ADVANTAGES OF THE COLOR OCTET GLUON PICTURE ☆

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It is pointed out that there are several advantages in abstracting properties of hadrons and their currents from a Yang–Mills gauge model based on colored quarks and color octet gluons.

In the discussion of hadrons, and especially of their electromagnetic and weak currents, a great deal of use has been made of a Lagrangian field theory model in which quark fields are coupled symmetrically to a neutral vector "gluon" field. Properties of the model are abstracted and assumed to be true for the real hadron system. In the last few years, theorists have abstracted not only properties true to each order of the coupling constant (such as the charge algebra $SU_3 \times SU_3$ and the manner in which its conservation is violated) but also properties that would be true to each order only if there were an effective cutoff in transverse momentum (for example, Bjorken scaling, V-A light cone algebra, extended V-A-S-T-P light cone algebra with finite quark bare masses, etc.).

We suppose that the hadron system can be described by a theory that resembles such a Lagrangian model. If we accept the stronger abstractions like exact asymptotic Bjorken scaling, we may have to assume that the propagation of gluons is somehow modified at high frequencies to give the transverse momentum cutoff. Likewise a modification at low frequencies may be necessary so as to confine the quarks and antiquarks permanently inside the hadrons.

The resulting picture could be equivalent to that emerging from the bootstrap-duality approach (in which quarks and gluons are not mentioned initially), provided the baryons and mesons then turn out to behave as if they were composed of quarks and gluons.

We assume here the validity of quark statistics (equivalent to para-Fermi statistics of rank three, but with restriction of baryons to fermions and mesons to bosons). The quarks come in three "colors", but all physical states and interactions are supposed to be singlets with respect to the SU_3 of color. Thus, we do not accept theories in which quarks are real, observable particles; nor do we allow any scheme in which the color non-singlet degrees of freedom can be excited. Color is a perfect symmetry. (We should mention that even if there is a fourth "charmed" quark u′ in addition to the usual u, d, and s, there are still three colors and the principal conclusions set forth here are unaffected.)

For a long time, the quark-gluon field theory model used for abstraction was the one with the Lagrangian density

$$L = -\bar{q}\,[\gamma_\alpha(\partial_\alpha - igB_\alpha\lambda_0) + M]\,q + L_B. \qquad (1)$$

Here M is the diagonal mechanical mass matrix of the quarks and L_B is the Lagrangian density of the free neutral vector field B_α, which is a color singlet. Recently, it has been suggested [1] that a different model be used, in which the neutral vector field $B_{A\alpha}$ is a color octet ($A = 1 \dots 8$) and we have

$$L = -\bar{q}\,[\gamma_\alpha(\partial_\alpha - igB_{A\alpha}\chi_A) + M]\,q + L_B \text{ (Yang–Mills)}, \qquad (2)$$

where χ_A is the color SU_3 analog λ_i. In this communication we discuss the advantages of abstracting properties of hadrons from (2) rather than (1).

We remember, of course, that the real description of hadrons may involve a mysterious alteration of L_B to $\hat{L}_B$ or of L_B(Y-M) to $\hat{L}_B$(Y-M), where the new

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Lagrangian has the needed properties at high and low frequencies to give scaling and confinement respectively. No convincing example of such a situation has ever been given. In ref. [1], it was suggested the required new gluon propagation might be supplied in a model where $B_{A\mu}$ appears as one mode of a quantized string in a multilocal field theory version of a dual picture for the glue. (The mass-shell version of such a dual scheme, for particles treated as real, is known to reduce to a Yang–Mills theory as the slope parameter α' for Regge trajectories tends toward zero.) Another suggestion [2] is that somehow the free gluon propagator contains, instead of the factor $1/q^2$, a factor μ^2/q^4, where μ is some mass. All such suggestions are, for the moment, mere speculations.

It may be, of course, that there is no modification at high frequencies, in which case we would probably not have exact asymptotic Bjorken scaling. Also, modification at low frequencies may not be necessary for confinement.

A modified theory would clearly have an operator term δ in the energy density that violates scale invariance but not $SU_3 \times SU_3$, while the unmodified one would either lack δ or generate it spontaneously. A theory with $\delta = 0$ would have a massless scalar dilaton as $M \to 0$.

The simplest and most obvious advantage of (2) over (1) is that the gluons are now just as fictitious as the quarks. The color octet gluon field $B_{A\alpha}$ does not communicate with any physical channel, since the physical states are all color singlets; in contrast, the color singlet gluon field B_α would have the same quantum numbers as the baryon current, the ϕ meson, and so forth. Since in (2) the gluon is unphysical, we have no objection to the occurrence of long-range forces in its fictitious channel, produced either by massless gluons in the unmodified version or by the noncanonical glue propagation in the modified version. These fictitious long-range forces and the associated infrared divergencies could provide a mechanism for confining all color nonsinglets permanently. They would not be present in physical hadronic interactions, where long-range forces are know to be absent.

The second advantage is that we can see in (2) a *hint* as to why Nature selects color singlets. Looking at the crudest nonrelativistic, weak-coupling approximation to (2), we find a potential

$$g^2 (2\pi)^{-1} \sum_{i \neq j} r_{ij}^{-1} C_{iA} C_{jA} ,$$

where the C_{iA} are the color octet SU_3 charges of the various quarks, antiquarks, and gluons. Then it is easy to envisage a situation in which the only states with deep attraction would be the color singlets. (We suppose that in the true theory the other states become completely unphysical.)

Recently, this point has been given publicity by Lipkin [3], who treats, however, a Han–Nambu picture in which color nonsinglets can be physically excited by electromagnetism and in which there are three triplets of real quarks with integral charges that average to 2/3, −1/3, and −1/3. We have rejected such a picture. In fact, a serious argument against it is the clash between the color octet Yang–Mills gauge on the one hand and the electromagnetic gauge or the Yang–Mills gauge of unified weak and electromagnetic interactions on the other. Since, in our work, the weak and electromagnetic currents form color singlets, we encounter no such difficulty.

A third and very important advantage of the color octet gluon scheme has been pointed out by L.B. Okun in a private communication to H. Pagels. Okun's point is that in (1) there is no distinction between ordinary SU_3 and the SU_3 of color in the limit $m_u = m_d = m_s$, and thus we would have the symmetry of SU_9 (or of SU_6 for $m_u = m_d$) where these groups combine color SU_3 and ordinary SU_3. No evidence of such extended symmetries exists. In (2), of course, these annoying symmetries are not present.

A fourth apparent advantage of the color octet gluon scheme has recently been demonstrated [4] using the asymptotic perturbation theory method of Gell-Mann and Low. Assuming that the method is valid (sum of asymptotic forms of orders of perturbation theory equaling asymptotic form of sum), one can have a situation in which the bare coupling constant is zero, there are no anomalous dimensions for color singlet quantities, and the behavior of light cone commutators comes closer to scaling behavior than in the color singlet vector gluon case (1). However, actual Bjorken scaling does not occur; instead, each moment $\int F_2(\xi)\,\xi^n\,d\xi$ of the Bjorken scaling function appears multiplied, in the Bjorken limit, by a distinct power $(\ln q^2)^{p_n}$, where $-q^2$ is the virtual photon mass squared.

That sort of violation of Bjorken scaling is not contradicted by present experiments. Furthermore, many sum rules and symmetry principles of light cone current algebra would be preserved.

For us, the result that the color octet field theory model comes closer to asymptotic scaling than the color singlet model is interesting, but not necessarily conclusive, since we conjecture that there may be a modification at high frequencies that produces true asymptotic scaling.

There is one more advantage of the color octet gluon scheme over the color singlet scheme, and it is the main point we wish to stress in this communication. In either scheme, there is an anomalous divergence of the axial vector baryon current $F^5_{i\alpha}$. While, for the other eigth axial vector currents $F^5_{i\alpha}(i = 1 \dots 8)$, we have simply

$$\partial_\alpha F^5_{i\alpha}(x) = \mathcal{D}(x, x, \mathrm{i}\gamma_5 \{\tfrac{1}{2}\lambda_i, M\}), \tag{3}$$

the divergence equation for $F^5_{0\alpha}$ is [5]

$$\partial_\alpha F^5_{0\alpha} = \mathcal{D}(x, x, \mathrm{i}\sqrt{\tfrac{2}{3}}M\gamma_5) + \sqrt{6}g^2(8\pi^2)^{-1}G_{\mu\nu}G^*_{\mu\nu}, \tag{4}$$

where $\mathcal{D}(x,y,G)$ is the physical operator that corresponds in a free quark model to $\bar{q}(x)Gq(y)$, and $G_{\mu\nu} = \partial_\mu B_\gamma - \partial_\nu B_\mu$ for the color singlet case, while $G_{A\mu\nu} = \partial_\mu B_{A\nu} - \partial_\nu B_{A\mu} + gf_{ABC}B_{B\mu}B_{C\nu}$ for the color octet case.

Here the extra term in (4) arises from a several-gluon effect in the strong interaction analogous to the two-photon effect in the familiar electromagnetic triangle anomaly [6], which contributes a term $e^2(16\pi^2)^{-1}F_{\mu\nu}F^*_{\mu\nu}$ to the divergence of F^5_3.

It was shown [6] that in renormalizable gluon models the anomalous divergence arises essentially from the lowest order triangle diagram.

Wilson has demonstrated [7] that the anomaly is the consequence of a singularity in coordinate space. In field theory models this singularity comes from low order quark loop diagrams, since higher order corrections are less singular and do not contribute. Therefore, in a theory in which the gluon propagation is less singular at small distances than in the canonical one, the anomaly coefficient will be unchanged, since the quark propagation is left canonical.

In the color singlet gluon picture, the anomalous divergence term in (4) is necessarily associated [5] with an anomalous singularity in the bilocal current $F^5_{0\alpha}(x,y)$ as $z^2 = (x-y)^2$ tends to zero:

$$F^5_{0\alpha}(x,y) \mathrel{\hat{=}} 3\,\mathrm{i}(2\pi^2)^{-1}g\,G^*_{\alpha\beta}z_\beta(z^2)^{-1}. \tag{5}$$

The existence of such a term, while not contradicted by experiment so far, would destroy the light cone algebra *as a system* since one of the bilocal currents arising from commutation of two physical currents would be infinite on the light cone. In any case, we have assumed that the full light cone algebra is correct or at most violated by powers of logarithms, and we therefore cannot tolerate the term (5).

In ref. [5], this situation was posed as a puzzle: how to get rid of the anomalous singularity in $F^5_{0\alpha}(x,y)$, while retaining the anomalous divergence term for $\partial_\alpha F^5_{0\alpha}(x)$ given by triangle diagram.

The color octet gluon scheme solves the puzzle. The anomalous divergence term in $\partial_\alpha F^5_{0\alpha}(x)$ is unchanged, except for replacing $G_{\mu\nu}G^*_{\mu\nu}$ by $G_{A\mu\nu}G^*_{A\mu\nu}$, but it is now associated with a singularity as $z^2 \to 0$ not in $F^5_{0\alpha}(x,y)$, but in a different formal quantity, the corresponding color octet operator, which we may call $F^5_{0A\alpha}(x,y)$:

$$F^5_{0A\alpha}(x,y) \mathrel{\hat{=}} 3\,\mathrm{i}(2\pi^2)^{-1}g\,G^*_{A\alpha\beta}z_\beta(z^2)^{-1}. \tag{6}$$

Since $F^5_{0A\alpha}(x,y)$ is not a physical operator, being a color octet, we can have no objection to its being singular on the light cone.

To summarize, then, the fifth advantage of the color octet gluon scheme is that we get rid of the unacceptable anomalous singularity (5) in $F^5_{0\alpha}(x,y)$.

Now we can believe and make use of the anomalous divergence term in (4). This term looks as if it could be very useful in connection with the PCAC idea. Let us assume that the strong form of PCAC is correct [8]. Formally, we mean by this that as the bare quark masses tend to zero and the generators of $SU_3 \times SU_3$ become conserved, the conservation occurs according to the Nambu–Goldstone pattern, with eight massless pseudoscalar mesons. Physically, we mean that the real world of hadrons is not terribly far from such a situation, and not far at all from a situation with $SU_2 \times SU_2$ conserved and three massless pions. The bare quark masses are such that $m_\mathrm{u} \approx m_\mathrm{d} \ll m_\mathrm{s}$ and the ratios $M^2_\pi : M^2_\mathrm{K} : M^2_\eta$ are not very different from $0:1:4/3$.

It has always been a great mystery why, if we abstract relations from a field theory model like (1) or (2), we do not have in the limit $M \to 0$ the conservation of nine axial vector currents and the existence of nine massless pseudoscalar mesons. Turning on the quark bare masses, with $m_u \approx m_d \ll m_s$, we would have four nearly massless pseudoscalar mesons instead of three, in bad disagreement with observation. To put in another way, as m_u and m_d tend to zero, we would have $U_2 \times U_2$ conservation and four massless pseudoscalar mesons.

The mystery might appear to be resolved, since the anomalous term in (4) breaks the conservation of $F^5_{0\alpha}$ even in the limit $\mathcal{M} \to 0$ and so in that limit it looks as if there need not be a ninth massless pseudoscalar meson ‡, and in the limit $m_u \to 0$, $m_d \to 0$ it looks as if there need not be a fourht one.

Unfortunately, the extra term in (4) is itself a divergence of another (non-gauge invariant) pseudovector, and thus as $\mathcal{M} \to 0$ we still have the conservation of a modified axial vector baryon charge; we must still explain why this new ninth charge seems to correspond neither to a parity degeneracy of levels nor to a massless Nambu–Goldstone boson as $\mathcal{M} \to 0$.

It is important to find the explanation ‡. Assuming that strong PCAC does not fail, we conjecture that the question is closely related to the question of whether there are modifications of Yang–Mills gluon propagation and, if so, what is the nature of those modifications.

‡ In ref. [5], the authors, appalled at the anomalous singularity that accompanied the anomalous divergence in the color singlet gluon case, discussed the possibility of somehow getting rid of the anomalous divergence and finding a different explanation of the absence of a ninth pseudoscalar meson as $\mathcal{M} \to 0$. The alternative explanation tentatively offered was that F^5_0, cummuting with $SU_3 \times SU_3$, could vanish in the limit $\mathcal{M} \to 0$ when applied to "single particle states" instead of giving either parity doubling or a ninth massless pseudoscalar meson. However, using the full group $(SU_6)_{W,\text{currents}}$ of the light-like vector, axial vector, and tensor charges, we wee that F^5_0 fails to commute with the tensor charges T_{ix} and T_{iy}, and all matrix elements of those charges would have to vanish between "single particle states". The same is true of the modified F^5_0 that includes the effect of the anomalous divergence. It seems unlikely that all "single particle" matrix elements of T_{ix} and T_{iy} vanis as $\mathcal{M} \to 0$.

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