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The Max-Min Principle of Product Differentiation: An Experimental Analysis

Paolo Patelli

Scuola Superiore S. Anna[a]

Santa Fe Institute [b]

Andrea Mangani

Università di Siena [c]

[a]via Carducci 40, 56100 Pisa, Italy. Email: paolop@sssup.it

[b]1399 Hyde Park Rd., 87501 Santa Fe, NM, USA. Email: paolo@santafe.edu

[c]Piazza San Francesco 7, 53100 Siena, Italy. Email: mangania@unisi.it

1 Introduction

- A major issue in Industrial Organization literature is to determine firms' strategies when technology allows for product differentiation.
- A typical example of vertical characteristic is quality. With products defined by one characteristic only (vertical or horizontal), research has determined that the principle of minimum differentiation (Hotelling, 1929) does not hold when models are "well-behaved" (D'Aspremont et al, 1979, Shaked, Sutton, 1983).
- Recently, authors have explored the case of multidimensional product differentiation, for which products can be defined by two or more characteristics. Several models have been developed.
- Most of this work suggests that, in equilibrium, firms' strategies will result in products maximally differentiated along one dimension,and minimally differentiated among the other characteristics.
- Notwithstanding, found equilibria are not unique. The theoretical predictions on firms "strategies" in a context of multidimensional product differentiation are difficult to test empirically.
- In this paper we take in to consideration the model of multidimensional product differentiation proposed by Irmen, Thisse (1998), in which products can be differentiated along two horizontal dimensions and we test the max-min product differentiation outcome.

2 The Model

- There are two firms in the market, A and B. Firm's A location is described by a vector $\boldsymbol{a} = (a_1, a_2)$, $\boldsymbol{a} \in [0,1]^2$; B's location is $\boldsymbol{b} = (a_1, a_2)$
- There is a continuum of consumers distributed over the characteristics' unit square $C = [0,1]^2$ according to a nonnegative continuous density function $g(z)$, where $\boldsymbol{z} = (z_1, z_2)$ is a consumer's address, so that $\int_{\mathbb{R}^2} g(z)dz = N$ is the total population.
- Consumers have a conditional indirect utility function $V_i(z)$, $i = A, B$: a consumer buying at A enjoys an utility equal to

$$V_A = S - p_A - \sum_{j=1}^{2} t_j (z_j - a_j)^2 \tag{1}$$

  where S gross surplus from consuming either variant, p_A is the price of variant A. The last term is the square of the weighted Euclidean distance between consumer's ideal point and location of variant A; t_j salience coefficient of characteristic j, $(t_j = t = 0.5, \forall j)$.
- Simulated consumers have unit demands. The demand for variant A is then defined by the mass of consumers for whom variant A is weakly preferred to B:

$$D_A = \int_{\{z; V_A(z) \geq V_B(z)\}} g(z)dz. \tag{2}$$

- Consumers indifferent between purchasing product A or B are located on a line defined (in terms of z_j) by

$$p_A + \sum_{j=1}^{2} t_j (z_j - a_j)^2 = p_B + \sum_{j=1}^{2} t_j (z_j - b_j)^2. \tag{3}$$

2.1 Model Prediction

In the general case of n dimensions, the following results are showed by Irmen, Thisse (1998).

- When all weights (t_j) are equal, there are n local equilibria in which firms choose maximum differentiation along one characteristic and minimum differentiation along the remaining ones.
- When there is a dominant characteristic, the Nash equilibrium involves maximum differentiation along the dominant characteristic only.

Irmen, Thisse (1998) show that, assuming $t_k = t$ for all t, for each $k = 1, \ldots, n$, there exists $\epsilon > 0$ such that

$$\boldsymbol{a}^* = (1/2, \ldots, 0, 1/2, \ldots, 1/2), \ \boldsymbol{b}* = (1/2, \ldots, 1, 1/2, \ldots, 1/2)$$

is the only equilibrium of the first stage of the game, if deviations by firm A (resp. B) are restricted in a particular domain defined by

$$\begin{cases} \frac{1}{2} - \epsilon < a_i < \frac{1}{2} + \epsilon & (\text{resp. } \frac{1}{2} - \epsilon \leq b_i < \frac{1}{2} + \epsilon) \quad \forall i \neq k \\ 0 \leq a_i < \epsilon & (\text{resp. } 1 - \epsilon < b_i \leq 1) \quad \text{if } i = k. \end{cases}$$

3 Experimental design

- 60 students in economics were recruited at the University of Trento, divided in 5 groups of 12 people each.
- Individuals could not talk with each other. Pairs were randomized. Each session lasted about 1 hour including instruction time.
- For each group of subjects three different treatments:

  FT : subjects had to choose two varieties of the product: $v_1 \in [0, 100]$ and $v_2 \in [0, 100]$. After this choice, subjects were made aware about the location of their opponent. At the second stage of the game, subjects were asked to price their product, with $p \in [1, 100]$. After the price stage, the results in term of market share, profits, and opponent choices were disclosed. This treatment lasted for 10 periods.

  ST : subjects were told to make the same decisions (v_1, v_2 and, at the second stage, price), but this time, after the first period, there were 3 periods in which they could change only the level of price, maintaining the same location. This treatment lasted for 10 periods (10 choices of location and 40 choices of price).

  TT : subjects chose simultaneously product location (v_1 and v_2) and price for ten periods. Also this treatment lasted for 10 periods.

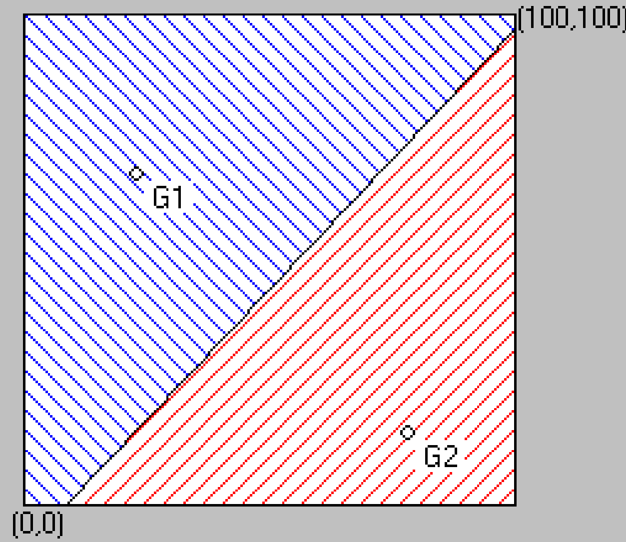
4 Theoretical prediction

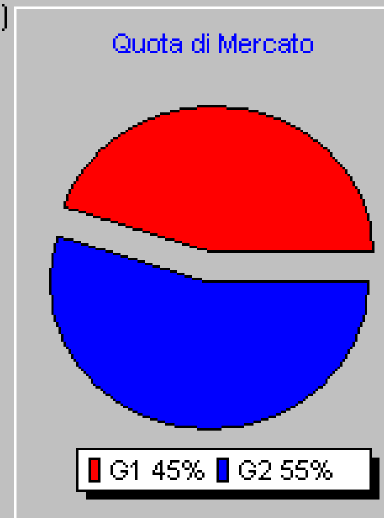
- In the FT firms' location strategies: subjects are expected to locate in order to maximize differentiation along one dimension and differentiate along the other dimension. A backward induction process is assumed, even though it has been shown that individuals rarely adopt it.
- In the ST, predictions remain unchanged: the game theoretical solution for a finite repetition of the game would, by the backward induction argument, prescribe in each repetition the same behavior as the subgame perfect equilibrium solution of the source game.
- In the TT, nothing can be said about theoretical predictions, because locations are chosen at the last (and unique) stage of the game. In fact, as explained in Economides (1987), *any game structure where locations are chosen in the last stage does not have a (subgame-perfect) equilibrium.*
- Since product differentiation along at least one characteristic should relax price competition, we expect the higher the product differentiation, the higher should be the level of prices. Again, we measure product differentiation as the Euclidean distance between firms.
- In the theoretical equilibrium, when $t_k = t$ for all t, in each n local equilibrium prices are expected to be equal to (or near to) $t = 0.5$ and firms are expected to earn the same profits. This prediction derives directly from the theoretical model (Irmen, Thisse, 1998).

Hotel

Tu sei G 1

	G 1	G 2
V 1	23	78
V 2	67	15
P	54	52
Prof.	241	288





(23,23) 23 Varietà bene 1

(67,67) 67 Varietà bene 2

(1,100) Prezzo

✔ OK

2 Fai la tua Scelta

paolo@santafe.edu

5 Results

	AvD	Sd	Mode	Median	AvP	Avπ
Nash Eq.	100	0	100	–	1	–
FT	31.74	21.53	0	27.69	39.74	161.63
ST	24.41	19.74	0	20.61	16.57	64.93
TT	18.50	18.36	0	13.47	16.92	64.07

Table 1: Summary statistics, data pooled by treatment

	AvD	Sd	Mode	AvP	Avπ
s1	25.82	20.33	0	19.29	80.54
s2	29.74	23.14	0	17.14	66.98
s3	23.60	21.11	0	14.68	52.22
s4	25.26	19.64	0	33.60	135.25
s5	20.00	17.35	0	17.83	69.51

Table 2: Summary statistics, data pooled by session

5.1 Differentiation

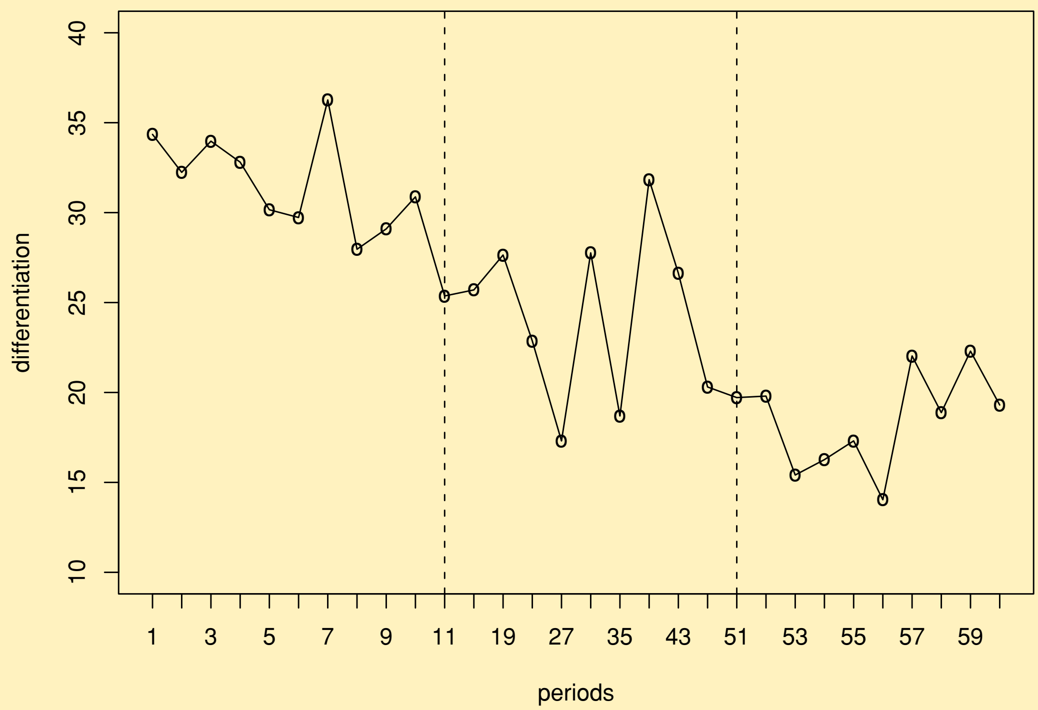


Figure 1: Average of total differentiation. Data are aggregated by session and by treatment. The vertical lines delimit the treatments.

5.2 Prices

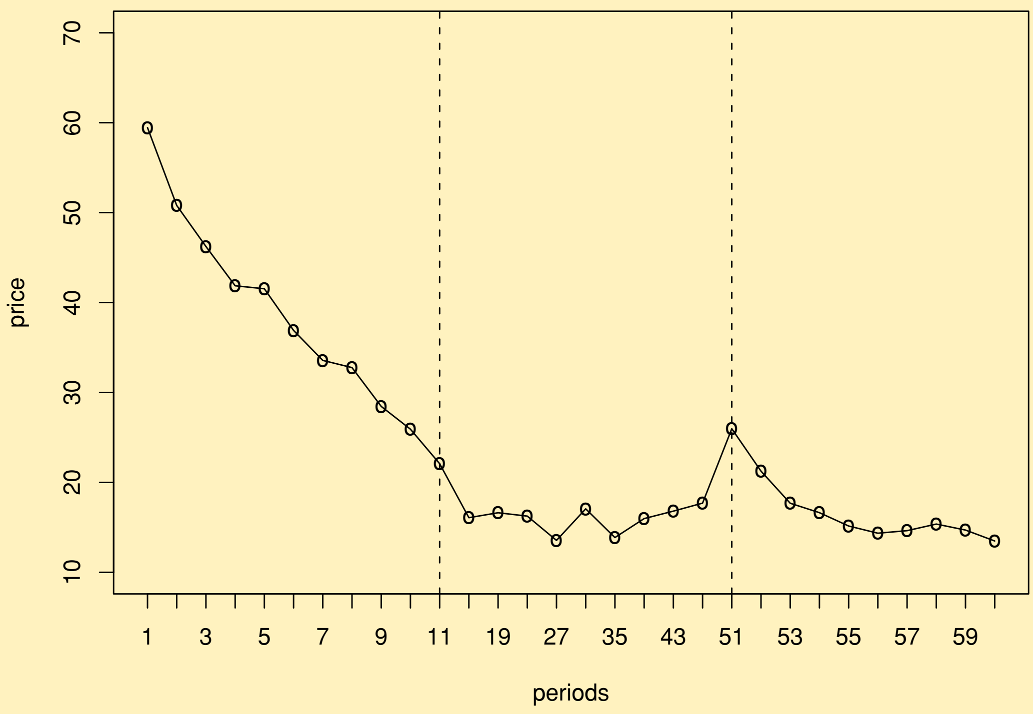


Figure 2: Price average per period. Data are aggregated by session and by treatment. The vertical lines delimit the treatments.

5.3 Profits

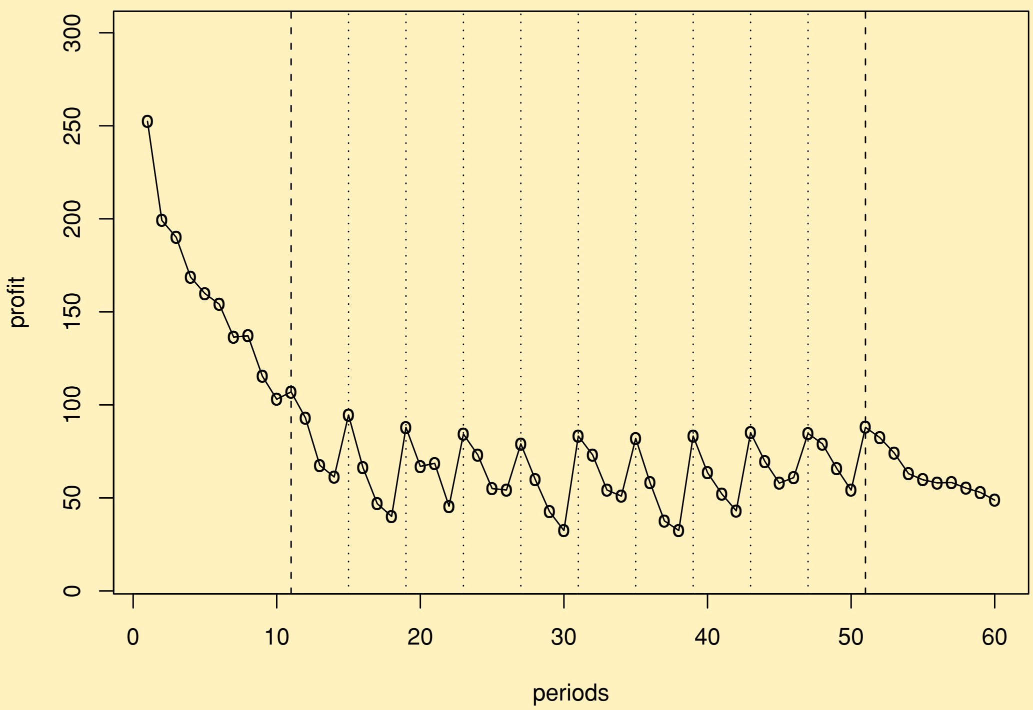


Figure 3: Average profits per period. Data are aggregated by session and by treatment. The vertical lines delimit the treatments.

5.4 Locations

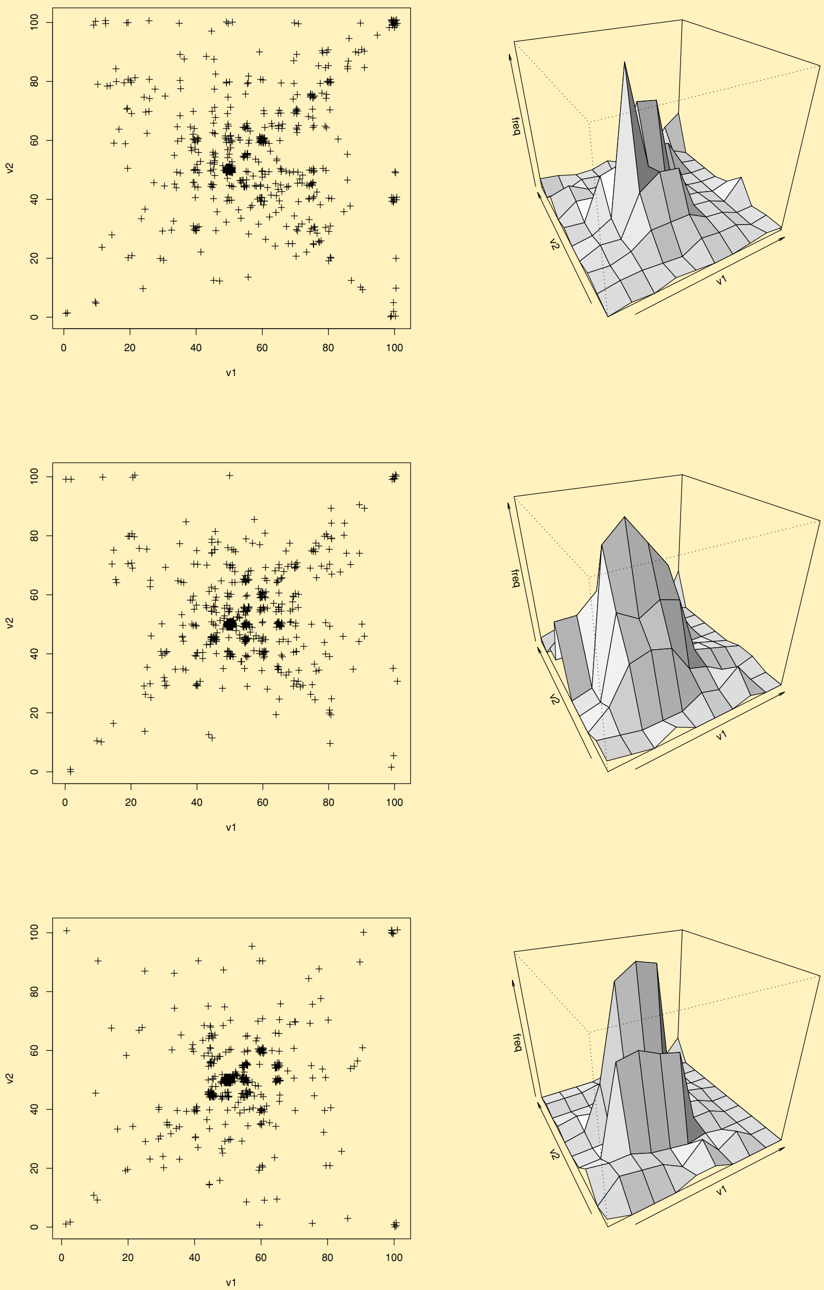


Figure 4: Locations - FT, ST, TT. In the left plot a small random number is added in order to allow the visualization of overlapping points. In the right plot data are binned in 10 by 10 classes.

5.5 Variety distribution

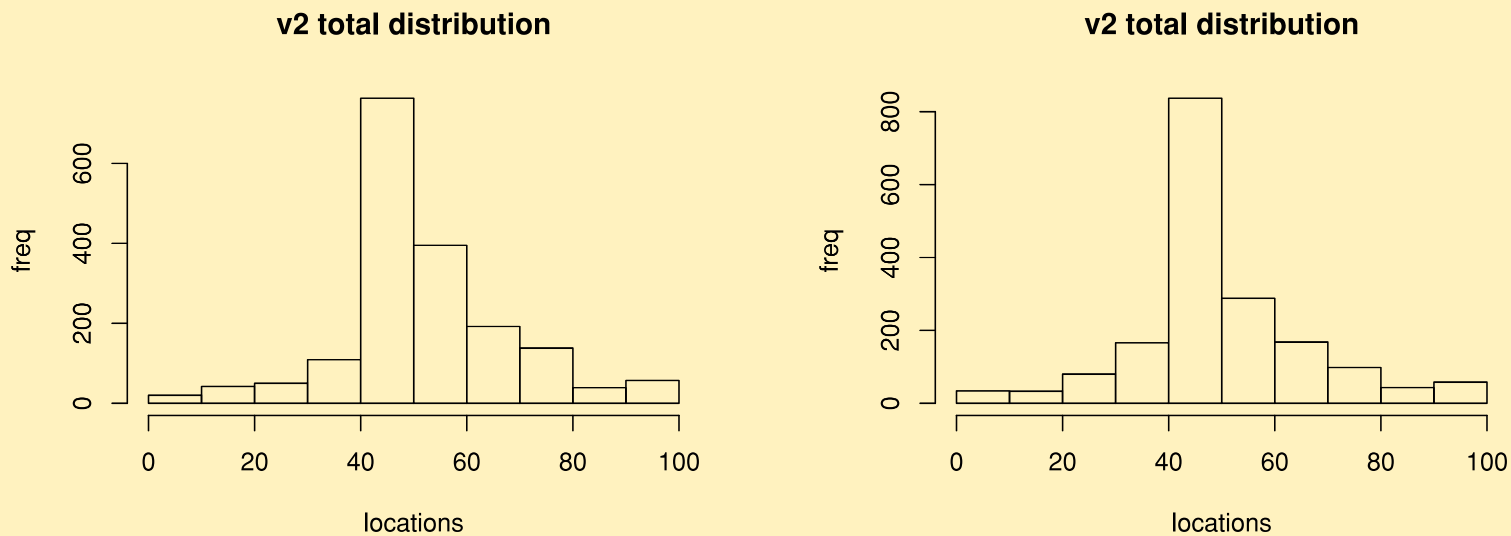


Figure 5: Variety distributions: the left histogram shows the v_1 variety distribution, the right one the v_2.

5.6 Differentiation distribution

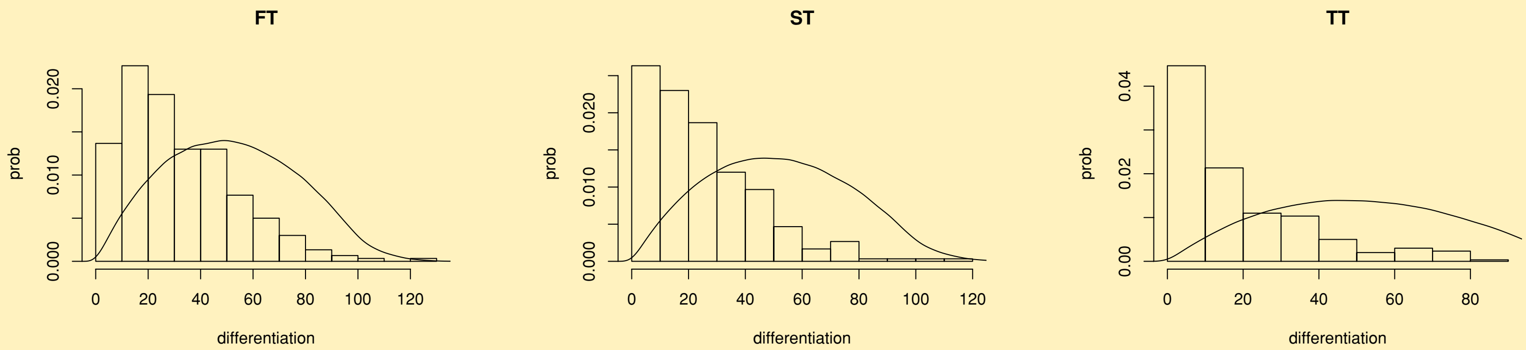


Figure 6: Distribution of variety differentiation in the three traitments. The line represent the theoretical variety distribution when agents choose randomly.

5.7 Price and differentiation

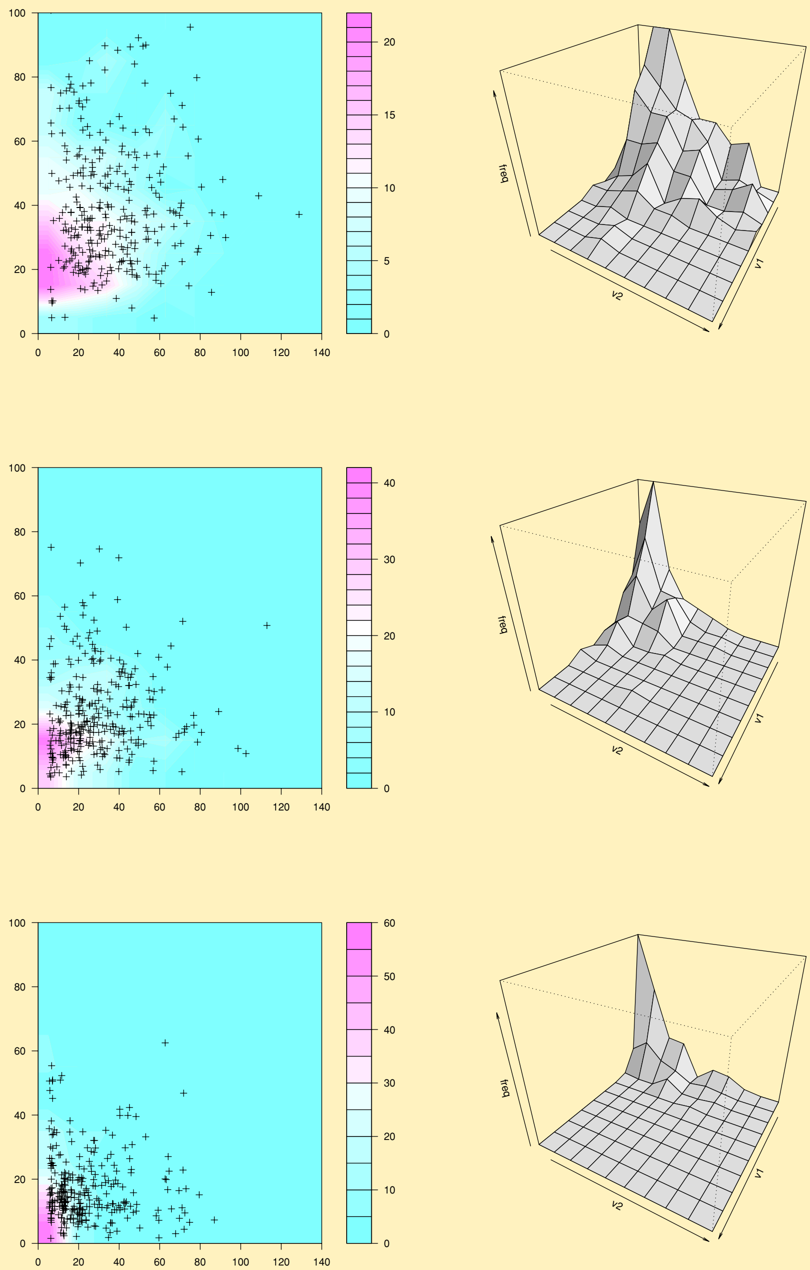


Figure 7: Price and differentiation in FT, ST,TT . In the left plot a small random number is added in order to allow the visualization of overlapping points. In the right plot data are binned in 10 by 10 classes.

5.8 Prices vs profits

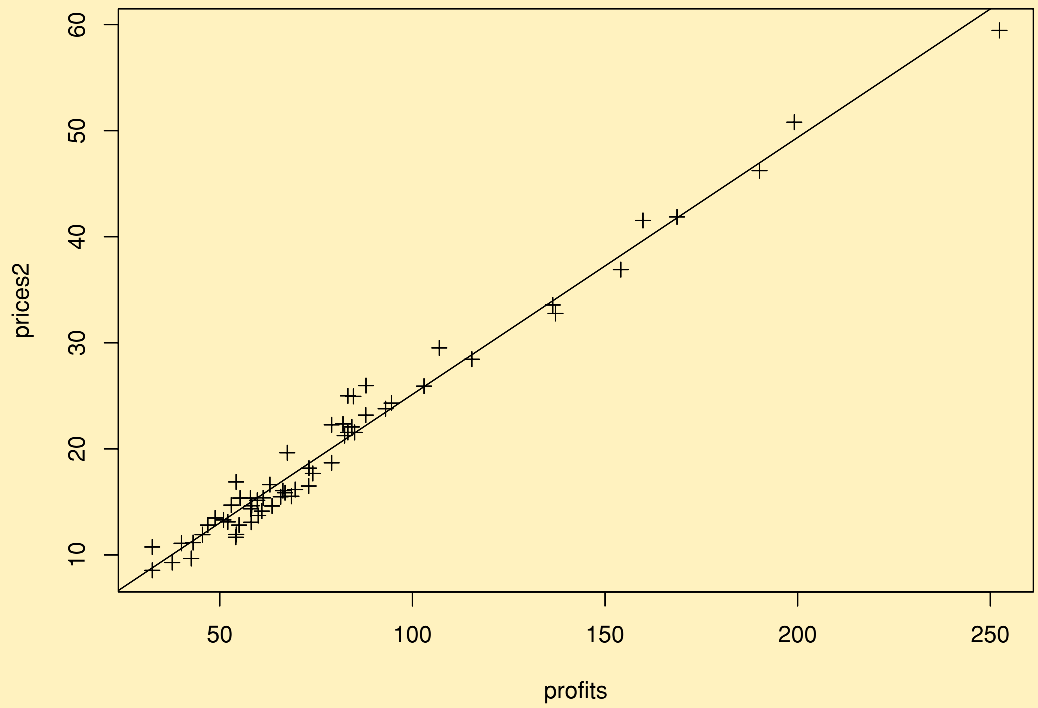


Figure 8: Price and Profits. Linear model: adjusted R-squared = 0.9777.

6 Conclusions

- Our results do not confirm the theoretical predictions formulated by Irmen, Thisse (1998) and Ansari et al (1998), but they resemble previous experiments on location and pricing: firms tend to agglomerate towards the center of consumers' distribution, in order to exploit the advantages of a central location.
- Naturally, this is an experimental result obtained in a controlled environment, and nothing can be said about how firms behave in real world. However, the majority of other experimental results confirm our outcomes, and these experimental result can be helpful in exploring the fundamental issue of multidimensional product differentiation.

7 Future development

- Extend the theoretical framework trough:
  - relaxation of some standard assumption
  - introduction of an evolutionary environment
- Detailed exploration of the effects of the experimental setting on the subjects behavior.