Econophysics

Master curve for price-impact function

The price reaction to a single transaction depends on transaction volume, the identity of the stock, and possibly many other factors. Here we show that, by taking into account the differences in liquidity for stocks of different size classes of market capitalization, we can rescale both the average price shift and the transaction volume to obtain a uniform price-impact curve for all size classes of firm for four different years (1995–98). This single curve collapse of the price-impact function suggests that fluctuations from the supply-and-demand equilibrium for many financial assets, differing in economic sectors of activity and market capitalization, are governed by the same statistical rule.

Our results complement previous efforts by using huge amounts of data, by looking at the short-term response to a single trade, and by measuring time in units of transactions rather than in seconds. We used the Trade and Quote database as our data source and studied the 1,000 largest stocks for 1995 (black), 1996 (green), 1997 (blue) and 1998 (red). The black dashed line is the power-law best fit for all points.

The results obtained for 1995 use all of the available data (Fig. 1a). On a double-logarithmic scale, the slope of each curve varies from roughly 0.5 for small transactions in higher-capitalization stocks, to about 0.2 for larger transactions in lower-capitalization stocks. When we repeat this for the years 1996–98, the results are similar, except that the slopes become slightly flatter with time, ranging from roughly 0.4 to 0.1 in 1998.

Higher-capitalization stocks tend to have smaller price responses for the same normalized transaction size. To explain this observation, we carried out a best fit of the impact curves for small values of the normalized transaction size with the functional form \( \Delta p = \omega \left( \frac{\Delta P}{\omega C} \right)^{\gamma} \), where \( \gamma \) is the liquidity and \( \omega \) is +1 or -1 for buying and selling, respectively. For all four years, the liquidity of each group increases as roughly 0.4, where \( C \) is the average market capitalization of each group (Fig. 1b, inset).

We make use of this apparent scaling to collapse the data shown in Fig. 1a into a single curve. We assume that \( \Delta p = \omega \left( \frac{\Delta P}{\omega C} \right)^{\gamma} \), where \( \gamma \) and \( \delta \) are constants, and we rescale the \( \omega \) and \( \Delta P \) axes of each group according to the transformations \( \omega \rightarrow \omega C^\delta \) and \( \Delta P \rightarrow \Delta P C^\gamma \). We then search for the values of \( \delta \) and \( \gamma \) that place all of the points most accurately on a single curve. In all the years that we investigated, there is a clear minimum for \( \delta = \gamma = 0.3 \). The resulting rescaled price-impact curves for buys in 1995 are shown in Fig. 1b.

We have investigated demand-and-supply fluctuations in a way that is complementary to the traditional approach in economics. The mechanism for making transactions has recently been theoretically modelled by assuming that order placement and cancellation are largely random, which results in predictions of price impact that are qualitatively consistent with those made here. Our findings show that it is important to model financial institutions in detail, and that it may be more useful to model human behaviour as random rather than rational for some purposes.

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brief communications

Two-phase behaviour of financial markets

Buying and selling in financial markets is driven by demand, which can be quantified by the imbalance in the number of shares transacted by buyers and sellers over a given time interval. Here we analyse the probability distribution of demand, conditioned on its local noise intensity, and discover the surprising existence of a critical threshold, \( \Sigma_c \). For \( \Sigma < \Sigma_c \), the most probable value of demand is roughly zero; we interpret this as an equilibrium phase in which neither buying nor selling predominates. For \( \Sigma > \Sigma_c \), two most probable values emerge that are symmetrically around zero demand, corresponding to excess demand and excess supply; we interpret this as an out-of-equilibrium phase in which the market behaviour is mainly buying for half of the time, and mainly selling for the other half.

We use the Trade and Quote database to analyse each and every transaction of the 116 most actively traded stocks in the two-year period 1994–95. We quantify demand by computing the volume imbalance, \( \Omega(t) \), defined as the difference between the number of shares, \( Q_s \), traded in buyer-initiated transactions and the number, \( Q_o \), traded in seller-initiated transactions in a short time interval, \( \Delta t \).

\[
\Omega(t) = Q_o - Q_s = \sum_{i=1}^{N} a_i q_i a_i
\]

where \( i = 1, \ldots, N \) labels each of the \( N \) transactions in the time interval \( \Delta t \), \( q_i \) denotes the number of shares traded in transaction \( i \), and \( a_i = \pm 1 \) denotes buyer-initiated and seller-initiated trades, respectively.

We also calculate, for the same sequence of intervals, the local noise intensity, \( \Sigma(t) = (\langle q_i - \langle q_i \rangle \rangle) \), where \( \langle ... \rangle \) denotes the local expectation value, computed from all transactions of that stock during the time interval \( \Delta t \).

We find (Fig. 1a) that for small \( \Sigma \), the conditional distribution, \( P(\Omega|\Sigma) \), is single-peaked, displaying a maximum at zero demand, \( \Omega = 0 \). For large \( \Sigma \), above the critical threshold, \( \Sigma_c \), the behaviour of \( P(\Omega|\Sigma) \) undergoes a qualitative change, becoming double-peaked with a pair of maxima appearing at non-zero values of demand, \( \Omega = \Omega_c \), where \( \Omega = \Omega_c \), and \( \Omega = -\Omega_c \), which are symmetrical around \( \Omega = 0 \).

Our findings suggest that there is a link between the dynamics of a human system with many interacting participants (the financial market) and the ubiquitous phenomenon of phase transitions that occur in physical systems with many interacting units. Physical observables associated with phase transitions undergo large fluctuations that display power-law behaviour, so our results raise the possibility that volatile market movements and their empirically identified power-law behaviour are related to general aspects of phase transitions.

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