Coral Gables Conference on

FUNDAMENTAL INTERACTIONS
AT HIGH ENERGY

CENTER FOR THEORETICAL STUDIES
JANUARY 22-24, 1969 UNIVERSITY OF MIAMI

Timm Gudehus, Geoffrey Kaiser,
and Arnold Perlmutter

EDITORS

GORDON AND BREACH, SCIENCE PUBLISHERS
New York London Paris
CERTAIN THEORIES OF CP-VIOLATION*

Murray Gell-Mann

California Institute of Technology, Pasadena, California

Attempts to explain CP-violation, namely the effect discovered by Pich, Cronin (FC) and collaborators, can be classified as follows:

1. Superweak Theories

These theories I believe to be the most elegant because they make no observable positive predictions other than about those things that experimentalists are already studying. A ΔS = 2 interaction is postulated to explain how \( K_1^0 \) can decay into \( K_2^- \) and thence into \( 2\pi^0 \), the FC effect. The strength of this effect is \( \sim 10^{-3} G \), where \( G \) is the weak coupling constant. The superweak coupling constant then has to be \( \sim 10^{-3} G^2 \), a number so small that it shows up nowhere other than in phenomena directly related to the FC effect.

2. "A Little Weaker than weak" Theories

R. J. Oakes will speak about this later in the Conference; such theories have been discussed by Zachariasen and Zweig, among others. Here we postulate an interaction with strength \( \sim 10^{-3} G \), which is both CP violating and P violating, and that gives the FC effect directly. This kind of theory predicts no large CP violating effect, at least at moderate

---


380
energies, but typically predicts many new effects of the same order as FC.

3. Theories of Fairly Large CP Violation at Moderate Energies

Here there is a C violating, CP violating effect of strength \( \sim 10^{-2} \) or \( 10^{-3} \); together with the P violating, CP conserving weak interaction with strength \( \sim G \), this leads to an FC effect with strength \( \sim 10^{-3} G \). These theories have the feature that large CP violations, with strength \( \gtrsim 10^{-2} \) or \( 10^{-3} \), are predicted in certain processes. We can classify such theories under two headings:

3(a). At moderate energies, these large CP violating effects are restricted to reactions involving certain new particles, not so far observed, so that it is not necessarily true that we can go out and search for 1% CP violations in already familiar reactions.

3(b) At moderate energies, these large CP violating effects occur in reactions involving familiar particles.

In the context of classes 3(a) and 3(b), I would like to discuss a few possibilities which concern the hypothetical intermediate vector boson for weak interactions, \( X^\pm \) (sometimes called \( W^\pm \)), which might be connected with CP violations in a theory of type 3(a) or type 3(b). It is instructive to consider the possible theories in which one attempts to connect the CP violation with such intermediate particles. I have not studied all of the literature on the subject, so I shall not be reporting on all theories; for example, Okubo and others have some interesting ideas that I shall not discuss.

One example of theories of type 3(a) is about a year old. I discussed it at the dedication of the Center for Theoretical Physics at M.I.T. We consider the possibility that there is a group of strongly interacting particles, strongly interacting among themselves, which we call primed hadrons \( h' \); they may obey the usual bootstrap requirements as well as, or even rather than, the ordinary hadrons \( h \). The strong
interactions of these particles conserve a quantum number \( n_X \), basically the difference between the numbers of \( X^+ \) and \( X^- \) particles. Of course, we need not discuss our theory in terms of intermediate particles of spin one since we may simply use quantum numbers, so that \( n_X \) is an abstract quantity. The semi-weak interaction, whose square produces the ordinary weak interaction, may be written

\[
G_m \left( J^+_\mu - J^-\mu \right) \]  

where

\[
J^\mu = J^{(e)}_\mu + J^{(h)}_\mu \]  

is the usual weak current with leptonic \((e)\) and hadronic \((h)\) parts. \( m \) is some mass. \( L^+\mu \) changes \( n_X \) by +1, that is, it is like a creation operator for \( X^+ \) particles. \( L^-\mu \) changes \( n_X \) by -1. In order to connect this with CP violation, we introduce the following hypothesis. We suppose that the strong interactions of \( h' \) particles are strongly CP violating. There is no operator analogous to CP that is conserved in strong interactions among \( h' \) particles. We note that, in order \( G \), the weak interaction is essentially

\[
G J^\mu J^\nu \left\langle \left( L^+\mu L^\nu \right) \right\rangle_{\text{vac}} \]  

The vacuum expectation value of the ordered product \( \left( L^+\mu L^\nu \right) \) is roughly a propagator. The \( L^\mu \) are not free fields any more, but are more general four-vector operators. The Fourier transform of the propagator is

\[
\mathcal{F}(q^2) \delta_{\mu\nu} + 
\mathcal{F}(q^2) q^\mu q^\nu \]  

simply by Lorentz invariance. There is nothing here to violate CP and we will obtain no FC effect. We must go to some higher order process. We will obtain the largest effect by including a single electromagnetic perturbation. A real photon gives order \( Ge^2 \), a virtual photon order \( Ge^2 \). We write the electromagnetic current
\[ J^\lambda + \tilde{J}^\lambda \]  \hspace{1cm} (5)

\[ J^\lambda \] is the ordinary electromagnetic current for hadrons and leptons. \[ \tilde{J}^\lambda \] refers to the world of h' particles. In order Ge², for example, we obtain an effective interactions as follows:

\[ J_\mu \gamma^\mu J^\lambda \left\langle \left( \mathcal{L}_\mu + \mathcal{L}_\nu \tilde{J}^\lambda \right) \right\rangle_{\text{vac}}. \]  \hspace{1cm} (6)

Lorentz invariance allows this to have many more terms than (4), and a number of these give CP violation. In order Ge², we obtain, the FC effect; electromagnetic and weak cooperation. In producing FC has been studied by T. D. Lee and collaborators, but there is a grave defect in many such theories because they cannot suppress a neutron static electric dipole (EL) moment of order Ge. The reason is that with a substantial electromagnetic violation of CP and the weak interaction violation P, both P and CP are violated in order Ge. The effect should be of the order of \( 10^{-20} \) e (cm). Experimentally, the number is \( \lesssim 10^{-23} \) e (cm). We can suppress this large EL moment by making two further assumptions:

1. The strong interaction among the h' particles is P conserving. This forbids anything similar to the mechanism (discussed by the Salzmanns) in which the intermediate spin-one boson has itself an electric dipole moment.

\[ \partial_\mu \mathcal{L}_\mu = 0 \]  \hspace{1cm} (7)

\[ \partial_\mu \mathcal{L}_\mu^+ = 0 \]

These assumptions turn out to remove the possibility of a neutron EL moment to order Ge. (I have not worked out in what order we first encounter this dipole moment.) These assumptions have some interesting consequences. Define
\[ L = \int L_0 \, d^3x \]  \hspace{1cm} (8)

Then \[ \dot{L} = 0 \quad \text{and} \quad \dot{L}^+ = 0 \]  \hspace{1cm} (9)

There are then two possibilities:

(A) \( L = 0 \), \( L^+ = 0 \). Superficially, this seems to be excluded, since in the propagator, written as \( \langle (\mathcal{L}_\mu, \mathcal{L}_\nu^+) \rangle_{\text{vac}} \), the static limit \( f(0) \) of the coefficient of \( \delta_{\mu\nu} \) would vanish, and the whole static weak interaction, in \( \beta \) decay for example, would disappear. Actually, because of gradient terms in equal-time commutators and because of possible terms of order \( G \) like "sea-gull" interactions (i.e., of the type \( e^2 A_\mu A_\nu \phi^+ \phi \)), the expression in terms of a vacuum expectation value that we have given for the propagator is a bit too simple and may not even have exactly the necessary covariance properties. Thus theories are conceivable in which \( L \) and \( L^+ \) vanish without destroying \( f(0) \). Such theories should be studied carefully.

Of course, in any such theory, the expression for the electromagnetic vertex in terms of a vacuum expectation value of a triple product must also be corrected.

(B) \( L \neq 0 \), \( L^+ \neq 0 \) and they are conserved. We note that \( n_\chi \) and \( Q_\chi \) are also conserved. Possibly \( Q_\chi = n_\chi \), but not necessarily. What algebra is generated by these conserved quantities for strong interactions among the \( h' \) particles? If the primed hadrons exist, they have their own strong symmetries, like ordinary hadrons \( h \).

The smallest candidate is \( SU(2) \) with generators \( L \), \( L^+ \), and \( Q_\chi \neq n_\chi \). That is, the \( \chi \) particles would be a set of hadrons without strangeness, with all the isotopic spin multiplets centered on zero charge. This is no good as a theory because it suppresses \( C \) violation in the vertex

\[ \langle (\mathcal{L}_\mu, \mathcal{L}_\nu^+, \mathcal{G}_\chi) \rangle_{\text{vac}} \]  \hspace{1cm} (10)

That happens because \( L \), \( L^+ \), and \( Q_\chi \) are just the isotopic
raising operator, the isotopic lowering operator, and the isotopic z-component respectively, and $L_\mu$, $L_\nu^+$, and $\gamma_\lambda$ are their currents. These form the components of an isovector. In forming an isoscalar under SU(2) the vertex function (10) has to be antisymmetric in all three currents and it has just the symmetry to be C-conserving.

We could try SU(3). Then

$$Q_x = L_z + Y'/2$$

(11)

$L_z$ is the commutator of $L$ and $L^+$ and $Y'$ is an isoscalar hypercharge operator for the $h'$ particles. Therefore $Q_x$ and hence $\gamma_\lambda$ has an isoscalar component, which gives a symmetrical 'Q-violating' part to the vertex (10). It follows, of course, that there exist strange $h'$ particles; these cannot decay since the weak and electromagnetic interactions we have given do not change $Y'$ and the strong interactions conserve $Y'$ by hypothesis. Since it is unlikely that such stable particles exist, we must either introduce a new coupling or look for a higher algebra. I have not checked all possible algebras; it is an amusing mathematical problem to determine if there are any stable $h'$ particles, using just the given interactions. This kind of theory is interesting (though perhaps not to be taken too seriously) because it does show that an FC effect of order GeV does not necessarily lead to a neutron El moment of order GeV.

Theories classified under 3(b) have been studied extensively in the literature. We have large, e.g., electromagnetic order, CP violations in processes involving well-known particles. Nevertheless, we can still discuss the possibility that CP violation is connected with the existence of the intermediate bosons, X. Work on this subject has been done by T. D. Lee, by Tuan, and by Okun', among others. We consider the possibility that X particles can be produced strongly in pairs by ordinary hadrons (as Glashow suggested many years ago), that is, X particles are now ordinary hadrons but having $n_X \neq 0$. (all the familiar hadrons have $n_X = 0$). $n_X$ is conserved by strong interactions. This
would perhaps explain the Utah result that there are muons in cosmic rays at very high energies (10^{12} \text{ ev}) that do not appear to come from pion decay. We could have pair production of X's which then decay rapidly (in 10^{-16} \text{ sec}) into \mu \nu (as a prominent decay mode). We suppose that there is no C symmetry conserved by strong interactions except one that carries \( n_X \) into itself, i.e.,

\[ C n_X C^{-1} = + n_X \]  

(12)

That leads to some funny symmetries. It implies duplication of strongly interacting particles spaced two in charge apart when \( n_X = +1 \) and -1, and spaced four in charge apart when \( n_X = +2 \) and -2.

This theory leads to an FC effect from cooperating weak and electromagnetic interactions because the electromagnetic current for particles with \( n_X \neq 0 \) clearly has unusual charge conjugation properties. Okun' and Kobzarev have pointed out that we also obtain, in all models so far studied, a strong FC effect with no electromagnetic coupling, of order \( G \alpha \) (strong coupling constant). T. D. Lee has proposed as an example a Lagrangian that all other workers in the field have subsequently used.

\[ L = L_{\text{strong}[SU(3) \text{ conserving}]} + L_{\text{strong}[SU(3) \text{ violating}]} + L_{\text{free X particles}} + G m_X \bar{u} \gamma_\alpha (1 + \gamma_5)(d \cos \theta + s \sin \theta) + \text{h.c.} + \text{i} f \left( \bar{u} \gamma_\alpha \gamma_5 u + \bar{d} \gamma_\alpha \gamma_5 d + \bar{s} \gamma_\alpha \gamma_5 s \right) \]

\[ + \epsilon_{\alpha \beta \gamma} \Phi_{X} X_{\beta} \partial_{\gamma} X_{\alpha}^{\dagger} \]  

(13)

The fourth term in \( L \) contains \( X_{\alpha} \), the field operator for X particles, multiplied into the ordinary weak current formulated in terms of quarks. \( u \) and \( d \) form a quark isotopic doublet; \( s \) is an isosinglet and \( \theta \) is the Cabibbo angle. This fourth term is a schematic way of representing
Is there a small parameter?

The PC effect.

Neutron E1 moment.

Explanation (if needed) of Utah results on \( \mu \) in cosmic rays.

Explanation (if needed) of possible asymmetry of \( \eta \to 3\pi \) (Columbia).

Explanation (if needed) of \( \gamma d \) not obeying detailed balance (Princeton).

<table>
<thead>
<tr>
<th>Type 3(a)</th>
<th>Type 3(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>( f \ 10^{-3} )</td>
</tr>
<tr>
<td>( G_{e^2} )</td>
<td>( G_{e^f} )</td>
</tr>
<tr>
<td>None to order ( G_e ) OK.</td>
<td>Gef, OK, with ( f \ 10^{-3} )</td>
</tr>
<tr>
<td>None</td>
<td>None, with ( f \ 10^{-3} )</td>
</tr>
<tr>
<td>None</td>
<td>None, with ( f \ 10^{-3} )</td>
</tr>
</tbody>
</table>

A comparison of the two types of theory

(Of course, if the factor \( 10^{-3} \) can be explained as an accident of size of matrix element, these comparisons come out differently.)
the semi-weak interaction. The last term in $\mathcal{L}$ is the important one -- it is a hadronic interaction which does not admit of $C$ conservation unless $G$ carries $X$ into $X$ and $X^+$ into $X^+$. $u$ is carried into $u^+ (\text{with a Majorana representation for the gamma matrices}); \ d \to d^+$
and $s \to s^+$. We have $X \to \frac{\pi}{2} X$ and $X^+ \to \frac{\pi}{2} X^+$. Under these conditions, Okun' and Kobzarev have pointed out that there is an FC effect of order $G^6$ not involving electromagnetism. There has been some controversy with the Hawai'i group on this point, but the conclusion seems to be correct that to explain the smallness (10^{-8}G) of the FC effect, we should have $f \sim 10^{-3}$. Many of the statements above, based on $f \sim 1$, are then no longer true.

In conclusion, let us compare theories of type 3(a) and type 3(b), as in the accompanying figure (see overleaf).

(Of course, if the factor 10^{-8} can be explained as an accident of size of matrix element, these comparisons come out differently.)

The main difference in the theories lies in the appearance of a small parameter $f$ in those of type 3(b). This means that there are a great many predictions of order $f$ for processes involving familiar particles, whereas in theories of type 3(a) there are no such large CP violating effects. The chances that either theory is right are probably not very great, however.
DISCUSSION

COESTER Concerning theories of type 3(a), the operator $\mathcal{L}_\mu$ is some local operator which has the properties which you mention. Why do you have to assume that there are any particles associated with it?

GELL-MANN It has to change some conserved quantum numbers. Otherwise, if it didn't, if it just operated on ordinary hadrons and didn't change anything except electric charge, then there would be no reason for the leptonic and non-leptonic weak interactions to be of the same order; the strong interaction would do what the operator $\mathcal{L}_\mu$ does and make a difference of several orders of magnitude between the non-leptonic and leptonic weak interactions. You must conclude that $\mathcal{L}_\mu$ changes not only the electric charge, but also some other property. In that case, there must be particles to take up that property. The operator must carry you from some sector of Hilbert space to some other sector.

COESTER In other words, it's a fundamental dogma that there are no pieces of Hilbert space that are not describing particles.

GELL-MANN What is there in that part of Hilbert space in which there are no particles? What do you have there?

COESTER Possibly nothing observable.

GELL-MANN For example, you can try writing a quark operator like that, assuming there are no real quarks in the world; but that would be a very peculiar operator. If it acts on the vacuum, it carries you to no state of any kind. You're talking about a local operator which, when acting on a vacuum, carries you to no physical state, and the mathematics of that is not understood. All I can say is that I don't know anything about such operators. I don't know what they look like. Don't you have to have a certain spectrum of Fourier variables which describe the energy and momentum of the created states?

FRONSDAL I wonder if you could tell me, with respect to theories of the second type, what are the predicted effects? Are there any you could hope to measure at all apart from the Fitch-Cronin effect?
GELL-MANN: I think so, with some effort.
FRONSDAL: Which ones?
GELL-MANN: It depends entirely on the detailed theory. But some of them have predictions of leptonic decays of strange particles with neutral lepton pairs. Some of them have predictions of small CP-violations in weak processes, almost any weak processes, of around one percent; but there will be discussions of such theories in the next few days. Many of them have positive predictions which can be tested over a period of years.
OAKES: Could you clarify the electromagnetic interactions in this first kind of theory that could or could not be an intermediate boson theory? You said the strange particles couldn't decay at all, not even in order $G e^2$. Is that true?
GELL-MANN: I was considering a class of theories of type 3(a) in which the X particles or something like them have strong interactions of their own. Those are the source of CP-violation and there is big CP-violation. In order to get rid of the neutron electric dipole moment in order G, we had to introduce conserved quantum numbers $L$ and $L^+$ but, in doing so, we ran into an interesting problem which is, what is the algebra generated by $L$, $L^+$ and this number $n_x$, which $L$ and $L^+$ change by one unit? SU(2) was not that algebra because it ended up forbidding what we had set out to explain. SU(3) would work and some higher algebras would work, but in the ones I investigated, including SU(3), none of the interactions we had written down led to a change in certain quantum numbers, for example, this $Y'$. $Y'$ is not altered by any interaction that we have written down, not the semi-weak, not the electromagnetic, and not the strong. Therefore, we have not written down anything that would make a particle with $Y'$ different from zero decay. Either they are perfectly stable particles that we have never heard of, or, what is more likely, if such a theory were to be right, we would have to introduce an extra term of some kind into the interaction to make them decay. Another possibility, which I have not completely excluded, is that if we go to a complicated enough algebra, we can get one such that the operators, chosen in some funny skew manner
from among the generators, actually generate the whole algebra. I don't know if such a thing exists.

OAKES The X particles themselves couple weakly to ordinary particles.

GELL-MANN Semi-weakly.

OAKES Can't their strange counterparts couple semi-weakly to ordinary particles?

GELL-MANN They could, yes. That would be an extra interaction.

OAKES Then they would decay.

GELL-MANN If you simply add an interaction to the current $\mu$, then you are just performing a rotation in the space of generators; you're just performing a rotation like the one that we perform in the ordinary SU(3) space. Of course, this doesn't change the problem. It's redefining operators and we're off again.

OAKES So there's some conserved quantum number in any event.

TELLER Would you be prepared to say that under some circumstances CP-violation may become the dominant process?

GELL-MANN I don't know.

TELLER In those parts of physics that are understood, do you have any effects which are small and always remain small? For example, relativistic deviations from classical mechanics are sometimes small, but we know exactly where to go to make them large.

GELL-MANN Sometimes you have to go to great lengths to make these effects large. For example, in gravitation, you have to go to big objects to make a large effect. You have a feeling that there must be some domain of science in which this effect becomes very large.

TELLER That is correct.

GELL-MANN I think it is not unlikely this is true. I think it would be fun if the reverse happened and if there were something so small that the experimentalists couldn't measure anything but the Fitch-Cronin effect for generations. But I did keep emphasizing that I was talking about smallness at moderate energies. None of us know what will happen at very high energies.