Phase-shift analysis of pion-proton scattering

Table 3. Track II

<table>
<thead>
<tr>
<th>Energy MeV</th>
<th>$\alpha_1$</th>
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<th>$\alpha_3$</th>
<th>$\alpha_5$</th>
<th>$\alpha_13$</th>
<th>$\alpha_{11}$</th>
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<td>-4</td>
<td>-2</td>
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Table 4. Track III

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<th>$\alpha_5$</th>
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<td>7</td>
<td>49</td>
<td>12</td>
<td>1</td>
<td>2</td>
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</tbody>
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On the scattering of $\gamma$-rays by protons

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Reported by M. GELL-MANN

The authors have discussed, in a recent publication (GELL-MANN et al., Phys. Rev., in press), the application of the principle of causality to the forward scattering of photons. General formulae were derived, similar to the dispersion formulae of KRAMERS (1927) and KRONIG (1946) and of TOLL and WHEELER,* relating forward-scattering amplitude to total cross-section for photons incident on various kinds of scatterers, particles of spin $\frac{1}{2}$ in particular. Some remarks were made about the specific problem of the scattering of $\gamma$-rays from protons.

It is our purpose here to amplify those remarks and to present a concise summary of our results up to date. Let us first present the consequences of the causality principle.

Denote by $f(\nu)$ the forward-scattering amplitude (in the laboratory system) for light of angular frequency $\nu$ incident on a proton at rest. Then $f$ may be divided into contributions arising from scattering with and without proton spin-flip:

$$f(\nu) = f_1(\nu)e^\nu + f_2(\nu)e^{-\nu}.$$  \hspace{1cm} (1)

Here \( e \) and \( e' \) are the initial and final polarization vectors of the photon and \( \sigma \)
is the proton spin operator.

Similarly we may write the total cross-section \( \sigma(v) \) for unpolarized light of
frequency \( v \) incident upon a proton at rest in the form

\[
\sigma(v) = \frac{\sigma_p + \sigma_d}{2}.
\]

where we define \( \sigma_p \) and \( \sigma_d \) as follows:

Consider circularly polarized light incident upon a proton with its spin
direction quantized parallel or antiparallel to the direction of motion of the
photon. Then the polarizations of the photon and proton may be parallel or
antiparallel. We denote by \( \sigma_p \) and \( \sigma_d \) respectively the total cross-sections in
these two cases. Let us further define

\[
\tilde{\sigma}(v) = \frac{\sigma_p - \sigma_d}{2}.
\]

Now we postulate:

I. The relativistic form of the causality principle—that no signal may be
propagated by light-waves, even in the presence of the scatterer, at a velocity
greater than \( c \), the velocity of light in vacuum.
II. The invariance of physical laws under the reversal of time.
III. That the total cross-sections \( \sigma_p \) and \( \sigma_d \) do not become more than
logarithmically infinite at infinite frequency.

It follows that

(a) \( \text{Re} f_1(v) \) and \( \text{Re} f_2(v) \) are even and odd functions of frequency respectively; and

(b) the amplitudes \( f_1 \) and \( f_2 \) are given by the formulae

\[
f_1(v) = A_0 + \frac{\gamma^2}{2\pi\kappa} \int \frac{dv' a(v')}{v'^2 - v^2 - it}
\]

and

\[
f_2(v) = A_1 v + \frac{\gamma^2}{2\pi\kappa} \int \frac{dv' \tilde{a}(v')}{v'(v'^2 - v^2 - it)}
\]

Here \( \epsilon \) is a small positive quantity which we need to tell us how to go around
the poles in the integrals.

The constants \( A_0 \) and \( A_1 \) require some discussion. As to \( A_0 \), the scattering
amplitude at zero frequency, all physical intuition instructs us to put it equal
to the classical Thomson amplitude

\[
A_0 = -\frac{\epsilon^2}{Me^2}.
\]

That (6) in fact holds true for calculations to all orders in present-day field
theories has been demonstrated by Thirring (1950) and by Knoll and
Ruderer (1954).

We may gain some insight into the significance of the other constant \( A_1 \) by
calculating, in the lowest order of perturbation theory, the forward amplitude
for light scattered by a Dirac particle with an anomalous Pauli magnetic
moment \( \mu \). We find

\[
f(v) = -\frac{\gamma}{Me^2} e' e - \frac{2\mu^2}{\kappa\epsilon^4} \hat{\kappa} \times e \times e
\]

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which leads us to put

$$A_1 = \frac{-2\mu^2}{R^3}.$$  \hspace{1cm} (8)

(Note that the Dirac magnetic moment does not enter into (7) and (8), but only the anomalous moment.) Now we have shown that equation (8) remains true in existing meson theories when the anomalous magnetic moment is attributed to the effects of a meson cloud rather than a phenomenological interaction of the Pauli type. Our demonstration is valid to all orders in the meson-nucleon coupling constant and to second order in the charge of the proton.

If we now wish to calculate $f_1$ and $f_2$ using formulas (4) and (5), we must know

what to insert for $\sigma$ and $\tilde{\sigma}$. The largest contribution to $\sigma$ is made by the cross-section for the creation of electron pairs in the Coulomb field of the proton. This gives rise to the Delbrück scattering of light, which has been calculated, using this method, by Toll (Dissertation, Princeton Univ.), and by Boerhavc and Glueckstim (1952). While the Delbrück scattering is larger in the forward direction than all other effects at energies greater than a few MeV, it vanishes rapidly at angles greater than zero, and is therefore not of great experimental interest at the moment. We shall therefore ignore it henceforth.

The principal contribution to $\sigma$ and $\tilde{\sigma}$ (at least at energies up to 5000 MeV) then comes from the photopion effect. Using recent experimental data\footnote{Gell-Mann, M., and Goldhaber, M. L.; to be published.} for $\sigma(\nu)$ we obtain from formula (4) the curves in Fig. 1 for $\text{Re} f_1(\nu)$ and $\text{Im} f_1(\nu)$ as functions of $\nu$. We are unable to proceed in the same straightforward manner in the case of $\tilde{\sigma}(\nu)$, since without polarization measurements we know only the inequality

$$|\tilde{\sigma}(\nu)| \leq \sigma(\nu).$$  \hspace{1cm} (9)

However, on the basis of preliminary phenomenological analyses of the angular distribution of the photopion effect,\footnote{For references see M. Gell-Mann and K. M. Watson, Annual Review of Nuclear Science (1956).} we can arrive at an educated guess for $\tilde{\sigma}(\nu)$; we then obtain the curves in Fig. 2 for $\text{Re} f_2(\nu)$ and $\text{Im} f_2(\nu)$. In Fig. 3 we give the differential scattering cross-section in the forward direction (as computed from Figs. 1 and 2) plotted against energy.
So far we have considered only the forward-scattering amplitude. At energies of several hundred MeV, we are unable to proceed further without detailed calculation. However, at low energies (say < 70 MeV) we can derive from our knowledge of the forward scattering considerable information about the scattering at other angles. The reason is that at low energies the photon wave-

![Graph showing scattering amplitude](image)

length is large compared to the dimension of the meson cloud about the proton and therefore only a few multipole orders contribute appreciably to the scattering. At very low energies, moreover, the angular distribution is probably determined by general considerations. It has been shown\(^1\) that at zero energy

![Graph showing angular distribution](image)

the scattering amplitude to second order in the proton charge and to all orders in the meson-nucleon coupling constant is given by \(\frac{-e^4}{M^2} e'\). e not only in the forward direction but at all angles. We conjecture that in a similar fashion

\(^1\) Y. Nambu, unpublished; see also M. Gell-Mann and M. L. Goldberger, unpublished.
On the scattering of γ-rays by protons

the scattering amplitude calculated for a Dirac particle with a Pauli moment is accurate, up to terms linear in the photon frequency, for all angles and not just for the forward direction. This conjecture has been checked in the lowest order of perturbation theory; an effort is now being made to determine whether it is true to all orders in the coupling constant.

References

Double meson scattering

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We wish to calculate the cross-section for the process

\[ \pi^+ + p \rightarrow 2\pi^0 + n \]

and we attempt to do this using the Tamm-Dancoff method as modified by Dyson.

Using charge symmetric FS(PS)-meson theory, we consider, in the notation of Dyson, the the amplitude \( \psi^t(p, k) = \sum_{\mu} \psi_{\mu}^a b_{\mu} a_{k} \psi^a \) describing a positive meson of momentum \( k \) in the state \( \Psi \) and a proton or anti-proton of momentum \( p \). The integral equation for this amplitude is set up in lowest T.D. approx., i.e., all amplitudes describing four or more particles are neglected; nucleon pair-formation in the intermediate states is also neglected. Neglecting pair-formation in the original formulation of the T.D. method would eliminate all matrix elements between -ve and +ve energy nucleon states; however such matrix elements are still present in the modified theory due to the fact that the nucleon can be present in the state \( \Psi \) or the state \( \Psi^\prime \). The equation obtained is

\[
\begin{align*}
&\sum_{\nu} \frac{G^2}{V \sqrt{\omega_\nu \omega_{\nu'}}} \gamma \Lambda_a (p - k') \gamma \\
&= \frac{\epsilon - \eta(p) (E_{\nu'} - \omega_\nu) - \omega_{\nu'}}{\epsilon - \eta(p) (E_{\nu'} + \omega_{\nu'}) - \omega_{\nu'}} \psi^t(p + k - k', k') \\
&+ \sum_{\nu} \frac{3G^2}{2V \omega_{\nu'}} \gamma (\epsilon - \eta(p - k') (E_{\nu'} + \omega_{\nu'}) - \omega_{\nu'}) \gamma \psi^t(p, k)
\end{align*}
\]

The second term on the R.H.S. is the nucleon self-energy to second order in \( G \); it contains the usual divergences and, as a first approximation to the calculation, it is neglected.

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