

# Role of design complexity in technology improvement

James McNerney<sup>a,b</sup>, J. Doyne Farmer<sup>a,c</sup>, Sidney Redner<sup>a,b,d</sup>, and Jessika E. Trancik<sup>a,e,1</sup>

<sup>a</sup>Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501; <sup>b</sup>Department of Physics, Boston University, Boston, MA 02215; <sup>c</sup>Libera Università Internazionale degli Studi Sociali, Guido Carli, Viale Pola 12, 00198 Rome, Italy; <sup>d</sup>Center for Polymer Studies, Boston University, Boston, MA 02215; and <sup>e</sup>Engineering Systems Division, Massachusetts Institute of Technology, Cambridge, MA 02139

Edited by Hans-Joachim Schellnhuber, Potsdam Institute for Climate Impact Research (PIK), Potsdam, Germany, and approved March 8, 2011 (received for review November 22, 2010)

We study a simple model for the evolution of the cost (or more generally the performance) of a technology or production process. The technology can be decomposed into  $n$  components, each of which interacts with a cluster of  $d - 1$  other components. Innovation occurs through a series of trial-and-error events, each of which consists of randomly changing the cost of each component in a cluster, and accepting the changes only if the total cost of the cluster is lowered. We show that the relationship between the cost of the whole technology and the number of innovation attempts is asymptotically a power law, matching the functional form often observed for empirical data. The exponent  $\alpha$  of the power law depends on the intrinsic difficulty of finding better components, and on what we term the design complexity: the more complex the design, the slower the rate of improvement. Letting  $d$  as defined above be the connectivity, in the special case in which the connectivity is constant, the design complexity is simply the connectivity. When the connectivity varies, bottlenecks can arise in which a few components limit progress. In this case the design complexity depends on the details of the design. The number of bottlenecks also determines whether progress is steady, or whether there are periods of stasis punctuated by occasional large changes. Our model connects the engineering properties of a design to historical studies of technology improvement.

design structure matrix | experience curve | learning curve | performance curve

The relation between a technology's cost  $c$  and the cumulative amount produced  $y$  is often empirically observed to be a power law of the form

$$c(y) \propto y^{-\alpha}, \quad [1]$$

where the exponent  $\alpha$  characterizes the rate of improvement. This rate is commonly termed the progress ratio  $2^{-\alpha}$ , which is the factor by which costs decrease with each doubling of cumulative production. A typical reported value (1) is 0.8 (corresponding to  $\alpha \approx .32$ ), which implies that the cost of the 200th item is 80% that of the 100th item. Power laws have been observed (or at least assumed to hold), for a wide variety of technologies (1–3), although other functional forms have also been suggested and in some cases provide plausible fits to the data\*. We give examples of historical performance curves for several different technologies in Fig. 1.

The relationship between cost and cumulative production goes under several different names, including the “experience curve,” the “learning curve,” or the “progress function.” The terms are used interchangeably by some, whereas others assign distinct meanings (1, 4). We use the general term performance curve to denote a plot of any performance measure (such as cost) against any experience measure (such as cumulative production), regardless of the context. Performance curve studies first appeared in the 19th century (5, 6), but their application to manufacturing and technology originates from the 1936 study by Wright on aircraft production costs (7). The large literature on this subject spans engineering (8), economics (4, 9), management science (1), organizational learning (16), and public policy (17). Performance

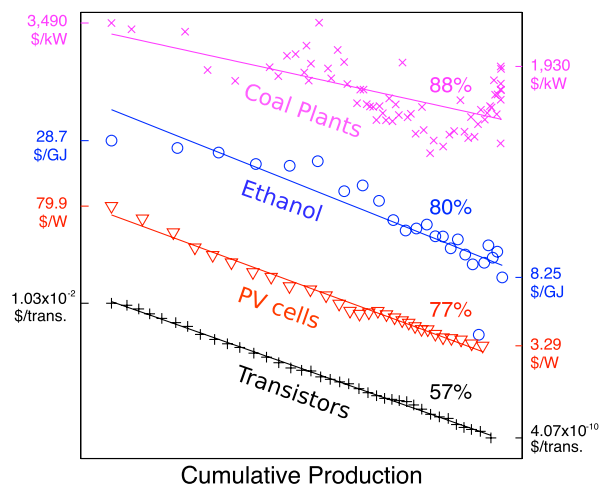


Fig. 1. Four empirical performance curves. Each curve was rescaled and shifted to aid comparison with a power law. The  $x$ - and  $y$ -coordinates of each series  $i$  were transformed via  $\log x \rightarrow a_i + b_i \log x$ ,  $\log y \rightarrow c_i + d_i \log y$ . The constants  $a_i$ ,  $b_i$ ,  $c_i$ , and  $d_i$  were chosen to yield series with approximately the same slope and range, and are given in *SI Text*. Tick marks and labels on the left vertical axis show the first price (in real 2000 dollars) of the corresponding time series, and those of the right vertical axis show the last price. Lines are least-squares fits to a power law. Percentages are the progress ratios of the fitted power laws. Source: coal plants (10), ethanol (11), photovoltaic cells (12, 13, 14), transistors (15).

curves have been constructed for individuals, production processes, firms, and industries (1).

The power law assumption has been used by firm managers (18) and government policy makers (17) to forecast how costs will drop with cumulative production. However, the potential for exploiting performance curves has so far not been fully realized, in part because there is no good theory explaining the observed empirical relationships. Why do performance curves tend to look like power laws, as opposed to some other functional form? What factors determine the exponent  $\alpha$ , which governs the long-term rate of improvement? Why are some performance curves steady and others erratic? By suggesting answers to these questions, the theory we develop here can potentially be used to guide investment policy for technological change.

Author contributions: J.M., J.D.F., S.R., and J.E.T. performed research and wrote the paper.

The authors declare no conflict of interest.

This article is a PNAS Direct Submission.

Freely available online through the PNAS open access option.

\*Koh and Magee (35, 36) claim an exponential function of time (Moore's law) predicts the performance of several different technologies. Goddard (34) claims costs follow a power law in production rate rather than cumulative production. Multivariate forms involving combinations of production rate, cumulative production, or time have been examined by Sinclair et al. (38) and Nordhaus (37).

<sup>1</sup>To whom correspondence should be addressed. E-mail: trancik@mit.edu.

This article contains supporting information online at [www.pnas.org/lookup/suppl/doi:10.1073/pnas.1017298108/-DCSupplemental](http://www.pnas.org/lookup/suppl/doi:10.1073/pnas.1017298108/-DCSupplemental).

An example of the possible usefulness of such a theory is climate change mitigation. Good forecasts of future costs of low-carbon energy technologies could help guide research and development funding and climate policy. Our theory suggests that based on the design of a technology we might be able to better forecast its rate of improvement, and therefore make better investments and better estimates of the cost of achieving low-carbon energy conversion.

There have been several previous attempts to construct theories to explain the functional form of performance curves (19–21). Muth constructed a model of a single-component technology in which innovation happens by proposing new designs at random (21). Using extreme value theory he derived conditions under which the rate of improvement is a power law. An extension to multiple components, called the production recipe model, was proposed by Auerswald et al. (19). In their model each component interacts with other components, and if a given component is replaced, it affects the cost of the components with which it interacts. They simulated their model and found that under some circumstances the performance curves appeared to be power laws. Other models include Bendler and Schlesinger, who derive a power law based on the assumption that barriers to improvement are distributed fractally (22), and Huberman, who represents the design process as a random evolving graph (20). More recently Frenken has used the Auerswald model to interpret and address questions such as the efficacy of outsourcing (23, 24). Other related models that use random search to model technological progress (but which do not directly address performance curves) are those of Silverberg and Verspagen (25, 26) and Thurner et al. (27).

In this paper we both simplify and extend the production recipe model of Auerswald et al. (19). The simplifications allow us to derive the emergence of a power law, and most importantly, to derive its exponent  $\alpha$ . We find that  $\alpha = 1/(\gamma d^*)$ , where  $\gamma$  measures the intrinsic difficulty of finding better components and  $d^*$  is what we call the design complexity. When the connectivity of the components is constant the design complexity  $d^*$  is equal to the connectivity. When connectivity is variable, the complexity can also depend on the detailed properties of the design, in ways that we make clear. We also show that when costs are spread uniformly across a large number of components, the whole technology undergoes steady improvement. In contrast, when costs are dominated by a few components, the total cost undergoes erratic improvement. Our theory thus potentially gives insight into how to design a technology so that it will improve more rapidly and more steadily.

We should emphasize that many factors besides design can affect costs—for example, the cost of input materials or fuels may change due to market dynamics rather than technology design (10). Furthermore, design is generally focused not just on reducing costs, but also on improving other properties such as environmental performance or reliability. The variable “cost” in the theory here can be interpreted as any property that depends on technology design.

### The Model

The production design consists of  $n$  components, which can be thought of as the parts of a technology or the steps in an industrial process<sup>1</sup>. Each component  $i$  has a cost  $c_i$ . The total cost  $\kappa$  of the design is the sum of the component costs:  $\kappa = c_1 + c_2 + \dots + c_n$ . A component’s cost changes as new implementations for the component are found. For example, a component representing the step “move a box across a room” may initially be implemented by a forklift, which could later be replaced by a conveyor belt.

<sup>1</sup>The original production recipe model (19) contained 6 parameters. We eliminated four of them as follows: length of production run  $T \rightarrow \infty$ ; output-per-attempted-recipe-change  $B \rightarrow 1$ ; available implementations per component  $s \rightarrow \infty$ ; search distance  $\delta \rightarrow 1$ .

Cost reductions occur through repeated changes to one or more components.

Components are not isolated from one another, but rather interact as parts of the overall design. Thus changing one component not only affects its cost, but also the costs of other dependent components. Components may be viewed as nodes in a directed network, with links from each component to those that depend on it. The relationship between the nodes and links can alternatively be characterized by an adjacency matrix. In systems engineering and management science this matrix is known as the design structure matrix (DSM) (28–30). A DSM is an  $n \times n$  matrix with an entry in row  $i$  and column  $j$  if a change in component  $j$  affects component  $i$  (Fig. 2). The matrix is usually binary (31, 32); however, weighted interactions have also been considered (33). DSMs have been found to be useful in understanding and improving complex manufacturing and technology development processes.

The model is simulated as follows:

1. Pick a random component  $i$ .
2. Use the DSM to identify the set of components  $\mathcal{A}_i = \{j\}$  whose costs depend on  $i$  (the outset of  $i$ ).
3. Determine a new cost  $c'_j$  for each component  $j \in \mathcal{A}_i$  from a fixed probability distribution  $f$ .
4. If the sum of the new costs,  $a'_i = \sum_{j \in \mathcal{A}_i} c'_j$ , is less than the current sum,  $a_i$ , then each  $c_j$  is changed to  $c'_j$ . Otherwise, the new cost set is rejected.

This process is repeated for  $t$  steps. The costs are defined on  $[0,1]$ . We assume a probability density function that for small values of  $c_i$  has the form  $f(c_i) \propto c_i^{\gamma-1}$ ; i.e., the cumulative distribution  $F(c_i) = \int_0^{c_i} f(c)dc \propto c_i^\gamma$ . The exponent  $\gamma$  specifies the difficulty of reducing costs of individual components, with higher  $\gamma$  corresponding to higher difficulty. This functional form is fairly general in that it covers any distribution with a power-series expansion at  $c = 0$ .

### Independent Components

We first consider the simple but unrealistic case of a technology with  $n$  independent components. This generalizes the one component case originally studied by Muth (21). The cost of a given component at time  $t$  is equivalent to the minimum of  $t$  independent, identically distributed random variables. In *SI Text*, we use extreme value theory to show that to first order in  $n/t$  the expected cost  $E[\kappa(t)]$  is

$$E[\kappa(t)] = \Gamma\left(1 + \frac{1}{\gamma}\right) \left(\frac{t}{n}\right)^{-1/\gamma}, \quad [2]$$

where  $\Gamma(a)$  is Euler’s gamma function.

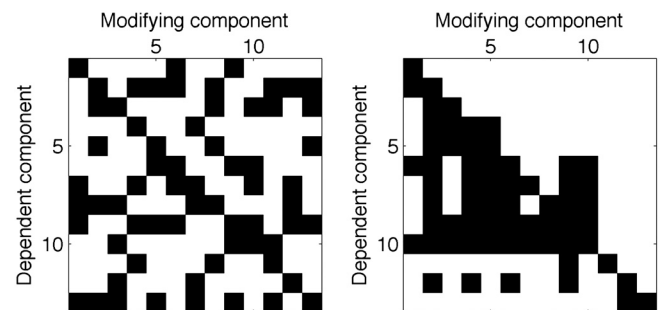


Fig. 2. Example design structure matrices (DSMs) with  $n = 13$  components. Black squares represent links. The DSM on the left was randomly generated to have fixed out-degree for each component. The DSM on the right represents the design of an automobile brake system (31). All diagonal elements are present because a component always affects its own cost.



1. Pick  $i$  and improve cluster  $\mathcal{A}_i$ .
2. Pick component  $j$  in the inset of  $i$  and improve cluster  $\mathcal{A}_j$ .

From Eq. 7, if component  $i$  has a large out-degree  $d_i$ , it is relatively unlikely to be improved by process 1. Nonetheless, if  $j$  has low out-degree, then  $i$  will improve more rapidly via process 2. Let  $d_j^i$  be the out-degree of component  $j$ , which is in the inset of  $i$ . Then the overall improvement rate of component  $i$  is determined by  $d_i^{\min} = \min_j \{d_j^i\}$ ; i.e., it is driven by the out-degree of the component  $j$  in its inset whose associated cluster  $\mathcal{A}_j$  is most likely to improve. In *SI Text*, we demonstrate numerically that asymptotically  $E[c_i] \sim t^{-1/d_i^{\min}}$ . As  $t$  becomes large, the difference in component costs can become quite dramatic, with the components with the largest values of  $d_i^{\min}$  dominating. The overall improvement rate for the whole technology is then determined by the slowest-improving components, governed by the design complexity

$$d^* = \max_i \{d_i^{\min}\}. \quad [8]$$

We call any component with  $d_i^{\min} = d^*$  a bottleneck. When  $t$  is large one can neglect all but the bottleneck components, and as we show in *SI Text*, the average total cost scales as  $E[k] \sim t^{-1/d^*}$ . Note that in the case of constant out-degree  $d$  Eq. 8 reduces to  $d^* = d$ .

To test this hypothesis we randomly generated 90 different DSMs with values of  $d^*$  ranging from 1 to 9 and  $\gamma = 1$ , simulated the model 300 times for each DSM, measured the corresponding average rate of improvement, and compared with that predicted from the theory. We find good agreement in every case, as demonstrated in *SI Text*.

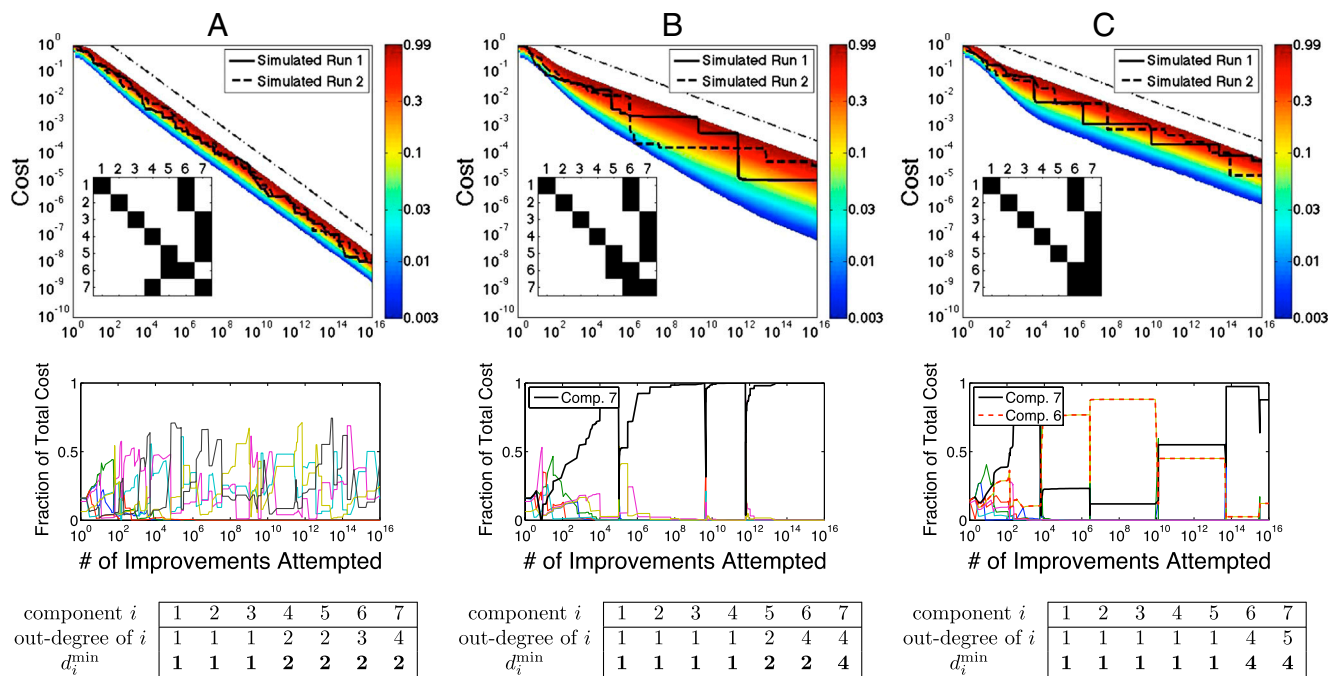
### Fluctuations

The analysis we have given provides insight not only about the mean behavior, but also about fluctuations about the mean. These can be substantial, and depend on the properties of the DSM. In Fig. 5 we plot two individual trajectories of cost vs. time for each of three different DSMs. The trajectories fluctuate in every case, but the amplitude of fluctuations is highly variable. In Fig. 5 *Left* the amplitude of the fluctuations remains relatively small and is roughly constant in time when plotted on double logarithmic scale (indicating that the amplitude of the fluctuations is always proportional to the mean). For Fig. 5 *Center* and *Right*, in contrast, the individual trajectories show a random staircase behavior, and the amplitude of the fluctuations continues to grow for a longer time.

This behavior can be explained in terms of the improvement rates  $d_i^{\min}$  for each component. The maximum value of  $d_i^{\min}$  determines the slowest-improving components. In Fig. 5 the maximum value of  $d_i^{\min} = 2$ . This value occurs for four components. After a long time these four components dominate the overall cost. However, because they have the same values of  $d_i^{\min}$  their contributions remain comparable, and the total cost is averaged over all four of them, keeping the fluctuations relatively small. (See Fig. 5 *Lower*.)

In contrast, in Fig. 5 *Center* we illustrate a DSM where the slowest-improving component (number 7) has  $d_7^{\min} = 4$  and the next slowest-improving component (number 6) has  $d_6^{\min} = 2$ . With the passage of time component 7 comes to dominate the cost. This component is slow to improve because it is rarely chosen for improvement. But in the rare cases that component 7 is chosen the improvements can be dramatic, generating large downward steps in its trajectory. The right case illustrates an intermediate situation where two components are dominant.

Another striking feature of the distribution of trajectories is the difference between the top and bottom envelopes of the plot



**Fig. 5.** Evolution of the distribution of costs. Each figure in the top row shows a simulated distribution of costs as a function of time using the DSM in the lower left corner of each plot. The upper dash-dot lines provides a reference with the predicted slope  $\alpha = 1/(\gamma d^*)$ , with  $\gamma = 1$ ; from left to right the slopes are  $-1/2$ ,  $-1/4$ , and  $-1/4$ . The data for each DSM are the result of 50,000 realizations, corresponding to different random number seeds. The distributions are color coded to correspond to constant quantiles; i.e., the fraction of costs less than a given value at a given time. The solid and dashed black curves inside the colored regions represent two sample trajectories of the total cost as a function of time. The DSMs are constructed so that in each case component 1 has the lowest out-degree and component 7 has the highest out-degree. Below each distribution we plot the fraction of the total cost contributed by each of the 7 components at any given time (corresponding to the first simulation run). The components in *B* and *C* with the biggest contribution to the cost in the limit  $t \rightarrow \infty$  are highlighted. The box at the bottom gives the value of  $d_i^{\min}$  for each component of the design.



any of them can potentially be described by the model we have developed.

Our analysis makes a unique contribution by connecting the literature on the historical analysis of performance curves to that on the engineering design properties of a technology. We make a prediction about how the features of a design influence its rate of improvement, focusing attention on the interactions of components as codified in the design structure matrix. Perhaps most importantly, we pose several falsifiable propositions. Our analysis illustrates how the same evolutionary process can display either historical contingency or steady change, depending on the design.

Our theory suggests that it may be possible to influence the long-term rate of improvement of a technology by reducing the connectivity between the components. Such an understanding of how the design features of a technology affect its evolution could aid engineering design, as well as science and technology policy.

**ACKNOWLEDGMENTS.** We thank Yonathon Schwarzkopf and Aaron Cluset for helpful conversations and suggestions, and Sidharth Rupani and Daniel Whitney for useful correspondence. J.M., J.D.F., and J.E.T. gratefully acknowledge financial support from National Science Foundation (NSF) Grant SBE0738187; S.R. acknowledges support from NSF Grant DMR0535503.

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