Quantum Mechanics in the Light of Quantum Cosmology

Murray Gell-Mann and James B. Hartle

California Institute of Technology, Pasadena, CA 91125, USA
Department of Physics, University of California, Santa Barbara, CA 93106, USA

We sketch a quantum-mechanical framework for the universe as a whole. Within that framework we propose a program for describing the ultimate origin in quantum cosmology of the "quasiclassical domain" of familiar experience and for characterizing the process of measurement. Predictions in quantum mechanics are made from probabilities for sets of alternative histories. Probabilities (approximately obeying the rules of probability theory) can be assigned only to sets of histories that approximately decohere. Decoherence is defined and the mechanism of decoherence is reviewed. Decoherence requires a sufficiently coarse-grained description of alternative histories of the universe. A quasiclassical domain consists of a branching set of alternative decohering histories, described by a coarse graining that is, in an appropriate sense, maximally refined consistent with decoherence, with individual branches that exhibit a high level of classical correlation in time. We pose the problem of making these notions precise and quantitative. A quasiclassical domain is emergent in the universe as a consequence of the initial condition and the action function of the elementary particles. It is an important question whether all the quasiclassical domains are roughly equivalent or whether there are various essentially inequivalent ones. A measurement is a correlation with variables in a quasiclassical domain. An "observer" (or information gathering and utilizing system) is a complex adaptive system that has evolved to exploit the relative predictability of a quasiclassical domain, or rather a set of such domains among which it cannot discriminate because of its own very coarse graining. We suggest that resolution of many of the problems of interpretation presented by quantum mechanics is to be accomplished, not by further scrutiny of the subject as it applies to reproducible laboratory situations, but rather by an examination of alternative histories of the universe, stemming from its initial condition, and a study of the problem of quasiclassical domains.

§1. Quantum Cosmology

If quantum mechanics is the underlying framework of the laws of physics, then there must be a description of the universe as a whole and everything in it in quantum-mechanical terms. In such a description, three forms of information are needed to make predictions about the universe. These are the action function of the elementary particles, the initial quantum state of the universe, and, since quantum mechanics is an inherently probabilistic theory, the information available about our specific history. These are sufficient for every prediction in science, and there are no predictions that do not, at a fundamental level, involve all three forms of information.

A unified theory of the dynamics of the basic fields has long been a goal of elementary particle physics and may now be within reach.

The equally fundamental, equally necessary search for a theory of the initial state of the universe is the objective of the discipline of quantum cosmology. These may even be related goals; a single action function may describe both the Hamiltonian and the initial state.\(^a\)

There has recently been much promising progress in the search for a theory of the quantum initial condition of the universe.\(^a\) Such diverse observations as the large scale homogeneity and isotropy of the universe, its approximate spatial flatness, the spectrum of density fluctuations from which the galaxies grew, the thermodynamic arrow of time, and the existence of classical spacetime may find a

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\(^a\) As in the "no boundary" and the "tunneling from nothing proposals" where the wave function of the universe is constructed from the action by a Euclidean functional integral in the first case or by boundary conditions on the implied Wheeler-DeWitt equation in the second. See, e.g., refs. 1 and 2.
unified, compressed explanation in a particular simple law of the initial condition.

The regularities exploited by the environmental sciences such as astronomy, geology, and biology must ultimately be traceable to the simplicity of the initial condition. Those regularities concern specific individual objects and not just reproduceable situations involving identical particles, atoms, etc. The fact that the discovery of a bird in the forest or a fossil in a cliff or a coin in a ruin implies the likelihood of discovering another similar bird or fossil or coin cannot be derivable from the laws of elementary particle physics alone; it must involve correlations that stem from the initial condition.

The environmental sciences are not only strongly affected by the initial condition but are also heavily dependent on the outcomes of quantum-probabilistic events during the history of the universe. The statistical results of, say, proton-proton scattering in the laboratory are much less dependent on such outcomes. However, during the last few years there has been increasing speculation that, even in a unified fundamental theory, free of dimensionless parameters, some of the observable characteristics of the elementary particle system may be quantum-probabilistic, with a probability distribution that can depend on the initial condition.⁹

It is not our purpose in this article to review all these developments in quantum cosmology.⁵ Rather, we will discuss the implications of quantum cosmology for one of the subjects of this conference—the interpretation of quantum mechanics.

§2. Probability

Even apart from quantum mechanics, there is no certainty in this world; therefore physics deals in probabilities. In classical physics probabilities result from ignorance; in quantum mechanics they are fundamental as well. In the last analysis, even when treating ensembles statistically, we are concerned with the probabilities of particular events. We then deal with the probabilities of deviations from the expected behavior of the ensemble caused by fluctuations.

When the probabilities of particular events are sufficiently close to 0 or 1, we make a definite prediction. The criterion for “sufficiently close to 0 or 1” depends on the use to which the probabilities are put. Consider, for example, a prediction on the basis of present astronomical observations that the sun will come up tomorrow at 5:59 AM ± 1 min. Of course, there is no certainty that the sun will come up at this time. There might have been a significant error in the astronomical observations or the subsequent calculations using them; there might be a non-classical fluctuation in the earth’s rotation rate or there might be a collision with a neutron star now racing across the galaxy at near light speed. The prediction is the same as estimating the probabilities of these alternatives as low. How low do they have to be before one sleeps peacefully tonight rather than anxiously awaiting the dawn? The probabilities predicted by the laws of physics and the statistics of errors are generally agreed to be low enough!

All predictions in science are, most honestly and most generally, the probabilistic predictions of the time histories of particular events in the universe. In cosmology we are necessarily concerned with probabilities for the single system that is the universe as a whole. Where the universe presents us effectively with an ensemble of identical subsystems, as in experimental situations common in physics and chemistry, the probabilities for the ensemble as a whole yield definite predictions for the statistics of identical observations. Thus, statistical probabilities can be derived, in appropriate situations, from probabilities for the universe as a whole.⁹

Probabilities for histories need be assigned by physical theory only to the accuracy to which they are used. Thus, it is the same to us for all practical purposes whether physics claims the probability of the sun not coming up tomorrow is $10^{-15}$ or $10^{-14}$, as long as it is very small. We can therefore conveniently consider approximate probabilities, which need obey the rules of the probability calculus only up to some standard of accuracy sufficient for all practical purposes. In quantum mechanics, as we shall see, it is likely that only by this means can probabilities be assigned to interesting histories at all.

§3. Historical

In quantum mechanics assigned a more clearly illusory event (Fig. we have not me passed through the screen, then probabilities to would be incorrect probability satisfied. Becauseity to arrive at probabilities to arrive per and the low probability $p(y)$,

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It is a general that one needs histories can b
The two-slit experiment. An electron gun at left emits an electron traveling towards a screen with two slits, its progress in space recapitulating its evolution in time. When precise detections are made of an ensemble of such electrons at the screen, it is not possible, because of interference, to assign a probability to the alternatives of whether an individual electron went through the upper slit or the lower slit. However, if the electron interacts with apparatus which measures which slit it passed through, then these alternatives decohere and probabilities can be assigned.

§3. Historical Remarks

In quantum mechanics not every history can be assigned a probability. Nowhere is this more clearly illustrated than in the two-slit experiment (Fig. 1). In the usual discussion, if we have not measured which slit the electron passed through on its way to being detected at the screen, then we are not permitted to assign probabilities to these alternative histories. It would be inconsistent to do so since the correct probability sum rules would not be satisfied. Because of interference, the probability to arrive at y is not the sum of the probabilities to arrive at y going through the upper and the lower slit:

\[ p(y) \neq p_U(y) + p_L(y) \quad (1) \]

because

\[ |\psi_U(y) + \psi_L(y)|^2 \neq |\psi_U(y)|^2 + |\psi_L(y)|^2. \quad (2) \]

If we have measured which slit the electron went through, then the interference is destroyed, the sum rule obeyed, and we can meaningfully assign probabilities to these alternative histories.

It is a general feature of quantum mechanics that one needs a rule to determine which histories can be assigned probabilities. The familiar rule of the “Copenhagen” interpretations described above is external to the framework of wave function and Schrödinger equation. Characteristically these interpretations, in one way or another, assumed as fundamental the existence of the classical domain we see all about us. Bohr spoke of phenomena that could be described in terms of classical language. Landau and Lifshitz formulated quantum mechanics in terms of a separate classical physics. Heisenberg and others stressed the central role of an external, essentially classical observer. A measurement occurred through contact with this classical domain. Measurements determined what could be spoken about.

Such interpretations are inadequate for cosmology. In a theory of the whole thing there can be no fundamental division into observer and observed. Measurements and observers cannot be fundamental notions in a theory that seeks to discuss the early universe when neither existed. There is no reason in general for a classical domain to be fundamental or external in a basic formulation of quantum mechanics.

It was Everett who in 1957 first suggested how to generalize the Copenhagen framework so as to apply quantum mechanics to cosmology. His idea was to take quantum mechanics seriously and apply it to the universe as a whole. He showed how an observer could be considered part of this system and how its activities—measuring, recording, and calculating probabilities—could be described in quantum mechanics.

Yet the Everett analysis was not complete. It did not adequately explain the origin of the classical domain or the meaning of the “branching” that replaced the notion of measurement. It was a theory of “many worlds,” (what we would rather call ‘many histories’), but it did not sufficiently explain how these were defined or how they arose. Also, Everett’s discussion suggests that a probability formula is somehow not needed in quantum
mechanics, even though a “measure” is introduced that, in the end, amounts to the same thing.

Here we shall briefly sketch a program aiming at a coherent formulation of quantum mechanics for science as a whole, including cosmology as well as the environmental sciences. It is an attempt at extension, clarification, and completion of the Everett interpretation. It builds on many aspects of the post-Everett developments, especially the work of Zeh, Žuček, and Joos and Zeh. In the discussion of history and at other points it is consistent with the insightful work (independent of ours) of Griffiths and Omnès. Our research is not complete, but we sketch, in this report on its status, how it might become so.

§4. Decoherent Sets of Histories
(a) A Caveat
We shall now describe the rules that specify which histories may be assigned probabilities and what these probabilities are. To keep the discussion manageable we make one important simplifying approximation. We neglect gross quantum variations in the structure of spacetime. This approximation, excellent for times later than $10^{-42}$ sec after the beginning, permits us to use any of the familiar formulations of quantum mechanics with a preferred time. Since histories are our concern, we shall often use Feynman’s sum-over-histories formulation of quantum mechanics with histories specified as functions of this time. Since the Hamiltonian formulation of quantum mechanics is in some ways more flexible, we shall use it also, with its apparatus of Hilbert space, states, Hamiltonian, and other operators. We shall indicate the equivalence between the two, always possible in this approximation.

The approximation of a fixed background spacetime breaks down in the early universe. There, a yet more fundamental sum-over-histories framework of quantum mechanics may be necessary. In such a framework the notions of state, operators, and Hamiltonian may be approximate features appropriate to the universe after the Planck era, for particular initial conditions that imply an approximately fixed background spacetime there. A discussion of quantum spacetime is essential for any detailed theory of the initial condition, but when, as here, this condition is not spelled out in detail and we are treating events after the Planck era, the familiar formulation of quantum mechanics is an adequate approximation.

The interpretation of quantum mechanics that we shall describe in connection with cosmology can, of course, also apply to any strictly closed sub-system of the universe provided its initial density matrix is known. However, strictly closed sub-systems of any size are not easily realized in the universe. Even slight interactions, such as those of a planet with the cosmic background radiation, can be important for the quantum mechanics of a system, as we shall see. Further, it would be extraordinarily difficult to prepare precisely the initial density matrix of any sizable system so as to get rid of the dependence on the density matrix of the universe. In fact, even those large systems that are approximately isolated today inherit many important features of their effective density matrix from the initial condition of the universe.

(b) Histories
The three forms of information necessary for prediction in quantum cosmology are represented in the Heisenberg picture as follows: The quantum state of the universe is described by a density matrix $\rho$. Observables describing specific information are represented by operators $\mathcal{O}(t)$. For simplicity, but without loss of generality, we shall focus on non-“fuzzy”, “yes-no” observables. These are represented in the Heisenberg picture by projection operators $P(t)$. The Hamiltonian, which is the remaining form of information, describes evolution by relating the operators corresponding to the same question at different times through

$$P(t) = e^{iHt/\hbar}P(0)e^{-iHt/\hbar}.$$  

An exhaustive set of “yes-no” alternatives at one time is represented in the Heisenberg picture by sets of projection operators $(P_1(t), P_2(t), \cdots)$. In $P_\alpha(t)$, $\alpha$ labels the set, $\alpha$ the particular alternative, and $t$ its time. An exhaustive set of exclusive alternatives satisfies

$$\sum_\alpha P_\alpha(t) = 1.$$  

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\[ \sum_a P_a^\alpha(t) = 1, \quad P_a^\alpha(t) P_b^\beta(t) = \delta_{ab} P_a^\alpha(t). \quad (4) \]

For example, one such exhaustive set would specify whether a field at a point on a surface of constant t is in one or another of a set of ranges exhausting all possible values. The projections are simply the projections onto eigenstates of the field at that point with values in these ranges. We should emphasize that an exhaustive set of projections need not involve a complete set of variables for the universe (one-dimensional projections)—in fact, the projections we deal with as observers of the universe typically involve only an infinitesimal fraction of a complete set.

Sets of alternative histories consist of time sequences of exhaustive sets of alternatives. A history is a particular sequence of alternatives, abbreviated \([P_a= (P^a_1(t_1), P^a_2(t_2), \ldots, P^a_n(t_n))\]. A completely fine-grained history is specified by giving the values of a complete set of operators at all times. One history is a coarse graining of another if the set \([P_a]\) of the first history consists of sums of the \([P_a]\) of the second history. The inverse relation is fine graining. The completely coarse-grained history is one with no projections whatever, just the unit operator!

The reciprocal relationships of coarse and fine graining evidently constitute only a partial ordering of sets of alternative histories. The arbitrary sets need not be related to each other by coarse/fine graining. The partial ordering is represented schematically in Fig. 2, where each point stands for a set of alternative histories.

Feynman’s sum-over-histories formulation of quantum mechanics begins by specifying the amplitude for a completely fine-grained history in a particular basis of generalized coordinates \(Q(t)\), say all fundamental field variables at all points in space. This amplitude is proportional to

\[ \exp \left( iS[Q(t)]/\hbar \right), \quad (5) \]

where \(S\) is the action functional that yields the hamiltonian, \(H\). When we employ this formulation of quantum mechanics, we shall introduce the simplification of ignoring fields with spins higher than zero, so as to avoid the complications of gauge groups and of fermion fields (for which it is inappropriate to discuss

Fig. 2. The schematic structure of the space of sets of possible histories for the universe. Each dot in this diagram represents an exhaustive set of alternative histories for the universe. (This is not a picture of the branches defined by a given set!) Such sets, denoted by \([\{P_a\}]\) in the text, correspond in the Heisenberg picture to time sequences \((P^a_1(t_1), P^a_2(t_2), \ldots, P^a_n(t_n))\) of sets of projection operators, such that at each time \(t_n\), the alternatives \(a_n\) are an orthogonal and exhaustive set of possibilities for the universe. At the bottom of the diagram are the completely fine-grained sets of histories each arising from taking projections onto eigenstates of a complete set of observables for the universe at every time. For example, the set \(\theta\) is the set in which all field variables at all points of space are specified at every time. This set is the starting point for Feynman’s sum-over-histories formulation of quantum mechanics. \(\theta\) might be the completely fine-grained set in which all field momenta are specified at each time, \(\theta\) might be a degenerate set of the kind discussed in §7 in which the same complete set of operators occurs at every time. But there are many other completely fine-grained sets of histories corresponding to all possible combinations of complete sets of observables that can be taken at every time.

The dots above the bottom row are coarse-grained sets of alternative histories. If two dots are connected by a path, the one above is a coarse graining of the one below—that is, the projections in the set above are sums of those in the set below. A line, therefore, corresponds to an operation of coarse graining. At the very top is the degenerate case in which complete sums are taken at every time, yielding no projections at all other than the unit operator! The space of sets of alternative histories is thus partially ordered by the operation of coarse graining.

The heavy dots denote the decoherent sets of alternative histories. Coarse grainings of decoherent sets remain decoherent. Maximal sets, the heavy dots surrounded by circles, are those decohering sets for which there is no finer-grained decoherent set.

eigenstates of the field variables.) The operators \(Q(t)\) are thus various scalar fields at different points of space.
Let us now specialize our discussion of histories to the generalized coordinate bases $Q(t)$ of the Feynman approach. Later we shall discuss the necessary generalization to the case of an arbitrary basis at each time $t$, utilizing quantum-mechanical transformation theory.

Completely fine-grained histories in the coordinate basis cannot be assigned probabilities; only suitable coarse-grained histories can. There are at least three common types of coarse graining: (1) specifying observables not at all times, but only at some times: (2) specifying at any one time not a complete set of observables, but only some of them: (3) specifying for these observables not precise values, but only ranges of values. To illustrate all three, we divide the $Q^i$ into up variables $x^i$ and $X^i$ and consider only sets of ranges $\{\Delta^i_k\}$ of $x^i$ at times $t_k$, $k=1, \ldots, n$. A set of alternatives at any one time consists of ranges $\Delta^i_k$, which exhaust the possible values of $x^i$ as $\alpha$ ranges over all integers. An individual history is specified by particular $\Delta^i_k$'s at the times $t_1, \ldots, t_n$. We write $[\Delta_\alpha]=(\Delta_{\alpha_1}^1, \ldots, \Delta_{\alpha_n}^n)$ for a particular history. A set of alternative histories is obtained by letting $\alpha_1, \ldots, \alpha_n$ range over all values.

Let us use the same notation $[\Delta_\alpha]$ for the most general history that is a coarse graining of the completely fine-grained history in the coordinate basis, specified by ranges of the $Q^i$ at each time, including the possibility of full ranges at certain times, which eliminate those times from consideration.

(c) Decohering Histories

The important theoretical construct for giving the rule that determines whether probabilities may be assigned to a given set of alternative histories, and what these probabilities are, is the decoherence functional $D[\text{history}]$, (history). This is a complex functional on any pair of histories in the set. It is most transparently defined in the sum-over-histories framework for completely fine-grained history segments between an initial time $t_0$ and a final time $t_f$, as follows:

$$D[Q'(t), Q(t)] = \delta(Q' - Q) \exp \left\{ i(S[Q'(t)] - S[Q(t)])/\hbar \right\} \rho(Q', Q).$$

(6)

Here $\rho$ is the initial density matrix of the universe in the $Q'$ representation, $Q'^i$ and $Q_i$ are the initial values of the complete set of variables, and $Q_i'$ and $Q_i$ are the final values. The decoherence functional for coarse-grained histories is obtained from (6) according to the principle of superposition by summing over all that is not specified by the coarse graining. Thus,

$$D([\Delta_\alpha], [\Delta_\beta]) = \int_{[\Delta_\alpha]} \delta Q' \int_{[\Delta_\beta]} \delta Q \delta(Q' - Q) \times e^{i(S[Q'] - S[Q])/\hbar} \rho(Q', Q).$$

(7)

More precisely, the integral is as follows (Fig. 3): It is over all histories $Q'(t)$, $Q(t)$ that begin at $Q'^i$, $Q_i$ respectively, pass through the ranges $[\Delta_\alpha]$ and $[\Delta_\beta]$ respectively, and wind up at a common point $Q_f$ at any time $t_f > t_i$. It is completed by integrating over $Q'^i$, $Q_i$, and $Q_f$. The connection between coarse-grained histories and completely fine-grained ones is transparent in the sum-over-histories formulation of quantum mechanics. However, the sum-over-histories formulation does not allow us to consider directly histories of the most general type. For the most general histories one needs to exploit directly the transformation theory of quantum mechanics and for this the Heisenberg picture is convenient. In the Heisenberg picture $D$ can be written

$$D([P_\alpha], [P_\beta]) = \text{Tr} \left\{ P_{\alpha_1}^{n}(t_n) \cdots P_{\alpha_1}(t_1) \rho P_{\beta_1}(t_1) \cdots P_{\beta_n}(t_n) \right\}.$$  

(8)

The projections of the expectation $D$ for every set of coarse-grained histories at $t_0$ is diagonal. (We assume $D$ bounded on every range of $P$.) The coarse-grained histories are specified by $Q'$, $Q_i$ and $Q_f$ at the beginning, end, and some intermediate times $t_i$.

Progressive construction of coarse-grained histories begins at the initial history $[\Delta_\alpha]$ and uses the serial connection of $[\Delta_\alpha]$, $[\Delta_\beta]$, $[\Delta_\gamma]$, $[\Delta_\delta]$, $[\Delta\varepsilon]$ of the $Q^i$ to form a history. The projection operator $D([P_\alpha], [P_\beta])$ is taken as the expectation of $P_\alpha$ on the operator $P_\beta$.

The projections the earliest on the right side of each set of coarse-grained histories are specified at $t_0$ and $t_f$.

In the most general case, the Heisenberg picture is more convenient than the coarse-grained histories. All histories are specified by the various variables at the initial time $t_0$. The coarse-grained histories are specified by $Q'^i$, $Q_i$, and $Q_f$ at the beginning, end, and some intermediate times $t_i$.
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Let us use the same notation \([A_{\alpha}^j]\) for the most general history that is a coarse graining of the completely fine-grained history in the coordinate basis, specified by ranges of the \( Q^j \) at each time, including the possibility of full ranges at certain times, which eliminate those times from consideration.

### Decohering Histories

The important theoretical construct for giving the rule that determines whether probabilities may be assigned to a given set of alternative histories, and what these probabilities are, is the decoherence functional \( D[\text{history}'(t), \text{history}'] \). This is a complex functional on any pair of histories in the set. It is most transparently defined in the sum-over-histories framework for completely fine-grained history segments between an initial time \( t_0 \) and a final time \( t_f \), as follows:

\[
D^\prime[Q'(t), Q(t)] = \delta(Q' - Q) \exp \{i(S[Q'(t)] - S[Q(t)])/\hbar\} \rho(Q', Q)
\]

Here \( \rho \) is the initial density matrix of the universe in the \( Q' \) representation, \( Q' \) and \( Q \) are the initial values of the complete set of variables, and \( Q' \) and \( Q \) are the final values. The decoherence functional for coarse-grained histories is obtained from (6) according to the principle of superposition by summing over all that is not specified by the coarse graining. Thus,

\[
\begin{align*}
D([A_{\alpha}], [A_{\beta}]) &= \int [A_{\alpha}] \int [A_{\beta}] \delta(Q' - Q) \\
& \times \exp \{i(S[Q'] - S[Q])/\hbar\} \rho(Q', Q) \\
& \int [A_{\alpha}] \int [A_{\beta}]
\end{align*}
\]

(7)

More precisely, the integral is as follows (Fig. 3): It is over all histories \( Q'(t) \), \( Q(t) \) that begin at \( Q', Q \) respectively, pass through the ranges \( [A_{\alpha}] \) and \( [A_{\beta}] \) respectively, and wind up at a common point \( Q' \) at any time \( t_f > t_0 \). It is completed by integrating over \( Q', Q \), and \( Q \).

The connection between coarse-grained histories and completely fine-grained ones is transparent in the sum-over-histories formulation of quantum mechanics. However, the sum-over-histories formulation does not allow us to consider directly histories of the most general type. For the most general histories one needs to exploit directly the transformation theory of quantum mechanics and for this the Heisenberg picture is convenient. In the Heisenberg picture \( D \) can be written

\[
D([P_{\alpha}], [P_{\beta}]) = \text{Tr} [P^i_{\alpha}(t_2) \cdots P^i_{\alpha}(t_1) \rho P^i_{\alpha}(t_1) \cdots P^i_{\alpha}(t_2)],
\]

(8)

In the most general case from the coarse-grained unitary All histories cancelling the various coarse grain wheels taking a cussed earlier. A general case where

\[
\sum_{\text{all } \rho} \rho = D\]

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The projections in (8) are time ordered with the earliest on the inside. When the \( P \)’s are projections onto ranges \( \Delta_0 \) of values of the \( Q \)’s, expressions (7) and (8) agree. From the cyclic property of the trace it follows that \( D \) is always diagonal in the final indices \( \alpha_0 \) and \( \alpha_0’ \). (We assume throughout that the \( P \)’s are bounded operators in Hilbert space dealing, for example, with projections onto ranges of the \( Q \)’s and not onto definite values of the \( Q \)’s).

Decoherence is thus an interesting notion only for strings of \( P \)’s that involve more than one time. Decoherence is automatic for “histories” that consist of alternatives at but one time.

Progressive coarse graining may be seen in the sum-over-histories picture as summing over those parts of the fine-grained histories not specified in the coarse-grained one, according to the principle of superposition. In the Heisenberg picture, eq. (8), the three common forms of coarse graining discussed above can be represented as follows: Summing on both sides of \( D \) over all \( P \)’s at a given time and using (4) eliminates those \( P \)’s completely. Summing over all possibilities for certain variables at one time amounts to factoring the \( P \)’s and eliminating one of the factors by summing over it. Summing over ranges of values of a given variable at a given time corresponds to replacing the \( P \)’s for the partial ranges by one for the total range. Thus, if \( \{ \bar{P}_d \} \) is a coarse graining of the set of histories \( \{ P_d \} \), we write

\[
D(\{ \bar{P}_d \}, \{ P_d \}) = \sum_{\text{all } \bar{P}_d \text{ not fixed by } \bar{P}_d} \sum_{\text{all } P_d \text{ not fixed by } \bar{P}_d} D(\{ P_d \}, \{ P_d \}).
\]  

(9)

In the most general case, we may think of the completely fine-grained limit as obtained from the coordinate representation by arbitrary unitary transformations at all times. All histories can be obtained by coarse-graining the various completely fine-grained ones, and coarse graining in its general form involves taking arbitrary sums of \( P \)’s, as discussed earlier. We may use (9) in the most general case where \( \{ \bar{P}_d \} \) is a coarse graining of \( \{ P_d \} \).

A set of coarse-grained alternative histories is said to decohere when the off-diagonal elements of \( D \) are sufficiently small:

\[
D(\{ P_d \}, \{ P_d \}) = 0, \quad \text{for any } \alpha_0' \neq \alpha_0.
\]  

(10)

This is a generalization of the condition for the absence of interference in the two-slit experiment (approximate equality of the two sides of (2)). It is a sufficient (although not a necessary) condition for the validity of the purely diagonal formula

\[
D(\{ \bar{P}_d \}, \{ P_d \}) \approx \sum_{\text{all } P_d \text{ not fixed by } \bar{P}_d} D(\{ P_d \}, \{ P_d \}).
\]  

(11)

The rule for when probabilities can be assigned to histories of the universe is then this: To the extent that a set of alternative histories decoheres, probabilities can be assigned to its individual members. The probabilities are the diagonal elements of \( D \). Thus,

\[
p(\{ P_d \}) = D(\{ P_d \}, \{ P_d \})
\]  

(12)

when the set decoheres. We will frequently write \( p(\alpha_0, \alpha_0', \cdots, \alpha_1, t_1) \) for these probabilities, suppressing the labels of the sets.

The probabilities defined by (11) obey the rules of probability theory as a consequence of decoherence. The principal requirement is that the probabilities be additive on “disjoint sets of the sample space.” For histories this gives the sum rule

\[
p(\{ \bar{P}_d \}) = \sum_{\text{all } \bar{P}_d \text{ not fixed by } \bar{P}_d} p(\{ P_d \}).
\]  

(13)

These relate the probabilities for a set of histories to the probabilities for all coarse-grained sets that can be constructed from it. For example, the sum rule eliminating all projections at only one time is

\[
\sum_{\alpha_0} p(\alpha_0, t_0, \cdots, \alpha_{k+1}, t_{k+1}, \alpha_{k+2} t_{k+2}, \cdots, \alpha_1 t_1)
\]  

(14)

These rules follow trivially from (11) and (12). The other requirements from probability theory are that the probability of the whole sample space be unity, an easy consequence of (11) when complete coarse graining is performed, and that the probability for an empty set be zero, which means simply that the probabili-
ity of any sequence containing a projection \( P = 0 \) must vanish, as it does.

The \( p([P_a]) \) are approximate probabilities for histories, in the sense of §2, up to the standard set by decoherence. Conversely, if a given standard for the probabilities is required by their use, it can be met by coarse graining until (10) and (13) are satisfied at the requisite level.

Further coarse graining of a decoherent set of alternative histories produces another set of decoherent histories since the probability sum rules continue to be obeyed. That is illustrated in Fig. 2, which makes it clear that in a progression from the trivial completely coarse graining to a completely fine graining, there are sets of histories where further fine graining always results in loss of decoherence. These are the maximal sets of alternative decohering histories.

These rules for probability exhibit another important feature: The operators in (12) are time-ordered. Were they not time-ordered (zig-zags) we could have assigned non-zero probabilities to conflicting alternatives at the same time. The time ordering thus expresses causality in quantum mechanics, a notion that is appropriate here because of the approximation of fixed background spacetime. The time ordering is related as well to the “arrow of time” in quantum mechanics, which we discuss below.

Given this discussion, the fundamental formula of quantum mechanics may be reasonably taken to be

\[
D([P_a], [P_a]) = \delta_{a_1\cdots a_n} \cdot p([P_a]) \tag{15}
\]

for all \([P_a]\) in a set of alternative histories. Vanishing of the off-diagonal elements of \( D \) gives the rule for when probabilities may be consistently assigned. The diagonal elements give their values.

We could have used a weaker condition than (10) as the definition of decoherence, namely the necessary condition for the validity of the sum rules (11) of probability theory:

\[
D([P_a], [P_a]) + D([P_a], [P_a]) = 0 \tag{16}
\]

for any \( \alpha_i \neq \alpha_k \), or equivalently

\[
\text{Re} \{D([P_a], [P_a])\} = 0. \tag{17}
\]

This is the condition used by Griffiths\(^{30}\) as the requirement for “consistent histories.” However, while, as we shall see, it is easy to identify physical situations in which the off-diagonal elements of \( D \) approximately vanish as the result of coarse graining, it is hard to think of a general mechanism that suppresses only their real parts. In the usual analysis of measurement, the off-diagonal parts of \( D \) approximately vanish. We shall, therefore, explore the stronger condition (10) in what follows. That difference should not obscure the fact that in this part of our work we have reproduced what is essentially the approach of Griffiths,\(^{31}\) extended by Omnès.\(^{22}\)

(d) Prediction and Retrodiction

Decoherent sets of histories are what we may discuss in quantum mechanics, for they may be assigned probabilities. Decoherence thus generalizes and replaces the notion of “measurement”, which served this role in the Copenhagen interpretations. Decoherence is a more precise, more objective, more observer-independent idea. For example, if their associated histories decohere, we may assign probabilities to various values of reasonable scale density fluctuations in the early universe whether or not anything like a “measurement” was carried out on them and certainly whether or not there was an “observer” to do it. We shall return to a specific discussion of typical measurement situations in §11.

The joint probabilities \( p(\alpha_n t_n, \ldots, \alpha_k t_1) \) for the individual histories in a decohering set are the raw material for prediction and retrodiction in quantum cosmology. From them, the relevant conditional probabilities may be computed. The conditional probability of, one subset \( \{\alpha_i t_i\} \), given the rest \( \{\alpha_i t_i\} \), is

\[
p(\{\alpha_i t_i\} | \{\alpha_i t_i\}) = \frac{p(\alpha_n t_n, \ldots, \alpha_k t_1)}{p(\{\alpha_i t_i\})}. \tag{18}
\]

For example, the probability for predicting alternatives \( \alpha_{k+1}, \ldots, \alpha_n \), given that the alternatives \( \alpha_1, \ldots, \alpha_k \) have already happened, is

\[
p(\alpha_n t_n, \ldots, \alpha_{k+1} t_{k+1} | \alpha_k t_k, \ldots, \alpha_1 t_1) = \frac{p(\alpha_n t_n, \ldots, \alpha_{k+1} t_{k+1}, \alpha_k t_k, \ldots, \alpha_1 t_1)}{p(\alpha_k t_k, \ldots, \alpha_1 t_1)}. \tag{19}
\]

The probability that \( \alpha_{n-1}, \ldots, \alpha_1 \) happened in the past, given present data summarized by an alternative \( \alpha_e \) at

\[
p(\alpha_{n-1} t_{n-1}, \ldots, t_1 | \alpha_e t_e) = \frac{p(\alpha_{n-1} t_{n-1}, \ldots, \alpha_e t_e)}{p(\alpha_e t_e)}.
\]

Decoherence emerges by (18)–(21) unity when our alternatives, because inspection of the situation in the past, gives rise to an amazement informed. If \( \rho_{eff} \) is defined by

\[
\rho_{eff} = \frac{P^e_{\alpha_n}(t_n) \cdots P^e_{\alpha_1}(t_1)}{\text{Tr} [P^e_{\alpha_n}(t_n) \cdots P^e_{\alpha_1}(t_1)]}
\]

then

\[
p(\alpha_n t_n, \ldots, \alpha_k t_1) = \frac{P(\alpha_n t_n, \ldots, \alpha_k t_1) \cdots P(\alpha_{k+1} t_{k+1}, \alpha_k t_k, \ldots, \alpha_1 t_1)}{P(\alpha_k t_k, \ldots, \alpha_1 t_1)}.
\]

By contrast, the matrix represents which probabilities are connected. As (20) show of both present of the universe.

Prediction an way. Because a trace in (8), any a probability contrast we expect decohere in the data and the histories of the different illustrate, natives in the probability in mechanics. For do decohere, the probabilities fall the sense of §2 that the initial set the interference past positions are the.

These differ from retrodiction are
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probabilities that occurs after new information is obtained. In a sequence of horse races, the joint probability for the winners of eight races is converted, after the winners of the first three are known, into a reassessed probability for the remaining five races by exactly this process. The main thing is that, because of decoherence, the sum rules for probabilities are obeyed; once that is true, reassessment of probabilities is trivial.

The only non-trivial aspect of the situation is the choice of the string of $P$'s in (8) giving a decoherent set of histories.

(e) Branches (Illustrated by a Pure $\rho$)

Decohering sets of alternative histories give a definite meaning to Everett's "branches". For a given such set of histories, the exhaustive set of $P_{\alpha_i}\lambda^{k}$ at each time $t_i$ corresponds to a branching.

To illustrate this even more explicitly, consider an initial density matrix that is a pure state, as in typical proposals for the wave function of the universe:

$$\rho = |\Psi\rangle\langle\Psi|.$$  \hspace{1cm} (23)

The initial state may be decomposed according to the projection operators that define the set of alternative histories

$$|\Psi\rangle = \sum_{\alpha_1,\ldots,\alpha_n} P^n_{\alpha_1}(t_n) \cdots P^1_{\alpha_1}(t_1)|\Psi\rangle = \sum_{\alpha_1,\ldots,\alpha_n} |P_{\alpha_1}, \Psi\rangle.$$  \hspace{1cm} (24)

The states $|P_{\alpha_1}, \Psi\rangle$ are approximately orthogonal as a consequence of their decoherence

$$\langle P_{\alpha'}, \Psi|P_{\alpha}, \Psi\rangle \approx 0, \text{ for any } \alpha' \neq \alpha.$$  \hspace{1cm} (25)

Equation (25) is just a recurrence of (10), given (23).

When the initial density matrix is pure, it is easily seen that some coarse graining in the present is always needed to achieve decoherence in the past. If the $P^n_{\alpha_1}(t_n)$ for the last time $t_n$ in (8) were all projections onto pure states, $D$ would factor for a pure $\rho$ and could never satisfy (10), except for certain special kinds of histories described near the end of §7, in which decoherence is automatic, independent of $\rho$. Similarly, it is not difficult to show that some coarse graining is required at any time in order to have decoherence of previous alternatives, with the same set of exceptions.

After normalization, the states $|P_{\alpha_1}, \Psi\rangle$ represent the individual histories or individual branches in the decohering set. We may, as for the effective density matrix of (d), summarize present information for prediction just by giving one of these states, with projections up to the present.

(f) Sets of Histories with the Same Probabilities

If the projections $P$ are not restricted to a particular class (such as projections onto ranges of $Q_i$ variables), so that coarse-grained histories consist of arbitrary exhaustive families of projections operators, then the problem of exhibiting the decohering sets of strings of projections arising from a given $\rho$ is a purely algebraic one. Assume, for example, that the initial condition is known to be a pure state as in (23). The problem of finding ordered strings of exhaustive sets of projections $|P_{\alpha_1}, \Psi\rangle$ so that the histories $P^n_{\alpha_1} \cdots P^1_{\alpha_1}|\Psi\rangle$ decohere according to (25) is purely algebraic and involves just subspaces of Hilbert space. The problem is the same for one vector $|\Psi\rangle$ as for any other. Indeed, using subspaces that are exactly orthogonal, we may identify sequences that exactly decohere.

However, it is clear that the solution of the mathematical problem of enumerating the sets of decohering histories of a given Hilbert space has no physical content by itself. No description of the histories has been given. No reference has been made to a theory of the fundamental interactions. No distinction has been made between one vector in Hilbert space as a theory of the initial condition and any other. The resulting probabilities, which can be calculated, are merely abstract numbers.

We obtain a description of the sets of alternative histories of the universe when the operators corresponding to the fundamental fields are identified. We make contact with the theory of the fundamental interactions if the evolution of these fields is given by a fundamental hamiltonian. Different initial vectors in Hilbert space will then give rise to decohering sets having different descriptions in terms of the fundamental fields. The probabilities acquire physical meaning.

Two different constructions from $c$ with different abilities. First situations of the $P$'s leave the initial

$$\rho = U[t] P_{i}(t):$$

If $\rho$ is pure then transformations only a single \history management made to identical decoherence constructed from one set decoherence abilities for the others.

In a similar way, abilities are realizations of long as they can projection operator changed as the Hamiltonian remains in the Hamiltonian (or time) is at some quantity.

The histories are transformations of a given set of different descriptions from that considering transformation (or alibi) set and hamiltonian (or alias) transformation, are projections onto spatial region is or by some reassignment of extraordinary coi fields and all mot universe! Histor onto values of time is thus different quantities.

In ordinary mechanics, two situations can correlate situations because
probabilities that occur after new information is obtained. In a sequence of horse races, the joint probability for the winners of eight races is converted, after the winners of the first three are known, into a reassessed probability for the remaining five races by exactly this process. The main thing is that, because of decoherence, the sum rules for probabilities are obeyed; once that is true, reassessment of probabilities is trivial.

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(c) Branches (Illustrated by a Pure \( \rho \))

Decohering sets of alternative histories give a definite meaning to Everett's "branches". For a given such set of histories, the exhaustive set of \( P_a^k \) at each time \( t \) corresponds to a branching.

To illustrate this more explicitly, consider an initial density matrix that is a pure state, as in typical proposals for the wave function of the universe:

\[
\rho = |\Psi\rangle\langle\Psi|.
\]

The initial state may be decomposed according to the projection operators that define the set of alternative histories

\[
|\Psi\rangle = \sum_{a_1 \cdots a_n} P_{a_1}(t_1) \cdots P_{a_n}(t_n) |\Psi\rangle = \sum_{a_1 \cdots a_n} |P_a\rangle, |\Psi\rangle.
\]

The states \( |P_a\rangle, |\Psi\rangle \) are approximately orthogonal as a consequence of their decoherence

\[
\langle P_a | |\Psi\rangle = 0, \text{ for any } \alpha_k \neq \alpha_k.
\]

Equation (25) is just a re-expression of (10), given (23).

When the initial density matrix is pure, it is easily seen that some coarse graining in the present is always needed to achieve decoherence in the past. If the \( P_a^k(t_a) \) for the last time \( t_a \) in (8) were all projections onto pure states, \( D \) would factor for a pure \( \rho \) and could never satisfy (10), except for certain special kinds of histories described near the end of §7, in which decoherence is automatic, independent of \( \rho \). Similarly, it is not difficult to show that some coarse graining is required at any time in order to have decoherence of previous alternatives, with the same set of exceptions.

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Two different constructs from a different alternative descriptions of the \( P_i \) leave the initial

\[
\rho = U \rho^0(t)
\]

If \( \rho \) is pure the transformation only a single histories made of identical decohered constructs from one set of decohered abilities for the same.

In a similar way, abilities are reassignments of the projection operators as the operation changes as one goes because in the Hamiltonian at any time is at any time a some quantity.

The histories transformations of a given set of different fields from that subjecting to transformations (or alibi) and Hamiltonian (or alias) transformed, a projections onto spatial region is or by any reassigned, trauriddoary co fields and all m universe! History onto values of times can thus different quantities.

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the probabilities ac-
quire physical meaning.
Two different simple operations allow us to
construct from one set of histories another set
with a different description but the same prob-
abilities. First consider unitary transfor-
mations of the \(P\)'s that are constant in time and
leave the initial \(\rho\) fixed:
\[
\rho = U \rho U^{-1},
\]
\[
P^\beta_\alpha(t) = U P^\beta_\alpha(t) U^{-1}.
\]
If \(\rho\) is pure there will be very many such
transformations; the Hilbert space is large and
only a single vector is fixed. The sets of
histories made up from the \(\{P^e_\beta\}\) will have an
identical decoherence functional to the sets
constructed from the corresponding \(\{P^e_\alpha\}\). If
one set decoheres, the other will and the proba-
bilities for the individual histories will be the
same.
In a similar way, decoherence and proba-
bilities are invariant under arbitrary
reassignments of the times in a string of \(P\)'s (as
long as they continue to be ordered), with the
projection operators at the altered times un-
changed as operators in Hilbert space. This is
because in the Heisenberg picture every projec-
tion is at any time a projection operator for
some quantity.
The histories arising from constant unitary
transformations or from reassignment of times
of a given set of \(P\)'s will, in general, have very
different descriptions in terms of fundamental
fields from that of the original set. We are con-
sidering transformations such as (27) in an ac-
tive (or alibi) sense so that the field operators
and hamiltonian are unchanged. (The passive
(or alias) transformations, in which these are
transformed, are easily understood.) A set of
projections onto the ranges of field values in a
spatial region is generally transformed by (27)
or by any reassignment of the times into an
extraordinarily complicated combination of all
fields and all momenta at all positions in the
universe! Histories consisting of projections
onto values of similar quantities at different
times can thus become histories of very
different quantities at various other times.
In ordinary presentations of quantum
mechanics, two histories with different descrip-
tions can correspond to physically distinct
situations because it is presumed that various
different hermitian combinations of field
operators are potentially measurable by
different kins of external apparatus. In quan-
tum cosmology, however, apparatus and
system are considered together and the notion
of physically distinct situations may have a
different character.

§5. The Origins of Decoherence

What are the features of coarse-grained sets
of histories that decohere, given the \(\rho\) and \(H\)
of the universe? In seeking to answer this ques-
tion it is important to keep in mind the basic
aspects of the theoretical framework on which
decoherence depends. Decoherence of a set of
alternative histories is not a property of their
operators alone. It depends on the relations of
those operators to the density matrix \(\rho\). Given
\(\rho\), we could, in principle, compute which sets
of alternative histories decohere.

We are not likely to carry out a compu-
tation of all decohering sets of alternative
histories for the universe, described in terms
of the fundamental fields, anytime in the near
future, if ever. However, if we focus attention
on coarse grainings of particular variables, we
can exhibit widely occurring mechanisms by
which they decohere in the presence of the ac-
tual \(\rho\) of the universe. We have mentioned in
§4(c) that decoherence is automatic if the pro-
jection operators \(P\) refer only to one time; the
same would be true even for different times if
all the \(P\)'s commuted with one another. Of
course, in cases of interest, each \(P\) typically
actors into commuting projection operators,
and the factors of \(P\)'s for different times often
fail to commute with one another, for exam-
ple factors that are projections onto related
ranges of values of the same Heisenberg
operator at different times. However, these
non-commuting factors may be correlated,
given \(\rho\), with other projection factors that do
commute or, at least, effectively commute in-
side the trace with the density matrix \(\rho\) in eq
(8) for the decoherence functional. In fact,
these other projection factors may commute
with all the subsequent \(P\)'s and thus allow
themselves to be moved to the outside of the
trace formula. When all the non-commuting
factors are correlated in this manner with effec-
tively commuting ones, then the off-diagonal
terms in the decoherence functional vanish, in
other words, decoherence results. Of course, all this behavior may be approximate, resulting in approximate decoherence.

This type of situation is fundamental in the interpretation of quantum mechanics. Noncommuting quantities, say at different times, may be correlated with commuting or effectively commuting quantities because of the character of $\rho$ and $H$, and thus produce decoherence of strings of $P$'s despite their non-commutation. For a pure $\rho$, for example, the behavior of the effectively commuting variables leads to the orthogonality of the branches of the state $|\Psi\rangle$, as defined in (24).

We shall see that correlations of this character are central to understanding historical records (§10) and measurement situations (§11).

As an example of decoherence produced by this mechanism, consider a coarse-grained set of histories defined by time sequences of alternative approximate localizations of the center of mass of a massive body such as a planet or a typical interstellar dust grain. As shown by Joos and Zeh, even if the successive localizations are spaced as closely as a nanosecond, such histories decohere as a consequence of scattering by the $3^\circ$ cosmic background radiation (if for no other reason). Different positions become correlated with nearly orthogonal states of the photons. More importantly, each alternative sequence of positions becomes correlated with a different orthogonal state of the photons at the final time. This accomplishes the decoherence and we may loosely say that such histories of the position of a massive body are “decohered” by interaction with the photons of the background radiation.

Other specific models of decoherence have been discussed by many authors, among them Joos and Zeh, Caldeira and Leggett, and Żurek. Typically these discussions have focussed on a coarse graining that involves only certain variables analogous to the position variables above. Thus the emphasis is on particular, non-commuting factors of the projection operators and not on correlated operators that may be accomplishing the approximate decoherence. Such coarse grainings do not, in general, yield the most refined approximately decohering sets of histories, since one could include projections onto ranges of values of the correlated operators without losing the decoherence.

The simplest model consists of a single oscillator interacting bilinearly with a large number of others, the coordinates of which are integrated over. Let $x$ be the coordinate of the special oscillator, $M$ its mass, $\omega_F$ its frequency renormalized by its interactions with the others, and $S_{\text{free}}$ its free action. Consider the special case where the density matrix of the whole system, referred to an initial time, factors into the product of a density matrix $\bar{\rho}(x', x)$ of the distinguished oscillator and another for the rest. Then, generalizing slightly a treatment of Feynman and Vernon, we can write $D$ defined by (7) as

$$D(\{x'_a\}, \{x_a\}) = \int_{\{x'_a\}} \delta x'(t) \int_{\{x_a\}} \delta x(t) \delta (x' - x) \times \exp \{i [S_{\text{free}}[x'(t)] - S_{\text{free}}[x(t)] + W[x'(t), x(t)]/\hbar] \bar{\rho}(x', x) \},$$

(28)

the intervals $[x_a]$ referring here only to the variables of the distinguished oscillator. The sum over the rest of the oscillators has been carried out and is summarized by the Feynman–Vernon influence functional $i W(x'(t), x(t))$. The remaining sum over $x'(t)$ and $x(t)$ is as in (7).

The case when the other oscillators are in an initial thermal distribution has been extensively investigated by Caldeira and Leggett. In the simple limit of a uniform continuum of oscillators cut off at frequency $\Omega$ and in the Fokker–Planck limit of $kT \gg \hbar \Omega \gg \hbar \omega_N$, they find

$$W[x'(t), x(t)] = -M\gamma \int dt [x' - x - x'] \times \gamma [x'(t) - x(t)]^2,$$

(29)

where $\gamma$ summarizes the interaction strengths of the distinguished oscillator with its environment. The real part of $W$ contributes dissipation to the equations of motion. The imaginary part squeezes the trajectories $x(t)$ and $x'(t)$ together, thereby providing approximate decoherence. Very roughly, primed and unprimed position intervals differing by distances $d$ on

$$d \geq \frac{1}{\gamma}$$

As stressed macroscopic $x$-for decoherent magnitude or dynamical time (The ratio is $T \sim 300^\circ K$, $d$—

The behavior native histories spaced far en onto ranges c roughly classic on positions $\beta$ but with the $\gamma$ disturbed by vi effect of quan dinate, (b) the of the other statistical fluct (b) when us see that the lar decoherence ti dinate resists classical behav

What the ab is that decoher universe fore variables. The one of us of wh in a quantum situ aions in its orb would rapidly c more detailed c

§6. Quasiclassic

As observers coarse graining limited sensory instruments, con in the end char ignorance. Yet unverse exhibit ing histories, i sort of “classic by classical la adapted while
Operators without losses consists of a single linearly with a large number of which be the coordinate of its mass, \(\omega\), its free Auiki's interactions with the free action. Consider density matrix of the time, \(\rho(x', t)\), oscillator and another straightening slightly a treatment, we can write

\[
\int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi\hbar}} \rho(x', t) \rho(x, t)
\]

As stressed by Žek, for typical macroscopic parameters, this minimum time for decoherence can be many orders of magnitude smaller than a characteristic dynamical time, say the damping time \(1/\gamma\). (The ratio is around \(10^{-45}\) for \(M \sim \text{gm}, T \sim 300^\circ\text{K}, d \sim \text{cm}\).)

The behavior of a coarse-grained set of alternative histories based on projections, at times spaced far enough apart for decoherence, onto ranges of values of \(x\) alone, is then roughly classical in that the successive ranges of positions follow roughly classical orbits, but with the pattern of classical correlation disturbed by various effects, especially (a) the effect of quantum spreading of the \(x\)-coordinate, (b) the effect of quantum fluctuations of the other oscillators, and (c) classical statistical fluctuations, which are lumped with (b) when we use the fundamental formula. We see that the larger the mass \(M\), the shorter the decoherence time and the more the \(x\)-coordinate resists the various challenges to its classical behavior.

What the above models convincingly show is that decoherence will be widespread in the universe for certain important “classical” variables. The answer to Fermi’s question to one of us of why we don’t see Mars spread out in a quantum superposition of different positions in its orbit is that such a superposition would rapidly decohere. We now proceed to a more detailed discussion of such decoherence.

\[\text{§6. Quasiclassical Domains}\]

As observers of the universe, we deal with coarse grainings that are appropriate to our limited sensory perceptions, extended by instruments, communication, and records, but in the end characterized by a great amount of ignorance. Yet we have the impression that the universe exhibits a finer-grained set of decohering histories, independent of us, defining a sort of “classical domain”, governed largely by classical laws, to which our senses are adapted while dealing with only a small part of it. No such coarse graining is determined by pure quantum theory alone. Rather, like decoherence, the existence of a quasiclassical domain in the universe must be a consequence of its initial condition and the Hamiltonian describing its evolution.

Roughly speaking, a quasiclassical domain should be a set of alternative decohering histories, maximally refined consistent with decoherence, with its individual histories exhibiting as much as possible patterns of classical correlation in time. Such histories cannot be exactly correlated in time according to classical laws because sometimes their classical evolution is disturbed by quantum events. There are no classical domains, only quasiclassical ones.

We wish to make the question of the existence of one or more quasiclassical domains into a calculable question in quantum cosmology and for this we need criteria to measure how close a set of histories comes to constituting a “classical domain”. We have not solved this problem to our satisfaction, but, in the next few sections, we discuss some ideas that may contribute toward its solution.

\[\text{§7. Maximal Sets of Decohering Histories}\]

Decoherence results from coarse grainings. As described in §4(b) and Fig. 2, coarse grainings can be put into a partial ordering with another. A set of alternative histories is a coarse graining of a finer set if all the exhaustive sets of projections \(\{P_x\}\) making up the coarser set of histories are obtained by partial sums over the projections making up the finer set of histories.

Maximal sets of alternative decohering histories are those for which there are no finer-grained sets that are decoherent. It is desirable to work with maximal sets of decohering alternative histories because they are not limited by the sensory capacity of any set of observers— they can cover phenomena in all parts of the universe and at all epochs that could be observed, whether or not any observer was present. Maximal sets are the most refined descriptions of the universe that may be assigned probabilities in quantum mechanics.

The class of maximal sets possible for the universe depends, of course, on the completely fine-grained histories that are presented by
the actual quantum theory of the universe. If we utilize to the full, at each moment of time, all the projections permitted by transformation theory, which gives quantum mechanics its protean character, then there is an infinite variety of completely fine-grained sets, as illustrated in Fig. 2. However, were there some fundamental reason to restrict the completely fine-grained sets, as would be the case if sum-over-histories quantum mechanics were fundamental, then the class of maximal sets would be smaller as illustrated in Fig. 4. We shall proceed as if all fine grainings are allowed.

If a full correlation exists between a projection in a coarse graining and another projection not included, then the finer graining including both still defines a decoherent set of histories. In a maximal set of decoherent histories, both correlated projections must be included if either one is included. Thus, in the mechanism of decoherence discussed in §5, projections onto the correlated orthogonal states of the 3°K photons are included in the maximal set of decohering histories along with the positions of the massive bodies. Any projections defining historical records such as we shall describe in §10, or values of measured quantities such as we shall describe in §11, must similarly be included in a maximal set.

More information about the initial ρ and H is contained in the probabilities of a finer-grained set of histories than in those of a coarser-grained set. It would be desirable to have a quantitative measure of how much more information is obtained in a further fine graining of a coarse-grained set of alternative histories. Such a quantity would then measure how much closer a decoherent fine graining comes to maximality in a physically relevant sense.

We shall discuss a quantity that, while not really a measure of maximality is useful in exploring some aspects of it. In order to construct this quantity the usual entropy formula is applied to sets of alternative decohering histories of the universe, rather than, as more usually, alternatives at a single time. We make use of the coarse-grained density matrix ̃ρ defined using the methods of Jaynes,36 but generalized to take account of the density matrix of the universe and applied to the probabilities for histories. The density matrix ̃ρ is constructed by maximizing the entropy functional

\[ S(\tilde{\rho}) = -\text{Tr} (\tilde{\rho} \log \tilde{\rho}) \]  

(31)

over all density matrices ̃ρ that satisfy the constraints ensuring that each

\[ \text{Tr} \left[ P_{a_i}(t_i) \cdots P_{a_j}(t_j) \tilde{\rho} P_{a_i}(t_i) \cdots P_{a_j}(t_j) \right] \]  

(32)

has the same value it would have had when computed with the density matrix of the universe, ρ, for a given set of coarse-grained histories. The density matrix ̃ρ thus reproduces the decoherence functional for this set of histories, and in particular their probabilities, but possesses as little information as possible beyond those properties.

A fine graining of a set of alternative histories leads to more conditions on ̃ρ of the form (32) than in the coarse-grained set. In non-trivial cases ̃S(̃ρ) is, therefore, lowered and ̃ρ becomes closer to ρ.

If the insertion of apparently new P's into a chain is redundant, then ̃S(̃ρ) will not be lowered. A simple example will help to illustrate this: Consider the set of histories consisting of projections P_{\alpha}(t_m) which project onto an orthonormal basis for Hilbert space at one time, t_m. Trivial further decoherent fine grainings can be constructed as follows: At each other time t_k introduce a set of projections P_{\alpha_k}(t_k) that, through the equations of motion, are identical to the set P_{\alpha_k}(t_m) are going through a completely fining all the time the projections. We thus have a histories that, if ̃ρ is exactly because. Indeed, in terms of maximality that at one time. To identify such time to redundancy

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at a single time, these techniques give a unified and generalized treatment of the variety of coarse grainings commonly introduced in statistical mechanics; and, as Jaynes and others have pointed out, the resulting $S(\hat{\rho})$'s are the physical entropies of statistical mechanics. Here, however, these techniques are applied to time histories and the initial condition is utilized. The quantity $S(\hat{\rho})$ is also related to the notion of thermodynamic depth currently being investigated by Lloyd.\textsuperscript{30}

§ 8. Classicality

Some maximal sets will be more nearly classical than others. The more nearly classical sets of histories will contain projections (onto related ranges of values) of operators, for different times, that are connected to one another by the unitary transformations $e^{-iHt_{1}}$ and that are correlated for the most part along classical paths, with probabilities near zero and one for the successive projections. This pattern of classical correlation may be disturbed by the inclusion, in the maximal set of projection operators, of other variables, which do not behave in this way (as in measurement situations to be described later). The pattern may also be disturbed by quantum spreading and by quantum and classical fluctuations, as described in connection with the oscillator example treated in § 5. Thus we can, at best, deal with quasiclassical maximal sets of alternative decohering histories, with trajectories that split and fan out at a result of the processes that make the decoherence possible. As we stressed earlier, there are no classical domains, only quasiclassical ones.

The impression that there is something like a classical domain suggests that we try to define quasiclassical domains precisely by searching for a measure of classicality for each of the maximal sets of alternative decohering histories and concentrating on the one (or ones) with maximal classicality. Such a measure would be applied to the elements of $D$ and the corresponding coarse grainings. It should favor predictability, involving patterns of classical correlation as described above. It should also favor maximal sets of alternative decohering histories that are relatively fine-grained as opposed to those which had to be carried to very coarse grainings before they would give
decoherence. We are searching for such a measure. It should provide a precise and quantitative meaning to the notion of quasiclassical domain.

§9. Quasiclassical Operators

What are the projection operators that specify the coarse graining of a maximal set of alternative histories with high classicity, which defines a quasiclassical domain? They will include, as mentioned above, projections onto comparable ranges of values of certain operators at sequences of times, obeying roughly classical equations of motion, subject to fluctuations that cause their trajectories to fan out from time to time. We can refer to these operators, which habitually decohere, as "quasiclassical operators". What these quasiclassical operators are, and how many of them there are, depends not only on $H$ and $\rho$, but also on the epoch, on the spatial region, and on previous branchings.

We can understand the origin of at least some quasiclassical operators in reasonably general terms as follows: In the earliest instants of the universe the operators defining spacetime on scales will above the Planck scale emerge from the quantum fog as quasiclassical. Any theory of the initial condition that does not imply this is simply inconsistent with observation in a manifest way. The background spacetime thus defined obeys the Einstein equation. Then, where there are suitable conditions of low temperature, etc., various sorts of hydrodynamic variables may emerge as quasiclassical operators. These are integrals over suitable small volumes of densities of conserved or nearly conserved quantities. Examples are densities of energy, momentum, baryon number, and, in later epochs, nuclei, and even chemical species. The sizes of the volumes are limited above by maximality and are limited below by classicity because they require sufficient "inertia" to enable them to resist deviations from predictability caused by their interactions with one another, by quantum spreading, and by the quantum and statistical fluctuations resulting from interactions with the rest of the universe. Suitable integrals of densities of approximately conserved quantities are thus candidates for habitually decohering quasiclassical operators. Field theory is local, and it is an interesting question whether that locality somehow picks out local densities as the source of habitually decohering quantities. It is hardly necessary to note that such hydrodynamic variables are among the principal variables of classical physics.\textsuperscript{35}

In the case of densities of conserved quantities, the integrals would not change at all if the volumes were infinite. For smaller volumes we expect approximate persistence. When, as in hydrodynamics, the rates of change of the integrals form part of an approximately closed system of equations of motion, the resulting evolution is just as classical as in the case of persistence.

§10. Branch Dependence

As the discussion in §5 and §9 shows, physically interesting mechanisms for decoherence will operate differently in different alternative histories for the universe. For example, hydrodynamic variables defined by a relatively small set of volumes may decohere at certain locations in spacetime in those branches where a gravitationally condensed body (e.g., the earth) actually exists, and may not decohere in other branches where no such condensed body exists at that location. In the latter branch there simply may be not enough "inertia" for densities defined with too small volumes to resist deviations from predictability. Similarly, alternative spin directions associated with Stern-Gerlach beams may decohere for those branches on which a photographic plate detects their beams and not in a branch where they recombine coherently instead. There are no variables that are expected to decohere universally. Even the mechanisms causing spacetime geometry at a given location to decohere on scales far above the Planck length cannot necessarily be expected to operate in the same way on a branch where the location is the center of a black hole as on those branches where there is no black hole nearby.

How is such "branch dependence" described in the formalism we have elaborated? It is not described by considering histories where the set of alternatives at one time (the $\kappa$ in a set of $P_\alpha$) depends on specific alternatives (the $\alpha$'s) of sets of earlier times. Such dependence we the probability formalism, the probability of the records of past events, together with the records of past events, describes the dependence of past events. A record is a high probability event in the relevant probabilistic description of the universe. The subject of discussion is the past, given the conditions at the time.

The branch becomes explicable considered the events in the past, the example a hydrodynamic not depending on the condensates that decohere in the condensate variables. H$\textsuperscript{3}$ smaller volumes with the record place and vice versa.

The branch provides the most plausible position that a branch is defined in terms of the $\kappa$ (e.g., values of the $\kappa$ in the classical branch). Any physical decoherence independent of the branch.

§11. Measure

When a measure of the number of variations of quasiclassical $\kappa$
dependence would destroy the derivation of the probability sum rules from the fundamental formula. However, there is no such obstacle to the set of alternatives at one time depending on the sets of alternatives at all previous times. It is by exploiting this possibility, together with the possibility of present records of past events, that we can correctly describe the sense in which there is branch dependence of decoherence, as we shall now discuss.

A record is a present alternative that is, with high probability, correlated with an alternative in the past. The construction of the relevant probabilities was discussed in §4, including their dependence on the initial condition of the universe (or at least on information that effectively bears on that initial condition). The subject of history is most honestly described as the construction of probabilities for the past, given such records. Even non-commuting alternatives such as a position and its momentum at different, even nearby times may be stored in presently commuting record variables.

The branch dependence of histories becomes explicit when sets of alternatives are considered that include records of specific events in the past. To illustrate this, consider the example above, where different sorts of hydrodynamic variables might decohere or not depending on whether there was a gravitational condensation. The set of alternatives that decohere must refer both to the records of the condensation and to hydrodynamic variables. Hydrodynamic variables with smaller volumes would be part of the subset with the record that the condensation took place and vice versa.

The branch dependence of decoherence provides the most direct argument against the position that a classical domain should simply be defined in terms of a certain set of variables (e.g., values of spacetime averages of the fields in the classical action). There are unlikely to be any physically interesting variables that decohere independent of circumstance.

§11. Measurement Situations

When a correlation exists between the ranges of values of two operators of a quasiclassical domain, there is a measurement situation. From a knowledge of the value of one, the value of the other can be deduced because they are correlated with probability near unity. Any such correlation exists in some branches of the universe and not in others; for example, measurements in a laboratory exist only in those branches where the laboratory was actually constructed!

We use the term “measurement situation” rather than “measurement” for such correlations to stress that nothing as sophisticated as an “observer” need be present for them to exist. If there are many significantly different quasiclassical domains, different measurement situations may be exhibited by each one.

When the correlation we are discussing is between the ranges of values of two quasiclassical operators that habitually decohere, as discussed in §9, we have a measurement situation of a familiar classical kind. However, besides the quasiclassical operators, the highly classical maximal sets of alternative histories of a quasiclassical domain may include other operators having ranges of values strongly correlated with the quasiclassical ones at particular times. Such operators, not normally decohering, are, in fact, included among the decohering set only by virtue of their correlation with a habitually decohering one. In this case we have a measurement situation of the kind usually discussed in quantum mechanics. Suppose, for example, in the inevitable Stern-Gerlach experiment, that $\sigma_z$ of a spin-1/2 particle is correlated with the orbit of an atom in an inhomogeneous magnetic field. If the two orbits decohere because of interaction with something else (the atomic excitations in a photographic plate for example), then the spin direction will be included in the maximal set of decoherent histories, fully correlated with the decohering orbital directions. The spin direction is thus measured.

The recovery of the Copenhagen rule for when probabilities may be assigned is immediate. Measured quantities are correlated with decohering histories. Decohering histories can be assigned probabilities. Thus in the two-slit experiment (Fig. 1), when the electron interacts with an apparatus that determines which slit it passed through, it is the decoherence of the alternative configurations of the apparatus that enables probabilities to
be assigned for the electron.

Correlation between the ranges of values of operators of a quasiclassical domain is the only defining property of a measurement situation. Conventionally, measurements have been characterized in other ways. Essential features have been seen to be irreversibility, amplification beyond a certain level of signal-to-noise, association with a macroscopic variable, the possibility of further association with a long chain of such variables, and the formation of enduring records. Efforts have been made to attach some degree of precision to words like "irreversible", "macroscopic", and "record", and to discuss what level of "amplification" needs to be achieved. While such characterizations of measurement are difficult to define precisely, some can be seen in a rough way to be consequences of the definition that we are attempting to introduce here, as follows:

Correlation of a variable with the quasiclassical domain (actually, inclusion in its set of histories) accomplishes the amplification beyond noise and the association with a macroscopic variable that can be extended to an indefinitely long chain of such variables. The relative predictability of the classical world is a generalized form of record. The approximate constancy of, say, a mark in a notebook is just a special case; persistence in a classical orbit is just as good.

Irreversibility is more subtle. One measure of it is the cost (in energy, money, etc.) of tracking down the phases specifying coherence and restoring them. This is intuitively large in many typical measurement situations. Another, related measure is the negative of the logarithm of the probability of doing so. If the probability of restoring the phases in any particular measurement situation were significant, then we would not have the necessary amount of decoherence. The correlation could not be inside the set of decohering histories. Thus, this measure of irreversibility is large. Indeed, in many circumstances where the phases are carried off to infinity or lost in photons impossible to retrieve, the probability of recovering them is truly zero and the situation perfectly irreversible—ininitely costly to reverse and with zero probability for reversal! Defining a measurement situation solely as the existence of correlations in a quasiclassical domain, if suitable general definitions of maximality and classicity can be found, would have the advantages of clarity, economy, and generality. Measurement situations occur throughout the universe and without the necessary intervention of anything as sophisticated as an "observer". Thus, by this definition, the production of fission tracks in mica deep in the earth by the decay of a uranium nucleus leads to a measurement situation in a quasiclassical domain in which the tracks directions decohere, whether or not these tracks are ever registered by an "observer".

§12. Complex Adaptive Systems

Our picture is of a universe that, as a consequence of a particular initial condition and of the underlying hamiltonian, exhibits at least one quasiclassical domain made up of suitably defined maximal sets of alternative histories with as much classicity as possible. The quasiclassical domains would then be a consequence of the theory and its boundary condition, not an artifact of our construction. How do we then characterize our place as a collectivity of observers in the universe?

Both singly and collectively we are examples of the general class of complex adaptive systems. When they are considered within quantum mechanics as portions of the universe, making observations, we refer to such complex adaptive systems as information gathering and utilizing systems (IGUSes). The general characterization of complex adaptive systems is the subject of much ongoing research, which we cannot discuss here. From a quantum-mechanical point of view the foremost characteristic of an IGUS is that, in some form of classical, formula, with theory of $\rho, H$ abilities of those for correlations external and the world perfect; notion! An ap is used to col of present d future perceptions (i.e., ex data, make fi
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d) An example of this occurs in the case of "null measurements" discussed by Renniger, and others. An atom decays at the center of a spherical cavity. A detector which covers all but a small opening in the sphere does not register. We conclude that we have measured the direction of the decay photon to an accuracy set by the solid angle subtended by the opening. Certainly there is an interaction of the electromagnetic field with the detector, but did the escaping photon suffer an "irreversible act of amplification"? The point in the present approach is that the set of alternatives, detected and not detected, exhibits decoherence because of the place of the detector in the universe.

e) Perhaps as complex adaptive systems can function in a is the case, they e or understand.
were significant: the necessary relation could ring histories. bility is large, e where the ity or lost in the probability and the situation costly to for reversal! tion solely as quasi-classical actions of many-worlds, would economy, and actions occur without the anything as Thus, this section tracks in the decay of a statement in which the ether or not ered by an ns, as a consequence of the habits at least up of suitably active histories possible. The can be a consequence of this. How can we adapt the ideas in a way that there are examples of adaptive behaviors described in the literature? (We refer to the information in IGUSes). The complex adaptive system is ongoing in the universe. From the point of view the IGUS is that in some form of approximation, however crude or classical, it employs the fundamental formula, with what amounts to a rudimentary theory of \( \rho, H \), and quantum mechanics. Probabilities of interest to the IGUS include those for correlations between its memory and the external world. (Typically these are assumed perfect; not always such a good approximation.) An approximate fundamental formula is used to compute probabilities on the basis of present data, make predictions, control future perceptions on the basis of these predictions (i.e., exhibit behavior), acquire further data, make further predictions, and so on.

To carry on in this way, an IGUS uses probabilities for histories referring both to the future and the past. An IGUS uses decohering sets of alternative histories and therefore performs further coarse graining on a quasi-classical domain. Naturally, its coarse graining is very much coarser than that of the quasi-classical domain since it utilizes only a few of the variables in the universe.

The reason such systems as IGUSes exist, functioning in such a fashion, is to be sought in their evolution within the universe. It seems likely that they evolved to make predictions because it is adaptive to do so.\(^e\) The reason, therefore, for their focus on decohering variables is that these are the only variables for which predictions can be made. The reason for their focus on the histories of a quasi-classical domain is that these present enough regularity over time to permit the generation of models (schemata) with significant predictive power.

If there is essentially only one quasi-classical domain, then naturally the IGUS utilizes further coarse grainings of it. If there are many essentially inequivalent quasi-classical domains, then we could adopt a subjective point of view, as in some traditional discussions of quantum mechanics, and say that the IGUS "chooses" its coarse graining of histories and, therefore, "chooses" a particular quasi-classical domain, or a subset of such domains, for further coarse graining. It would be better, however, to say that the IGUS evolves to exploit a particular quasi-classical domain or set of such domains. Then IGUSes, including human beings, occupy no special place and play no preferred role in the laws of physics. They merely utilize the probabilities presented by quantum mechanics in the context of a quasi-classical domain.

§13. Conclusions

We have sketched a program for understanding the quantum mechanics of the universe and the quantum mechanics of the laboratory, in which the notion of quasi-classical domains plays a central role. To carry out that program, it is important to complete the definition of a quasi-classical domain by finding the general definition for classicality. Once that is accomplished, the question of how many and what kinds of essentially inequivalent quasi-classical domains follow from \( \rho \) and \( H \) becomes a topic for serious theoretical research. So is the question of what are the general properties of IGUSes that can exist in the universe exploiting various quasi-classical domains, or the unique one if there is essentially only one.

It would be a striking and deeply important fact of the universe if, among its maximal sets of decohering histories, there were one roughly equivalent group with much higher classicalities than all the others. That would then be the quasi-classical domain, completely independent of any subjective criterion, and realized within quantum mechanics by utilizing only the initial condition of the universe and the Hamiltonian of the elementary particles.

Whether the universe exhibits one or many maximal sets of branching alternative histories with high classicalities, those quasi-classical domains are the possible arenas of prediction in quantum mechanics.

It might seem at first sight that in such a picture the complementarity of quantum mechanics would be lost; in a given situation, for example, either a momentum or a coordinate could be measured, leading to different kinds of histories. We believe that impression is illusory. The histories in which an observer, as part of the universe, measures \( \rho \) and the histories in which that observer measures \( x \) are decohering alternatives. The important point
is that the decoherent histories of a quasiclassical domain contain all possible choices that might be made by all possible observers that might exist, now, in the past, or in the future for that domain.

The EPR or EPRB situation is no more mysterious. There, a choice of measurements, say, \( \sigma_x \) or \( \sigma_y \) for a given electron, is correlated with the behavior of \( \sigma_x \) or \( \sigma_y \) for another electron because the two together are in a singlet spin state even though widely separated. Again, the two measurement situations (for \( \sigma_x \) and \( \sigma_y \)) decohere from each other. But here, in each, there is also a correlation between the information obtained about one spin and the information that can be obtained about the other. This behavior, although unfortunately called “non-local” by some authors, involves no non-locality in the ordinary sense of quantum field theory and no possibility of signaling outside the light cone. The problem with the “local realism” that Einstein would have liked is not the locality but the realism. Quantum mechanics describes alternative decohering histories and one cannot assign “reality” simultaneously to different alternatives because they are contradictory. Everett\(^6\) and others\(^\text{11}\) have described this situation, not incorrectly, but in a way that has confused some, by saying that the histories are all “equally real!” (meaning only that quantum mechanics prefers none over another except via probabilities) and by referring to “many worlds” instead of “many histories”.

We conclude that resolution of the problems of interpretation presented by quantum mechanics is not to be accomplished by further intense scrutiny of the subject as it applies to reproducible laboratory situations, but rather through an examination of the origin of the universe and its subsequent history. Quantum mechanics is best and most fundamentally understood in the context of quantum cosmology. The founders of quantum mechanics were right in pointing out that something external to the framework of wave function and Schrödinger equation is needed to interpret the theory. But it is not a postulated classical world to which quantum mechanics does not apply. Rather it is the initial condition of the universe that, together with the action function of the elementary par-

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**References**

For a subject as large as this one it would be an enormous task to cite the literature in any historically complete way. We have attempted to cite only papers that we feel will be directly useful to the points raised in the text. These are not always the earliest nor are they always the latest. In particular we have not attempted to review or to cite papers where similar problems are discussed from different points of view.

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30) S. Lloyd: private communication.


M. Namiki: I have two questions: (1) I think, the notion of "history" must be ascribed to a local system with a

finite space-time interval. Different local systems should have their own histories different from each other. This
idea is closely related to your coarse-graining procedure of defining the "history". What about this?
(2) You defined the "history", choosing a set of observables. Do we have the position dependent history and the momentum dependent history? They must be generally different from each other. (Your definition of the "history" may not be unique.) But this question may be related to the problem on how to make the "quasiclassical world". What about this?

J. B. Hartle: (1) In principle, given an initial condition for the universe and the hamiltonian we should be able to compute the maximal sets of coarse grained, alternative, decohering histories with high claclicity that define a quasiclassical world. The arguments of §5 and §9 support the idea that the results of such a calculation would include the histories of local sysytems. I agree that this must come out to be consistent with observations, but in a given theory we can calculate and see.

(2) We do not "choose" the histories of a quasiclassical world; we compute them as described above. A history in which, say, the position of an electron is measured and one in which the momentum is measured, are two decohering alternatives because of the decoherence of the alternative configuration of the apparatus or memory of an observer.

(3) We do not know if there is essentially one quasiclassical world or many, but in the context of quantum cosmology it is a decidable question.

J. Anandan: I am interested in the reason for your choice of quasiclassical variables. I believe that the world looks classical because the interactions, which we believe to be gravitational and gauge fields, are functions of space-time and therefore diagonal in basis of position eigenstates. But is your reason for choosing the quasiclassical variables that these variables decohere?

J. B. Hartle: We do not "choose" quasiclassical variables. We could, in principle, compute them from the initial condition, the hamiltonian, and the information about our specific history. All three forms of information are needed to determine them, not just how they enter into interactions. However, in §9 arguments are offered that the result of such a computation will include the local densities of conserved or approximately conserved quantities as well as the variables describing space-time above the Planck scale.

W. H. Zurek: In the context of the two previous questions it is perhaps useful to emphasize that the reason why the position may be selected as a preferred classical observable! The interactions usually depend on the position (i.e., potentials are \( V(x) \)). Therefore they commute with the position observable. Hence, it is the position observable which gets "monitored" by the environment. Consequently, it will inevitably decohere first. What I am suggesting is that IGUS'es have found it convenient to describe objects in the universe in terms of this observable which decoheres first, and began singling it out or position only "with the benefit of hindsight": They would have ended up calling "position" whatever observable happened to commute with the typical interaction hamiltonian.

P. Mittelsreedt: Comment: on the beginning of your lecture you mentioned that Copenhagen interpretation can not be applied to quantum cosmology. This statement should be specified since there are many faces of the Copenhagen interpretation. Let me consider the Neumann program: the quantum mechanical measuring process should be described in terms of quantum mechanics as an interaction between the object system and the measuring instrument. Many parts of this process are now very well understood. However there is an important missing link: The "objectification", i.e., the transition from a superposition to mixture of objectively decided outcomes has not yet been properly understood. In order to overcome this problem, Everett formulated the "many world interpretation". But even today we don't know any mechanism which could explain the objectification. For many practical situations the Copenhagen interpretation is sufficient, but the creation of the universe is not such a case. Hence the problems mentioned become relevant here.

Question: I have no doubt that the formalism which was presented here is very worthwhile and useful for the description of the early universe. However, I wonder whether your considerations can help to solve the open problems of the quantum theory of the measuring process. Can you explain by means of your results—at least partly—the transition from a superposition to a mixture which appears in any quantum mechanical measuring process?

J. B. Hartle: I believe that it is useful to distinguish two kinds of "problems" in quantum mechanics. First, there are the problems concerned with giving a coherent, consistent and precise formulation of the theory without special external assumptions. The questions of the definition and origin of a quasiclassical world that I have discussed here are the "problems" in this sense. Then there are problems concerned with the adequacy of quantum mechanics relative to standards for physical theory individual physicists may have. These I have not discussed. I think that your problem concerning "objectification" (which I hope I understand correctly as what is sometimes called the "reduction of the wave packet") falls into this latter category. For us, this reassessment of probabilities when new information is required is no more mysterious and no more quantum mechanical than the reassessment that occurs of the joint probability for the winners of eight horse races after the first five races have been run. We do not see it as a "problem" for quantum mechanics.

J.-P. Vigier: The problem with this way of presenting the problems of the history of the Universe since its origin is that we do not ever know (to quote Voltaire) that the Universe was created or not, i.e., it might have an infinite history. The various questions discussed in present day cosmology, the redshift controversy, the physical composition and distribution of the matter and of its dominant forms (plasma?) and the existence or not of a "big bang", are not introduced on this type of models. Unless those it is difficult to distinguish scientific speculation from science fiction.

J. B. Hartle: Thank you for your comment.

M. D. Levenson: This picture denotes the "observer" from the role of creator of classical reality through measurement to that of an "IGUS" that evolved by accident and i quasiclassical to the initial e IGUSes must make profiably cordantly. The preference of pi from the nor universe. Are tionary dead "IGUSes" the? Can any of th automata in a not still be the on the univers by our own IGUSes not cor EPR) correlat extent of tran J. B. Hartle: Y interesting quest (1) It is our quasiclassical universe. How ity of the ev sophisticated occurrance in t accident.

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quasiclassical "reality" produced by de-correlation due
to the initial conditions. The claim seems to be that an
"IGUS" must focus on classical aspects in order to
make profitable predictions and modify its behavior ac-
cordingly. This may not be true. In particular this con-
ference of professional quantum mechanics profits
from the non-classical aspects of the quasiclassical
universe. Are we a quantum-fluctuation or an evolu-
tionary dead end? Might there be even less classical
"IGUSes" than us? (Could we tell if they did exist?)
Can any of this be simulated, for example, by cellular
automata in a "quantum" artificial universe? Might it
not still be the case that the possible "initial conditions
on the universe" in this picture are entirely constrained
by our own present apparent existence? Are some
IGUSes not now learning how to use non-classical (e.g.,
EPR) correlations to modify the universe, at least to the
extent of translating 300 IGUSes to Kokubunji?

J. B. Hartle: You have asked several different and in-
teresting questions. Let me respond to them separately:
(1) It is our position that IGUSes evolve to exploit a
quasiclassical world presented by this particular
universe. However, some would estimate the probab-
ility of the evolution of IGUSes as high, even ones
sophisticated enough to use quantum mechanics. Their
occurrence in the universe may thus not be much of an
accident.
(2) In discussing the use by an IGUS of specifically quan-
tum mechanical phenomena, it is important to
remember that, with the definition of quasiclassical
world given here, measured quantum phenomena are in-
cluded in it. Quantum measurements are one reason
why we discuss a quasiclassical world and not a classical
one. A quasiclassical world consists of many alternative
histories. IGUSes carrying out quantum measurements
and reporting on them are thus using a quasiclassical
world.
(3) I think it would be very interesting to construct
models of the decoherence of histories and we are study-
ning this.
(4) I believe that it is very unlikely that the initial con-
ditions of the universe are determined by our existence.
We need so little to exist. One can ask oneself: "From
our existence, what can we predict for new observations
beyond the reach of present telescopes?" I think the
answer is "Almost nothing".

M. Peshkin: You never mentioned chaos. Is chaos central
to the overall strategy in your program, or does it mere-
ly play a technical role in some of the calculation?

J. B. Hartle: Chaos does not play a fundamental role in
the framework of quantum mechanics sketched here.
However, chaos is important in the universe, because it
is a mechanism for the amplification of small, prob-
abilistically distributed, fluctuations to events of
significance in the quasiclassical world. Thus, certain
chaotic phenomena may be ultimately traceable to quan-
tum fluctuations.