Supergravity and Superstrings

Murray Gell-Mann

California Institute of Technology
Pasadena, CA 91125

Over the last decade theorists have been searching hard for a unified quantum field theory of physical interactions with or without gravitation.

The "standard" SU(3) x SU(2) x U(1) theory has three independent coupling constants and numerous dimensionless parameters determining mass ratios, the weak coupling matrix, etc. The number of sets of Higgs bosons and the representation to which they belong appear arbitrary. The theory cries out to be embedded in a bigger and more nearly unified theory. The fact that the coupling constants appear to come together somewhere between 10^{15} GeV and the Planck mass \( \sim 2 \times 10^{19} \) GeV suggests a unified Yang-Mills theory, for example with SU(5) and three fermion families (fermions in \( 5 + \overline{10} \)). But such a theory, with one coupling constant, still has numerous dimensionless parameters and arbitrary Higgs boson behavior.

Renormalizability is assured, but the constants, although they absorb all the infinities, are not calculable and the unified Yang-Mills theory is still obviously incomplete. In particular, the very small and rather similar ratios \( \frac{\Lambda_{QCD}}{M_{\text{Unif}}} \) and \( \frac{M_{\text{Higgs}}}{M_{\text{Unif}}} \) must be explained. Of course, \( \frac{\Lambda_{QCD}}{M_{\text{Unif}}} \) is related to the value of the unified coupling constant near the unification mass needs explanation, as a modern version of the old problem of calculating \( \frac{1}{137} \).

Although there is as yet no direct evidence, an attractive modification involves switching to N=1 super-Yang-Mills theory, in which spin 1 bosons are accompanied by new spin 1/2 gluinos, photinos, etc., spin 1/2 fermions are accompanied by new spin zero squarks and sleptons, and spin zero higgsos are accompanied by new spin 1/2 higgsinos (ouch!). The super-Yang-Mills theory is more easily compatible with experiments on proton decay, and besides, it may provide the only way to keep some Higgs bosons light.

We may note here that there is a set of theories finite at least to two loops including (Hamidi and Schwarz, also Jones and Raby) just one of phenomenological interest, with SU(3) as the gauge group, three quark-squark, lepton-slepton families of \( 5 + \overline{10} \), and higgsos-higgsons as follows: one 24, four 5's, four \( \overline{5} \)'s. The theory has a number of parameters and can fit all experimental evidence to date. (The extra higgsos, mostly heavy, might be useful for producing enough baryons in the early universe.)

While N=1 supergravity, generalizing Einstein's gravity theory, is not necessarily very divergent itself (first divergences setting in perhaps at three loops), it is terribly divergent when coupled to external N=1 supermatter, such as N=1 super-Yang-Mills theory with N=1 supermultiplets of spin one-half and spin zero.

Nevertheless such N=1 supermatter coupled to N=1 supergravity with a cut-off has been used to show how important modifications are introduced by supergravity coupling into such quantities as the proton decay rate and branching ratios; but such a divergent theory does not really make sense.

We still require a finite, parameter-free theory and one that unifies all of physics, including Einsteinian gravitation.

Three paths are being explored in the search for the ultimate unified theory of physics. The first path involves N > 1 supergravity in four dimensions, without external supermatter, particularly the largest such theory, N=8 supergravity, where there is no room for external supermatter. The N=8 supergravity supermultiplet itself contains all the hadrons (fundamental fields of the theory).
N=8 Supergravity

Haptons:

\[
\begin{align*}
1 & \quad \text{graviton} & J_e \\
8 & \quad \text{gravitinos} & +\frac{3}{2} \\
28 & \quad \text{vector bosons} & +1 \\
56 & \quad \text{spin 1/2 fermions} & +\frac{1}{2} \\
70 & \quad \text{spin 0 bosons} & 0 \\
56 & \quad \text{spin 1/2 fermions} & -\frac{1}{2} \\
28 & \quad \text{vector bosons} & -1 \\
8 & \quad \text{gravitinos} & -\frac{3}{2} \\
1 & \quad \text{graviton} & -2 \\
\end{align*}
\]

Here, divergences probably do not set in until we get to three or perhaps seven loops, but there is no known reason to exclude them. N=8 supergravity has Newton’s constant \(k^2\) and may also have a dimensionless self-coupling parameter \(e\) for the gauge group SO(8) with 28 generators. Unfortunately SO(8) does not contain SU(3)xSU(2)xU(1). Moreover, the list of spin 1/2 particles, while long enough, does not quite match the list of quarks and leptons we know. If N=8 supergravity is to match experience, then the elementary particles we know cannot be the haptons of the theory, but must be composites and/or solitons.

The second and third paths involve higher-dimensional theories, with ten or eleven dimensions, where six or seven, respectively, are supposed to roll themselves up spontaneously into a small structure that is not observable directly but only through its effects on the spectrum and symmetries of the elementary particles and on very early cosmology.

Higher-dimensional theories that have been studied are either “superstring theories” in ten dimensions or else N=1 or N=2 supergravity theories in eleven and ten dimensions, respectively. The latter are much more recent, and, I believe, much less interesting, but have received much more attention.

If we perform trivial dimensional reductions (just circles for all the curled-up coordinates) then we obtain the relations:

\[
\begin{align*}
N=1, \ D=11 & \quad \text{supergravity} \\
N=2A, \ D=10 & \quad \text{supergravity} \\
N=2B, \ D=10 & \quad \text{supergravity} \\
\text{(non-chiral)} & \quad \text{(chiral)} \\
N=8, \ D=4 & \quad \text{supergravity} \\
\text{(non-chiral)} &
\end{align*}
\]

Ten-dimensional supergravity theories (N=1, N=2A, and N=2B) are all very singular even for one loop. Eleven-dimensional supergravity theory is trivially non-divergent in one loop because it is odd-dimensional, but in two loops it also must be highly divergent. Nevertheless, many brilliant theorists have spent a great deal of time on these theories, concentrating especially on generalizing the old Kaluza-Klein idea of getting electromagnetism from a fifth dimension.

Here they take the six or seven extra dimensions (index i) and our usual four dimensions (index µ) and use components \(g_{\mu i}\) of the metric tensor to construct gauge vector potentials, where the gauge symmetries are isometries of the 6- or 7- dimensional space.

While a trivial dimensional reduction of (just a product of 7 circles) turns N=1 supergravity into eleven dimensions into N=8 supergravity in four dimensions, with 28 vector bosons that can gauge SO(8), we notice that SO(8) does not contain SU(3)xSU(2)xU(1), and remark that a different, non-trivial dimensional reduction could lead, for example, to Kaluza-Klein bosons gauging SU(3)xSU(2)xU(1).

But this Kaluza-Klein program of accounting for gauge bosons is flawed. For one thing, it is very difficult, and perhaps impossible, to obtain a chiral theory with massless gauge bosons after dimensional reduction.

For another, the dimensional reduction over a space with non-trivial isometries tends to give a huge cosmological constant at the classical level. In any case, the higher-dimensional supergravity theories are no doubt hopelessly divergent.

The cosmological constant is proportional to the vacuum expected value of the energy density. Supersymmetry makes it vanish in the simplest theories, where there is no spontaneous violation. In realistic theories, where supersymmetry is gauged and spontaneously violated, it is by no means obvious how to get rid of the cosmological constant. Yet the effective cosmological constant in astronomy is less than \(10^{-118}\) times the natural unit involving the Planck mass. If the cosmological constant gets a large value at the classical level, then 1) we can try to cancel that against quantum corrections, hoping to do it convincingly to avoid introducing the biggest fudge factor in history, or 2) we can try to exploit (with Hawking) the possibility of quantum gravity giving a foaminess to space time, thus disconnecting the theoretical cosmological constant from the astronomical one.

Another promising strategy, however, is to have the cosmological constant vanish classically, and then try to make the corrections vanish too: The former is possible if we abandon Kaluza-Klein but the latter may be harder.

The third path, as mentioned above, involves superstrings. A superstring theory has an infinite number of fundamental boson and fermion states, of all spins, initially (as coupling starts from zero) stable and lying on an infinite number of straight Regge trajectories, so that states with spins increasing by two at a time, are spaced
equally in mass squared. The spacing is related to the "string tension." The coupling introduces level widths and modifies the trajectories.

Superstring theory was originally invented in 1971 by Neveu and Schwarz in an effort to describe hadrons. Unlike earlier string models, it incorporated not only a boson spectrum but also a fermion spectrum proposed by Ramond. The theory seems to require ten dimensions; of course, one hopes that six dimensions cancel spontaneously. In 1971 the theory possessed a kind of supersymmetry, noted by Gervais and Sakata. In 1976, Gliozzi, Scherk, and Olive noticed that the theory could be improved by the consistent omission of certain pieces, after which one could show that there were no ghosts or tachyons. The theory could then be shown to possess regular N=1 supersymmetry in ten dimensions and to contain N=1 supergravity in ten dimensions as a sector of the theory. This was after the invention of space-time supersymmetry and supergravity, which, if they had not been invented in other ways, could have been discovered in superstrings.

Meanwhile, QCD had given a presumably correct description of hadrons, with strings only an approximation. Thus, around 1974, Scherk and Schwarz suggested that the string theory be used as a unified theory of all physics, generalizing Einsteinian gravitation, with the trajectory slopes changed from $\frac{1}{(\text{GeV})^2}$ to $\frac{10^{-38}}{(\text{GeV})^2}$.

Original (Type I) superstring theory has closed strings, generalizing supergravity, and open strings, generalizing super-Yang-Mills. It is known how to construct a formal superstring theory when the gauge group is classical (i.e., unitary, orthogonal, or symplectic); simple arguments then eliminate the unitary groups. At first the choice of orthogonal or symplectic group seems arbitrary, but recently that has been found to be untrue.

Among the three constants, Newton’s G, string tension T, and super-Yang-Mills coupling constant g, there is one obvious relation and now another has been shown to exist. So the theory is parameter-free apart from $M_{\text{Planck}}$. As in any parameter-free theory, all the small quantities we know about must be calculable and must somehow come out small, in a way that depends on the nature of the spontaneous dimensional reduction and of the low-mass sector that results.

The initially massless states of the open strings in type I superstring theory belong to the adjoint representation of the gauge group. The states of the next excited level (initially around the Planck mass) belong to a different representation. The states continue to alternate between the two representations (for the classical groups, which are known to give formal superstring theories.) Type I superstring theory reduces, on truncation to the initially massless sector, to N=1 supergravity in ten dimensions, coupled to N=1 super-Yang-Mills in ten dimensions with the gauge group of the superstring theory, plus some interesting extra terms discussed later. The truncated theory is terribly divergent, of course, but shows the character of the superstring theory.

Extraction of results from superstring theory is crucially dependent on the nature of the spontaneous dimensional reduction. Trivial reduction of type I theory (10 dimensions reduced to four dimensions and the product of six circles) gives N=4 supergravity coupled to N=4 super-Yang-Mills as the initially massless sector.

N=4 supergravity, four dimensions:

\[
\begin{align*}
1 & \, \text{graviton} & -2 \\
4 & \, \text{gravitinos} & +2 \\
6 & \, \text{vector bosons} & +1 \\
4 & \, \text{spin} 1/2 \text{ fermions} & +1/2 \\
1 & \, \text{spin} 0 \text{ bosons} & 0 \\
6 & \, \text{vector bosons} & -1 \\
4 & \, \text{gravitinos} & -3/2 \\
1 & \, \text{graviton} & -2
\end{align*}
\]

N=4 super-Yang-Mills,

four dimensions:

\[
\begin{align*}
1 & \, \text{vector boson} & +1 \\
4 & \, \text{spin} 1/2 \text{ fermions} & -1/2 \\
6 & \, \text{spin} 0 \text{ bosons} & 0 \\
4 & \, \text{spin} 1/2 \text{ fermions} & -1/2 \\
1 & \, \text{vector boson} & -1
\end{align*}
\]

Breaking this last down to N=1 supersymmetry, we obtain:

\[
\begin{align*}
1 \, \text{super-Yang-Mills} & \quad +1/2 \\
3 \, \text{N}=1 \text{ matter multiplets} & \quad 0 \quad \text{x Adjunct}
\end{align*}
\]

A more complicated dimensional reduction will give different results and can break down the gauge group.

During the last couple of years Michael Green and John Schwarz have found that there are two more 10-dimensional super-
string theories IIA and IIB, with only closed strings. They reduce, on truncation to the initially massless sector, to N=2A and N=2B supergravity, respectively, in ten dimensions. But the superstring theories are finite to one loop instead of divergent like the corresponding supergravities.

We see that IIA and IIB superstrings, when truncated to the initially massless sector and trivially reduced to four dimensions, yield N=8 supergravity.

All three superstring theories, although they have the traditional description as "S-matrix" theories on the mass shell, can also be written as field theories (with fields as functionals of strings instead of functions of points) with local couplings. So far, the field description is not covariant. In the present description, there are just polynomial couplings like:

\[
\begin{array}{c}
\begin{array}{c}
\text{(Basic super-Yang-Mills coupling)}
\end{array}
\end{array}
\]

for open strings

or

\[
\begin{array}{c}
\begin{array}{c}
\text{(Basic self-coupling of supergravity)}
\end{array}
\end{array}
\]

for closed strings

Higher couplings like the fourth degree term in Yang-Mills theory come from summing over exchanges of heavy intermediate excited states.

Type I and Type IIB superstring theories and N=2B supergravity in ten dimensions appear at first to be threatened by the possible existence of "anomalies" (gravitational, Yang-Mills, and mixed) that would destroy gauge invariance and unitarity as well as finiteness or renormalizability. (The last applies to strings - ten-dimensional supergravity is divergent anyway.) IIA superstring theory, being non-chiral, has no anomalies vanishing trivially and is finite to one loop, but is less interesting than IIB for comparison with observation, just because it is non-chiral. Last year Witten and Alvarez-Gaumé were surprised to discover that all anomalies mysteriously cancel for N=2B ten-dimensional supergravity. The same result could be demonstrated for the IIB superstring. That made IIB theory, which is also finite to one loop, an attractive candidate for the universal theory. Type I, with its arbitrary gauge group, seemed less attractive.

In the last few weeks, however, Green and Schwarz have shown that all of the anomalies of type I string theory with a classical gauge group vanish if and only if the gauge group is SO(32) and also that the theory is finite rather than renormalizable to one loop if and only if the gauge group is SO(32). Here we have a very impressive form of unification, with (super)-gravitational, closed strings and (super)-Yang-Mills open strings implying each other and controlling each other's behavior.

The string theories IIA and IIB, as well as Type I with SO(32) as gauge group, are all finite to one loop and all anomaly-free. They seem increasingly desirable in this order, since Type IIA is non-chiral, while Type IIB lacks gauge bosons other than Kaluza-Klein ones, which give problems with chirality, on reduction to four dimensions.

There is one more gauge group, a non-classical one, that can yield finiteness and freedom from anomalies, provided a superstring theory can be built on it. I shall go into more detail below about that exciting possibility.

The finiteness of string theories in one loop is of a kind that suggests strongly the possibility of finiteness persisting to all orders, unlike the finiteness in one loop of N=1, D=11 supergravity, which appears to be merely a trivial consequence of D being odd and unlikely to persist in higher order. Also, the demonstration of the persistent absence of anomalies to all orders in string theory really requires this finiteness.

Where would the gauge bosons come in according to the different kinds of proposed unified theory?

N=8, D=4 supergravity: Composites or solitons.
N=1, D=11 supergravity (U(1)): Kaluza-Klein

Type I superstring, D=10: Haploids that survive non-trivial dimensional reduction.

Type IIB superstring, D=10: again composites or solitons. (This last according to Gell-Mann and Zweibel, who showed how in this theory two dimensions seem to want to roll up into a "teadrop" with a U(1) isometry, where the Kaluza-Klein boson acquires a mass and is useless as a gauge boson.)

It is presumably possible to integrate out all the initially massive states of a superstring theory and obtain a very complicated effective theory of the initially massless sector, not divergent if the superstring theory is finite. Expanding that effective theory at low energies, for type I superstring with SO(32), we obtain first the truncated theory with N=1 supergravity and N=1 super-Yang-Mills theory in ten dimensions, with a correction term that breaks supersymmetry but cancels anomalies, and then other terms that restore supersymmetry. The sum of these terms should give back the exact superstring theory with initially massive states integrated out. This kind of summation may not be unique, but it presumably always leads to a superstring.

Witten has shown that freedom from anomalies in ten dimensions for type I superstrings implies freedom from anomalies in four dimensions.

Kaluza-Klein aficionados want many isometries for their extra six- or seven-dimensional space in order to get massless gauge
bosons. Type I superstring fans want as few as possible. The field equations may require \( R_{ij}=0 \) (for the extra dimensions), where \( R_{ij} \) is the Ricci-Einstein tensor. (There can still be some Riemann-Christoffel curvature \( R_{ijkl}\neq 0 \).) The condition \( R_{ij}=0 \) (extra dimensions) will give no cosmological constant at the classical level and also no nontrivial isometries leading to extra massless gauge bosons. Even trivial U(1) isometries leading to massless Kaluza-Klein vector bosons may be eliminated by the requirement of surviving "massless" spin 1/2 fermions. Then there would be no Kaluza-Klein bosons at all. Note that in the dimensional reduction of a type I superstring if \( <R_{ij}>=0 \) then \( <G_{ij}>=0 \) (as in the presence of a generalized monopole). Spatial and Yang-Mills symmetries are violated together by related amounts.

Now there are many unanswered questions about superstring theories and their dimensional reduction; I shall briefly mention just a few.

If the field equations allow many possible dimensional reductions, which one wins? Of course, to discuss quantum cosmology, we must adopt the many-worlds approach to quantum mechanics, which is a good idea anyway. Then, do we, following Hawking, transform to a Euclidean space, with \( e^{\phi}=-e^{-\phi} \), and minimize the quantum-corrected action \( S \) so that by an inverse probability argument we have the most probable universe? Maybe the mathematics saves us the trouble and allows only one dimensional reduction!

What about sizes of extra dimensions? If we are using Kaluza-Klein ideas, then we are limited by the fact that the coupling parameters \( e \) are of order \( \frac{1}{R_{\text{Planck}}} \) for the Kaluza-Klein gauge bosons and these \( e \)'s would be too tiny if the \( R_{\text{extra dimensions}} \) were much larger than \( R_{\text{Planck}} \). With Type I superstrings we do not have this limitation. What are then the restrictions on \( R_{\text{extra dimensions}} \) from observation? Could we even have some of them very much larger than the Planck length?

In superstring theories (Type I or II) the closed strings generalize supergravity, which generalizes Einsteinian gravitation, and the superstrings must obey a generalization of general relativity. Here we have the reverse of Einstein's situation: theory first, principle afterward. What is the principle? It must be one of great importance.

If there is a Type I string theory (or a new type) based on the gauge group \( E_8 \times E_8 \) (which has dimension 496 and rank 16 like \( SO(32) \)), it also would give freedom from anomalies and finiteness to one loop. Theorists are now investigating whether there is some mathematical trick that makes possible an \( E_8 \times E_8 \) superstring. (We recall that existing methods permit only the classical orthogonal and symplectic groups, of which only \( SO(32) \) is allowed by anomaly considerations.)

If it is possible, \( E_8 \times E_8 \) would be highly desirable as the gauge group of a superstring:

1) With all initially massless spin 1, 1/2, 0 particles in the adjoint representation of the gauge group, it is very suitable to have a group with only one pair of low-dimensional representations \((1,248)\) and \((248,1)\) that serve, in a sense, as adjoint, fundamental, and spinor all at the same time.

2) The adjoint representation \((1,248)\) and \((248,1)\) contains a natural color \( SU_3 \) (that of the non-complex-number part of the underlying octonions, as pointed out by Gürsey) and the right kind of flavor variables (if there is a suitable reduction of symmetry) to describe the spin 1, 1/2, and 0 particles we know and love.

119