I. Utility and Indifference Curve

**Definition:** Utility Function, Indifference Curve, Marginal Rate of Substitution, Perfect Substitute, Perfect Complements.

1. Suppose your utility function is given by $U=4X+2Y$.
   (a) Consider the following commodity bundles; $A=(1,2)$, $B=(3,1)$, $C=(2, 5)$, $D=(3, 3)$. Rank them according to the utility levels.
   (b) Consider the following commodity bundles; $E=(3, 5)$, $F=(4, 3)$. Rank them according to the utility levels.
   (c) One unit of X good is always ( ) times as good as one unit of Y good. Suppose X is apple and Y is orange. For example, $G=(1, 50)$ and $H=(25, 2)$. Are two commodity bundles indifferent? At G, you have only one unit of apple and 50 units of orange. Here if you get one more unit of apple, then how many units of orange are you willing to give up? At H, you have 25 units of apple and only two units of orange. Here if you get one more unit of apple, then how many units of orange are you willing to give up? Does the relative importance between two goods vary?
   (d) Derive an indifference curves when your utility level is 20, 60, and 100.
   (e) Consider four commodity bundles; $(0, 30)$, $(5, 20)$, $(10, 10)$, $(14, 2)$. Do the four bundles give you the same utility? Are they on the same indifference curve? Draw indifference curve passing through the four commodity bundles. Calculate marginal rate of substitution at $(0, 30)$. Here if you get one more unit of X (apple), then how many oranges are you willing to give up? Calculate marginal rate of substitution at $(5, 20)$. Here if you get one more unit of X (apple), then how many oranges are you willing to give up? Repeat questions with $(10, 10)$ and $(14, 2)$. Is MRS constant?
   (f) 1 unit of X is a perfect substitute of ( ) unit(s) of Y. Therefore, we call these two good perfect substitute.

2. Suppose your utility function is given by $U=X+2Y$.
   (a) Consider the following commodity bundles; $A=(1,2)$, $B=(3,1)$, $C=(2, 5)$, $D=(3, 3)$. Rank them according to the utility levels.
   (b) Consider the following commodity bundles; $E=(5, 3)$, $F=(3, 4)$. Rank them according to the utility levels.
(c) One unit of X good is always ( ) times as good as one unit of Y good. Suppose X is apple and Y is orange. For example, G=(2, 50) and H=(100, 1). Are two commodity bundles indifferent? At G, you have only two units of apple and 50 units of orange. Here if you get one more unit of apple, then how many units of orange are you willing to give up? At H, you have 100 units of apple and only one unit of orange. Here if you get one more unit of apple, then how many units of orange are you willing to give up? Does the relative importance between two goods vary?

(d) Derive an indifference curves when your utility level is 20, 60, and 100.

(e) Consider four commodity bundles; (0, 30), (20, 20), (40, 10), (58, 1). Do the four bundles give you the same utility? Are they on the same indifference curve? Draw indifference curve passing through the four commodity bundles. Calculate marginal rate of substitution at (0, 30). Here if you get one more unit of X (apple), then how many oranges are you willing to give up? Calculate marginal rate of substitution at (20, 20). Here if you get one more unit of X (apple), then how many oranges are you willing to give up? Repeat questions with (40, 10) and (58, 1). Is MRS constant?

(f) 1 unit of X is a perfect substitute of ( ) unit(s) of Y. Therefore, we call these two good perfect substitute.

3. Suppose your utility function is given by U=X+Y.

(a) Consider the following commodity bundles; A=(1,2), B=(3,1), C=(2, 5), D=(3, 3). Rank them according to the utility levels.

(b) Consider the following commodity bundles; E=(5, 3), F=(3, 5). Rank them according to the utility levels.

(c) One unit of X good is always ( ) times as good as one unit of Y good. Suppose X is apple and Y is orange. For example, G=(1, 100) and H=(100, 1). Are two commodity bundles indifferent? At G, you have only one unit of apple and 100 units of orange. Here if you get one more unit of apple, then how many units of orange are you willing to give up? At H, you have 100 units of apple and only one unit of orange. Here if you get one more unit of apple, then how many units of orange are you willing to give up? Does the relative importance between two goods vary?

(d) Derive an indifference curves when your utility level is 20, 60, and 100.

(e) Consider three commodity bundles; (0, 60), (30, 30), (60, 0). Do the three bundles give you the same utility? Are they on the same indifference curve? Draw indifference curve passing through the four commodity bundles. Calculate marginal rate of substitution at (0, 60). Here if you get one more unit of X (apple), then how many oranges are you willing to give up? Calculate marginal rate of substitution at (30, 30). Here if you get one more unit of X (apple), then how many oranges are you willing to give up? Repeat questions with (60, 0). Is MRS constant?

(f) 1 unit of X is a perfect substitute of ( ) unit(s) of Y. Therefore, we call these two good perfect substitute.
4. Suppose your utility function is given by \( U = \min(X, Y) \). (That is, \( U \) value is equal to the minimum of \( X \) and \( Y \)).

(a) Interpret the above utility function.
(b) Consider the following commodity bundles; \( A = (1, 1), B = (3, 1), C = (1, 5), D = (2, 2) \). Rank them according to the utility levels.
(c) Draw indifference curves when your utility level is 3, 5, and 10.
(d) Calculate MRS at (3, 1). To answer this question, consider the following equivalent question; At (3, 1), if you get one more unit of \( X \), how many \( Y \) goods are you willing to give up? Calculate MRS at (1, 3). Again, to answer this question, consider the following equivalent question; At (1, 3), if you get one more unit of \( X \), how many \( Y \) goods are you willing to give up? Here is a more challenging question. Calculate MRS at (1, 1).
(e) What do you call these two goods?

5. Suppose your utility function is given by \( U = XY \).

(a) Consider the following commodity bundles; \( A = (1, 1), B = (3, 1), C = (1, 3), D = (2, 2) \). Rank them according to the utility levels.
(b) Suppose the following two bundles; \( E = (1, 100) \) and \( F = (100, 1) \). Are they indifferent? What is your utility level at \( E \) and \( F \)? At \( E \), if you get one more unit of \( X \), what is your utility level? If you get one more unit of \( Y \), what is your utility level? At \( E \), \( X \) is ( ) times as good as \( Y \). Now let’s move on to \( F \). At \( F \), if you get one more unit of \( X \), what is your utility level? If you get one more unit of \( Y \), what is your utility level? At \( F \), \( X \) is ( ) times as good as \( Y \). Note \( E \) and \( F \) are on the same indifference curve. That is both give you the same utility. However the relative importance between \( X \) and \( Y \) are different at \( E \) and \( F \). At \( E \), \( X \) is relatively more important than \( Y \) is while at \( F \), \( Y \) is relatively more important than \( X \). Therefore relative importance between \( X \) and \( Y \) varies. Compare this conclusion with the case of perfect substitute (1-(c), 2-(c) and 3-(c)).
(c) (optional) Mathematically, MRS is equal to \((\partial U/\partial X)/(\partial U/\partial Y)\). \((\partial U/\partial X)\) is the partial derivative of \( U \) with respect to \( X \) and \( \partial U/\partial Y \) is the partial derivative of \( U \) with respect to \( X \).
Since \( U = XY \), \( \partial U/\partial X = Y \) and \( \partial U/\partial Y = X \).
Therefore MRS is given by \((\partial U/\partial X)/(\partial U/\partial Y) = Y/X\). So MRS at \( E = (1, 100) \) is 100 and MRS at \( F = (100, 1) = 1/100 \).
(d) Draw indifference curves when utility levels are 10, 20 and 30.
(e) Is MRS in this case diminishing as \( X \) increases? Define diminishing marginal rate of substitution and explain what that means (in terms of relative importance between two goods).
6. Interpret the following indifference curves.

7. Prove the four properties of indifference curve

II. Budget Set or Economically Feasible Set.

Definition: Budget Set, Economically Feasible Set.

1. Various Economically Feasible Set.

   (1) Derive budget constraint equation and draw the budget line.

   (a) Income=100, Px=4, Py=2. What happen to budget line if income goes up? What happen to budget line if income goes down? What happen to budget line if the price of X goes up? What happen to budget line if the price of Y goes up?

   (b) Income=30, Px=3, Py=1. What happen to budget line if income goes up? What happen to budget line if income goes down? What happen to budget line if the price of X goes up? What happen to budget line if the price of Y goes up?
(c) Income=20, Px=5, Py=4. What happen to budget line if income goes up? What happen to budget line if income goes down? What happen to budget line if the price of X goes up? What happen to budget line if the price of Y goes up?

(2) Initial Endowment and Economically Feasible Sets

(a) Suppose you have A=(100, 0) as an initial endowment. Plot A.
(b) Suppose the price of X is 1 and the price of Y is 1. And you can sell some of your X goods and buy some amount of Y. Draw your economically feasible set.
(c) Suppose the price of X rises. Now Px=2, and Py=1. Draw your economically feasible set and compare it with the original one.
(d) Suppose the price of Y rises. Now Px=2, and Py=2. Draw your economically feasible set and compare it with the original one.

(3) Intertemporal Choice

(a) Suppose X is current consumption and Y is future consumption. Assume that you have $100 and your future earning is zero. Draw this situation.
(b) If there is no financial institutions. So if you save $1 for the future you will have $1 in the future. So if you spend $50 for current consumption (that means you save $50), when the future comes, you will have $50 in your pocket. Draw your economically feasible set.
(c) Now we have a financial institution. If you deposit $1 at the bank, you will receive $1.1 in the future. Now draw your economically feasible set and compare it with (b).

(4) Work-Leisure Choice

(a) Suppose the current hourly wage rate is $5. You have 24 hours a day and want to allocate 24 hours to work and to leisure. The time allocated to leisure is on X-axis and your income is on Y-axis. Draw economically feasible set. If you decide to work for 9 hours, how much is your income? Plot this choice on the graph.
(b) Assume that the current hourly wage rate has gone up to $10. Draw economically feasible set and compare it with (a).

(5) Kinked Budget Line

Suppose the price of good Y is equal to 1; the price of good x is P1 up to quantity x* but switches to P2 thereafter. M is your income. Draw the budget lines with the following assumptions.

(a) M=30, P1=2, P2=4, x*=5
III. Utility Maximization

1. Find utility maximizing point(s). (Draw graphs)

(a) \( U = X + Y, \text{Income}=50, P_x=4, P_y=2 \)
(b) \( U = 3X + Y, \text{Income}=30, P_x=1, P_y=1 \)
(c) \( U = 2X + 3Y, \text{Income}=30, P_x=2, P_y=3 \)
(d) \( U = \min\{X, Y\}, \text{Income}=30, P_x=2, P_y=3 \)
(e) \( U = \min\{2X, 3Y\}, \text{Income}=60, P_x=2, P_y=3 \)
(f) Suppose indifference curves are bowed into the origin. \( \text{Income}=30, P_x=3, P_y=2 \)
(g) In the case of (f), at the utility maximizing point, calculate MRS.
(h) (challenging question) \( U = XY, \text{Income}=10, P_x=1, P_y=1 \)
(i) (challenging question, optional) \( U = XY, \text{Income}=16, P_x=2, P_y=1 \)

2. Utility Maximization.
At the utility maximizing point, \( \text{MRS}=P_x/P_y \) (i.e. an indifference curve is tangent to the budget line). What happen if \( \text{MRS}>P_x/P_y \) or \( \text{MRS}<P_x/P_y \)?

IV. Reference

Chapter 2: 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9