A VECTOR-SPINOR BOOTSTRAP AS AN IDENTITY *

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It has been shown 1-3) in the first non-trivial approximation that the Lagrangian field theory of spin 1/2 fermions and neutral vector mesons interacting through a conserved current has the apparently miraculous property that the "elementary" spinor particle lies on a Regge trajectory.

We would like to point out here that this feature of the spinor-vector field theory, if it is in fact exactly true, also has interesting implications when phrased in terms of "S-matrix theory", in which it is assumed that there are no elementary objects and that all particles are created by a "bootstrap mechanism".

We are instructed by the bootstrap theory to believe that the spinor-vector scattering amplitudes, as a function of \( l = J - \frac{1}{2} \), where \( J \) is the angular momentum, have no \( \delta_{l0} \) terms but contain only Regge poles, and possibly also Regge cuts in the \( l \) plane. The scattering amplitudes, for the three channels with parity \((-)^l\), may thus be written as

\[
F^{l+} = \frac{1}{l - \alpha} \left( \begin{array}{ccc} \beta_{11} & \beta_{12} & \sqrt{l} \beta_{13} \\ \beta_{12} & \beta_{22} & \sqrt{l} \beta_{23} \\ \sqrt{l} \beta_{13} & \sqrt{l} \beta_{23} & \beta_{33} \end{array} \right) + \ldots, \quad (1)
\]

where the channel labelled 3 is the "nonsense" channel at \( l = 0 \) and the \( \sqrt{l} \) factors in the sense-nonsense amplitudes have been written explicitly. The +... refers to the presence of additional Regge poles, in form similar to the one explicitly indicated, and possibly also to Regge cuts.

The factorisation property of the residues of the Regge poles tells us that

\[
\beta_{ij} = \alpha \left( \beta_{i3} \beta_{j3} / \beta_{33} \right) \quad (i, j = 1, 2). \quad (2)
\]

Notice first that if \( \beta_{33} \) vanishes at some energy and the \( \beta_{ij} \) do not, then unless \( \alpha \) vanishes also, the \( \beta_{ij}(i, j = 1, 2) \) would all have poles at that energy, resulting in poles in \( F^{l+} \) at the same energy for all \( l \). Since singularities at a fixed energy for all \( l \) in \( F^{l} \) are presumably not to be tolerated by "S-matrix theory", we are led to the requirement that \( \alpha \) must vanish when \( \beta_{33} \) vanishes. We therefore get an actual particle with \( l = 0 \) \((J = \frac{1}{2})\) occurring at any energy where \( \beta_{33} = 0 \) and \( \beta_{ij} \neq 0 \), and conversely.

We now consider a very simple approximation for \( \beta \) and \( \alpha \), which we first illustrate in terms of a one-channel problem involving two spinless particles. Suppose the force between these particles is provided by the exchange of a single spinless particle with coupling constant \( g^2 \). We expand the \( \beta \)'s and \( \alpha \)'s in powers of \( g^2 \). The leading trajectory will have the form

\[
\beta = g^2 f(s) + \ldots, \quad \alpha = -1 + g^2 \frac{\pi}{s} \int \frac{d s'}{s - s'} k^l f(s') + \ldots
\]

A crude but recognizable version of a dynamical calculation (in particular, a bootstrap calculation) of a spinless bound state consists in setting this approximate \( \alpha \) equal to zero at the mass of the state. If we are concerned with a simple bootstrap, we require this mass to be the mass of the exchanged particle and require the residue of the resulting pole to be \( g^2 \). Both \( g^2 \) and the mass ratio are determined, even in this approximation.

Now imagine that we approach in the same spirit a model bootstrap calculation of a spinor particle (say a nucleon) as a bound state of itself and a neutral vector meson, with the force provided just by the exchange of the same nucleon with coupling constant \( \gamma^2 \) and with no anomalous magnetic term. In the presence of nucleon conservation, the identification of the nucleon-exchange term with a single diagram is subject to some ambiguities of gauge. A clear-cut prescription for the input force in Born approximation is to calculate the contribution of the sum of both Feynman diagrams, in any gauge, to the partial wave amplitudes and then to omit all terms in \( \delta_{l0} \), which are precisely the ones that contain a pole in energy at the mass of the nucleon in the sense-sense amplitudes. In the Lorentz gauge,
our prescription reduces to taking the nucleon exchange diagram only.

We now examine trajectories in this model to low order in $\gamma^2$. For the leading trajectory, $\alpha$ approaches 0 instead of -1 as $\gamma^2 \to 0$ and $\beta_{33}$, at least in order $\gamma^2$, vanishes when the total energy $W$ equals the nucleon mass $m$, for any value of $m$, while $\beta_{32}$ and $\beta_{31}$ are not zero at $W = m$. We then have, to order $\gamma^2$, the result that $\alpha = 0$ at $W = m$, and a spin $\frac{1}{2}$ particle of mass $m$ thus appears on the leading trajectory. Moreover, the poles in energy corresponding to this particle in the sense-sense amplitudes have just the right residues to reproduce the Born approximation nucleon pole terms that we discarded from the input. These results are mathematically equivalent to the work on Langrangian field theory 3) *.

Thus the same approximation that gave a crude bootstrap (determining mass ratio and coupling constant) in our spinless model gives an identity in the spinor-vector model **.

This result may indicate that the equations used in the bootstrap theory permit solutions in which few particles occur (such as a fermion and a neutral vector boson), and the mass ratios and coupling constants are not determined in a simple way by the usual bootstrap conditions. In fact, the couplings could be rather weak. Conceivably, then, something like the conventional Lagrangian field theory description of lepton electrodynamics *** is consistent with the mathematics of the bootstrap approach.

In the application of the bootstrap equations to the case for which they are intended, namely a description of the strongly interacting particles, there is no reason to take seriously a problem with just a vector meson and a nucleon and with just nucleon exchange. When other channels and other exchanges are included, we need no longer expect $\beta_{33}$ to vanish identically in $m$ at $W = m$, and thus we have no indication that the bootstrap will turn into an identity for the general problem of strong interactions.

References

* It should be acknowledged, of course, that it is not yet known in the Langrangian field theory whether $\beta_{33}$ vanishes identically at $W = m$ in higher order in $\gamma^2$. Correspondingly, in the spinor-vector bootstrap model, the vanishing of our $\beta_{33}$ at $W = m$ could be just an accidental feature of the approximation.

** It will be interesting to see if gravitation theory, for example, also exhibits the miracle of turning elementary particles into Regge poles.

*** There could, in principle, be other distinctions between a true bootstrap solution and the type of solution, resembling electrodynamics, the existence of which we have conjectured. For example, the Levinson criterion might indicate the presence of an "elementary particle" in the latter case but not in the former.

HIGHER SYMMETRIES OF STRONG INTERACTIONS AND ASYMPTOTIC RELATIONS BETWEEN MESON-BARYON SCATTERING CROSS SECTIONS

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After the work on the classification schemes of elementary particles by Gell-Mann and Nishijima, where the isospin invariance and hypercharge conservation hold, there were many attempts at building up schemes of strong interaction with higher symmetries. In particular, the unitary symmetry (the SU3 group) in the triplet model of Sakata 1-3) and in the octet model of Gell-Mann 4) and Ne'eman 5) and the symmetry group G2 in the seven-dimensional charge space of Behrends and Sirlin 6) have been discussed recently. The possible experimental tests of these symmetries have already been discussed in a series of papers. In particular, some relations between the cross sections of the meson-baryon and the baryon-baryon scattering processes in these models have been obtained in refs. 7-11.