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NOTE

Technical change and the profit rate: a simple proof of the Okishio theorem

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This note presents a simple geometrical proof of Okishio's celebrated theorem on the falling rate of profit (Okishio, 1961). My intent is simply to clarify how Okishio's theorem 'works'. I will thus refrain from comment on the general debate on the falling rate of profit in Marxian economics, on the limits of the general linear model used here, or on the importance and range of applicability of Okishio's contribution. To those unfamiliar with the general linear model, and the Frobenius theorems used here, I recommend reference to Brody (1970) or Pasinetti (1977).

The essential economic assumptions underlying the following proof are that there are no scarce non-produced means of production ('no land'), that each 'industry' produces a single product (no joint products), that labour is homogeneous, and that there are no fixed capital goods (the period of production of all goods and the turnover time of all capital goods is one period). These are Okishio's assumptions. Recent contributions (those of Roemer, 1979, for example) have generalized Okishio's result somewhat.

Using the usual notation I will let

A = the production matrix, whose elements, a_{ij}, represent the amount of commodity i required to produce a unit of commodity j;

l = the (row) vector of direct labour inputs per unit of output of each commodity' a_{ij};

b = the (column) vector of wage bundle elements, b_i, representing the units of commodity i consumed by a worker in return for an hour's labour time.

The augmented input matrix, M, is constructed by summing the direct commodity input coefficients with the commodity inputs via the wage bundle, or $M = A+bl$ such that the elements of M, $M_{ij} = a_{ij}+b_i a_{ij}$.

If the hourly wage rate is w, and the competitive rate of profit is r^0, the vector of prices of production, or long run competitive prices can be written

$$p^0 = (1+r^0)\,(p^0A^0+wl^0)$$

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or, because $w = p$b (workers do not save),

$$p^0 = (1+r^0)p^0M^0 \qquad (1)$$

or

$$p^0\,[I-(1+r^0)M^0] = 0$$

from which it can be seen that p^0 is the left eigenvector of M^0. The profit rate can therefore be expressed as

$$r^0 = \frac{1-\alpha^0}{\alpha^0}$$

where α^0 is the maximal eigenvalue of M^0.

Using the above familiar results, I can now prove Okishio's theorem, namely: that *if all goods exchange at their prices of production, and the wage bundle is unchanged, for any new technology for the production of good i whose introduction, at the prevailing prices of production, yields super profits to an individual capitalist, the effect of a general introduction of this technology and of the associated changes in prices throughout the economy will be to raise the competitive rate of profit.*

Proof

Consider some new technology producing commodity i. Represent it as a new vector m_i^n of the augmented input matrix, M. Represent the original sector i technology as m_i^0. If the new technology is to be introduced, it must generate super profits:

$$p^0{}_i - p^0m_i{}^n(1+r^0) > 0 \qquad (2)$$

where, as before, p_i^0 and the price vector p^0 represent prevailing prices defined by (1) and r^0 is the general prevailing rate of profit. Inequality (2) defines a 'profitable' innovation, i.e. one that an individual capitalist would introduce, or one that would yield super profits.

The definition of a profitable technological innovation may be illustrated in Figure 1. The original technology, is indicated by the ray m_i^0, with its per unit inputs at point m^0. Unit input costs (p^0m^0) are constant along the isocost line ab. Any new technology with less of at least one input per unit of output and more of none, say m_i^*, is clearly a profitable innovation, as it must lie inside the isocost line and thus will satisfy the profitability condition (2). Equivalently, we know (from a theorem of Frobenius) that reducing any element of M while increasing none will lower its maximal eigenvalue, thus raising the associated rate of profit. Thus by Frobenius' theorem, the general introduction of the technology whose unit inputs are indicated by point m^* in Figure 1 will raise the rate of profit.

But we cannot in general represent all 'profitable' innovations as a simple reduction in some element in M: the new technology, m_i^n, may have some element larger than m_i. This new technology, whose unit inputs are indicated in Figure 1 by point m^n, lies inside the isocost line ab and thus satisfies the profitability inequality (2). But because $m^n{}_{ji} > m^0{}_{ji}$, Frobenius' theorem does not directly apply.

To apply this theorem we construct a hypothetical technology, whose unit input requirements, indicated in Figure 1 by point m^1, have been constructed in the following way. Start with m^n, the actual new technology. Holding constant $m^n{}_{ki}$, increase m_{ji} until the profitability equation (2) is rendered an equality. This procedure produces the input requirements of the hypothetical technology, indicated by point m^1 lying on the isocost line *ab*.

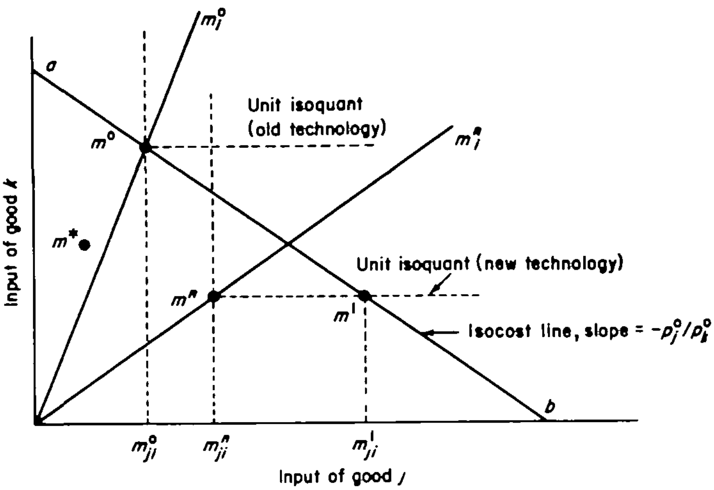

Fig. 1

The new price equations using m^1 are:

$$p^1 - p^1M^1\ (1+r^1) = 0 \tag{3}$$

But if m^1 satisfies (2) the super profit equation, at the *prevailing* prices and profit rate as an *equality* (by construction) we can write:

$$p^0 - p^0M^1\ (1+r^0) = 0 \tag{4}$$

Thus the introduction of the hypothetical technology m_i^1 does not disturb the prevailing prices (by construction) nor the prevailing profit rate. Thus $r^0 = r^2$. Why is this? Prices are defined such that profits on total capital invested is the same for each capitalist, and are therefore equal to total costs marked up by the going profit rate (1). If, by construction, costs for the ith capitalist do not change, the output price which maintains the going rate of profit is also unchanged (4). This is true even if the composition of costs changes. If the price of the ith output does not change, then the costs of production of all other capitalists remain constant and there is no price adjustment elsewhere in the system. Hence all prices remain unchanged at the going rate of profit.

Now M^1 differs from M^n only by the enlarged m^1_{ji} coefficient in the hypothetical m_i^1. Thus $\alpha^1 > \alpha^n$ and thus $r^1 < r^n$. But $r^0 = r^1$, so $r^0 < r^n$. QED

When m_i^n is adopted the new prices of production will be given by the solution of

$$p^nM^n(1+r^n) = p^n \tag{5}$$

Suppose at these new prices that

$$p^n - p^nM^0\ (1+r^n) > 0 \tag{6}$$

Then following the analysis above

$$r^0 < r^n < r^0$$

But this is impossible, for it contradicts Frobenius' theorem which demonstrates that for any M only the maximal eigenvalue (and hence only *one* value of r) is economically meaningful, i.e. has associated with it non-negative prices. Thus (6) will not occur and the choice of technique is unambiguous.

This result clearly does not disprove the tendency of the profit rate to fall. It does, however, show that under the assumptions used here, no pattern of technical change (whether labour saving or not) can produce a lower competitive rate of profit as long as commodities exchange at their prices of production and the wage bundle is unaffected. (If good i is non-basic, the profit rate will be unaffected.)

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