + \beta}} l \) so the elasticity of the job finding rate with respect to the Vacancies to Unemployed ratio is \( \frac{\beta}{\alpha + \beta} \). Using the above social network and firm size parameters, the implied elasticity is \(.33\), while the empirical elasticity (Shimer 2006) is \(.28\) (Using JOLTS) or \(.37\) (using help wanted). If instead we assume that exposure to unemployment is a result of city-sizes, then the implied elasticity is \(.5\), the upper bound of the range discussed in Petrongolo and Pissarides. Thus this simple, reduced form model comes surprisingly close to reproducing the elasticity of some matching function estimates. However, Nagypal (2006) finds that the elasticity with respect to vacancies is \(.67\), while Petrongolo and Pissarides estimate it at between \(.3\) and \(.5\) (Petrongolo and Pissarides (2001) Nagypal (2006)). Thus the range of estimates for the elasticity of the matching function is quite large. Important none of these papers, reject the assumption that the functional form of the matching function is Cobb-Douglas.

As an additional empirical check, we can look at the distribution of residuals from the help wanted data. In order to pin down what the distribution of residuals might look like, we can use the fact that the stable distributions for a normalized infinite sum of power-law distributed variables also has a power law tail.

Consider \( \lim_{t_0 \to \infty} P(l \leq l_0) = 1 - \frac{1}{l_0^{\alpha + \beta}} \) so \( l \) is approximately power-law (for large \( l_0 \)) with parameter \( \alpha + \beta \). Thus we get
\[ \log(f) = \log(\ln \frac{1}{\alpha + \beta}) + \frac{\beta}{\alpha + \beta} \log(V/U) \]  

(6)

I run this regression using quarterly data on the job finding rate, BLS unemployment, and the Conference Board help wanted index since 1951, all from Shimer(2006). Without detrending the data, the estimated elasticity is .38, and this is invariant to including year or quarter fixed-effects.

More interestingly, the distribution of (exponentiated) residuals should also be distributed with a power law tail. It is very difficult to formally test for the existence of a power-law, particularly with data that is both sparse and contains significant autocorrelation. This is because power-laws are an asymptotic property, and thus may only be revealed with large amounts of IID data. The existing statistical tests for power law tails are very unreliable when small samples are used\(^1\). Thus we rely here on informal graphical evidence, and consider these results purely suggestive.

Figure 1 shows that the distribution of residuals has a heavy right tail. Figure 2, a plot of log rank versus the residuals shows a linear curve for the large residuals, which is suggestive of a Pareto-like distribution.

4 Conclusion

In this paper I built an extremely simple model that delivers an aggregate Cobb Douglas matching function. The model is calibrated. It is shown to sit in the observed elasticity of the matching function estimated from Conference Board data. In addition, the distribution of residuals follows a power-law tail distribution as predicted by the model.

Heavy tailed distributions are finding increased use in economics. Many papers have used the theory of fat tail distributions to explain fluctuations in the stock market(Gabaix et al. 2003) as well as aggregate GDP volatility(Gabaix 2006). Using these insights to explain fluctuations in the labor market seems like a promising area for further research. This paper could be seen as an initial step in that direction.

However, more micro-founded models and richer data are needed to fully investigate this.

\(^1\)Farmer and Geanakoplos 2006, Unpublished Mimeo
Figure 1: Distribution of \( \text{exp}(\text{Residual}) \)

![Graph showing distribution of \( \text{exp}(\text{Residual}) \)](image)

References


Figure 2: Plot of log Rank versus Residuals