Optimal Incentives with State-Dependent Preferences

Sung-ha Hwang
Sogang University

Samuel Bowles
Santa Fe Institute

Abstract

In both experimental and natural settings, incentives sometimes underperform, generating smaller effects on the targeted behaviors than would be predicted for entirely self-regarding agents. A parsimonious explanation is that incentives that appeal to self-regarding economic motives may crowd out noneconomic motives such as altruism, reciprocity, intrinsic motivation, ethical values, and other social preferences, leading to disappointing and sometimes even counterproductive incentive effects. We present evidence from behavioral experiments that crowding may take two forms: categorical (the effect on preferences depends only on the presence or absence of the incentive) or marginal (the effect depends on the extent of the incentive). We extend an earlier contribution (Bowles and Hwang, 2008) to include categorical crowding, thus providing a more general framework for the study of optimal incentives and as a result, an expanded range of situations for which the sophisticated planner will (surprisingly) make greater use of

[Corrections added on August 27, 2014, after online publication: The authors were listing in incorrect order; the order is correct as it appears above.]

Sung-ha Hwang, School of Economics, Sogang University, 35, Baekbeom-ro, Seoul, Korea (sunghah@sogang.ac.kr). Samuel Bowles, Santa Fe Institute (samuel.bowles@gmail.com).

We thank Margaret Alexander, Lopamudra Banerjee, Ernst Fehr, Duncan Foley, John Geanakoplos, Bernd Irlenbusch, Suresh Naidu, Seung-Yun Oh, Carlos Rodriguez-Sickert, Sandra Polanía-Reyes, John Roemer, Bob Rowthorn, Paul Seabright, Rajiv Sethi, Joaquim Silvestre, Peter Skott, Joel Sobel, E. Somnathan, Tim Taylor, Elisabeth Wood, Giulio Zanella, and two anonymous referees for comments, and the Behavioral Sciences Program of the Santa Fe Institute and the U.S. National Science Foundation for support of this project. Sung-Ha Hwang would also like to thank the Korea Research Foundation Grant funded by the Korean Government (NRF-2012S1A5A8022822).

Received November 28, 2011; Accepted September 26, 2012.

© 2013 Wiley Periodicals, Inc.
incentives when incentives crowd out social preferences than when motivational crowding is absent.

1. Introduction

The standard problem facing a social planner seeking to induce a target population to act more prosocially (to contribute to a public good, for example) is to select an incentive, the effect of which will modify the material costs or benefits of the targeted activity so as to induce the desired behavior. But when incentives affect preferences, the sophisticated planner (aware of this complication) must also take account of the fact that incentives may adversely affect the targeted individuals’ noneconomic motivations that will also influence their actions, thus possibly reducing or even reversing the intended effect of the incentive. If the effect of the subsidy were separable in the incentives and the citizen’s noneconomic preferences, so that the use of an incentive had no effect on the target’s preexisting altruism or intrinsic pleasure in acting prosocially, then the presence of these noneconomic motivations would present no particular problem for the planner. But if the citizen’s noneconomic motivations are compromised by the implementation of the incentive, then the planner must take into account not only the direct effect of the incentives but also the possibly adverse indirect effects which occur when incentives crowd out social preferences. In this case, incentives and social preferences are not separable, but instead are substitutes.

Both experiments and natural observations provide evidence of this nonseparability of incentives and social preferences and suggest that the crowding out variant may be quite common (surveyed in Bowles 2008 and Bowles and Polanía-Reyes 2012). A growing empirical literature has explored the relationship between incentives and performance where the targets of the incentives are motivated by both conventional and social preferences (Bewley 1999; Young and Burke 2001; Bandiera, Barankay, and Rasul 2005; Fehr and Goette 2007; Fehr and Schmidt 2007). Moreover, a number of economic models have provided psychologically plausible mechanisms that may account for the crowding phenomenon (Bar-Gill and Fershtman 2004; Benabou and Tirole 2006; Falk and Kosfeld 2006). For example, Bar-Gill and Fershtman (2005) provide a model in which a subsidy crowds out altruism by triggering an endogenous preference change. Finally, the neurological pathways whereby economic incentives may diminish prosocial behavior are beginning to be identified (Li et al. 2009).

In contrast to this literature, here we do not address the extent or causes of nonseparability, but instead like Frey (1999), Diamond (2006), and Funfgelt and Baumgartner (2012) investigate the implications for the optimal use of explicit economic incentives (which we will call simply incentives). We are particularly interested in whether incentives should be used less when they crowd out social preferences, as is commonly thought.
A critical distinction that was absent from our earlier work (Bowles and Hwang 2008) is that between a categorical form of crowding out, in which the mere presence of the incentive adversely affects social preferences, and a marginal form of crowding out in which the citizen’s noneconomic motivations to contribute vary inversely with the level of the incentive. We will see that categorical and marginal crowding out have different effects on optimal incentives and that where (as seems plausible) the net benefit function is concave in the amounts contributed to the public good, in the presence of categorical crowding out (the case we did not address in our earlier work), the sophisticated planner will make more extensive use of incentives by comparison with the naive planner who thinks that preferences and incentives are separable. In this sense, our earlier work is incomplete because if the sophisticated planner considers only marginal crowding out she will miss a possibly large class of cases in which crowding out calls for more use of incentives rather than less. The current paper differs from our 2008 model additionally because we here consider an alternative planners’ objective function in which the planner takes account of the effects of motivational crowding out on the citizen’s behavior but otherwise abstracts from the incentive’s negative effects on the citizen’s pleasure in giving. We explain these alternative planners’ objective functions in Section 4, and show in the concluding section that our main results hold for both objective functions.

2. Categorical and Marginal Crowding Out: Evidence

Because of the importance of the categorical–marginal distinction, we begin with a recent experiment that allows an estimate of both categorical and marginal crowding. Irlenbusch and Ruchala (2008) implemented a public goods experiment in which subjects faced three conditions: no incentives to contribute and a bonus given to the highest contributing individual that was either high or low. Payoffs in the games were such that even with no incentive individuals would maximize their income by contributing 25 units. In the no-incentive case, contributions averaged 37 units, or 48% above what would have occurred if the participants had been motivated only by the material rewards of the game, suggesting substantial effects of social preferences. Contributions in the low-bonus case were not significantly different from the no-bonus treatment. In the high-bonus case, however, significantly higher contributions occurred, but the amount contributed (53 units) barely (and insignificantly) exceeded that predicted for self-interested subjects (50 units). Thus while the high incentive “worked” (it increased contributions 43% over the no incentive case) it appears to have done this by substituting own-income-maximizing preferences for social preferences.

Bowles and Polania-Reyes (2012) estimated the marginal effect of the bonus in the Irlenbusch and Ruchala (2008) experiment using the observed
behavior in the high and low bonus case along with the assumption that marginal crowding affects the slope of the citizens’ best response function by a given amount (so that the function remains linear). The results are shown in Figure 1.

Comparing the high and low bonus cases, they found that a unit increase in the bonus is associated with a 0.31 increase in contributions. This contrasts with the marginal effect of 0.42 that would have occurred under separability, that is, had own-income-maximizing subjects simply best responded to the incentive. Crowding out thus affected a 26% reduction in the marginal effect of the incentive. The estimated response to the incentive also gives us the level of categorical crowding out, namely the difference between the observed contributions in the absence of any incentive (37.04) and the predicted contributions had an arbitrarily small incentive been in effect (the vertical intercept of the “observed” line in Figure 1) or 34.55. The incentive thus categorically crowded out 21% of the effect of social preferences (measured by the excess in contribution levels above prediction for self-interested subjects, 12.04.)

Categorical crowding out is also evident in three experiments by Heyman and Ariely (2004). For example, reported willingness to help a stranger load a sofa into a van was much lower under a small money incentive than with no incentive at all, yet a moderate incentive increased the willingness.
to help (over the no incentive condition). Using these data as they did for the Irlenbusch and Ruchala study, Bowles and Polanía-Reyes estimated that the mere presence of the incentive reduced the willingness to help by 27% (compared to the no incentive condition).

3. The Effect of Incentives on Individual Choice and Nash Equilibrium Contributions

To formalize these categorical and marginal incentive effects, we need to model the influence of incentives on preferences. Incentives may change preferences, or may alter the motivational salience of a variety of preexisting preferences. The distinction may be clarified in the Irlenbusch–Ruchala setup: If a group of individuals accustomed to interacting under conditions similar to their high bonus were to occasionally find themselves in a no bonus situation, would they contribute the self regarding 25 units, because the exposure to the high bonus situation had given them self-regarding preferences? Or would they contribute 37 units, because in the no bonus situation individuals would consider acting on self-regarding preferences to be inappropriate? Here, we model the second case, namely, where incentives temporarily alter the salience of an individual’s heterogeneous motives. In this case, we say that preferences are state-dependent, with the presence or extent of the incentive defining distinct states.\(^1\)

The key psychological insight captured by our model is that individuals have a multiplicity of motives—self-regarding, altruistic, spiteful, and so on—the behavioral salience of which varies with the situation (Ross and Nisbett 1991). Incentives affect preferences in this case because they alter the situation, providing cues as to whether the setting is more like, say, shopping, or like interacting with a close friend or family member. A psychologist would call these preferences situation-dependent, with incentives constituting the situation. Thus, the preference function is an evaluation of states that the individual’s action may bring about that is itself dependent on the current situation of the individual, and the latter varies according to the nature of incentives present. The other mechanism by which incentives may affect preferences—endogenous preferences—involves learning, resulting in a durable (not temporary) change in the preference function itself so that an individual who has learned new preference will subsequently behave differently in a given state.

To model the effects of state-dependent preferences on optimal incentives, consider a community of identical individuals indexed by \(i = 1, \ldots, n\) who may contribute to a public project an amount \(a_i \in [0, 1]\). The total

\(^1\) In the first case above, incentives affect the process by which new preferences are learned, so that preferences are endogenous rather than state-dependent. We address this case in Hwang and Bowles (2012).
contributions, $\sum_j a_j$, result in a benefit to each citizen of $\phi(\sum_j a_j)$, where $\phi(0) = 0$ and $\phi' > 0$. Each individual experiences the cost of contribution $g(a_i)$ where $g(0) = g'(0) = 0$, $g' > 0$, and $g'' > 0$. Incentives are provided in the form of a subsidy $s$ that is proportional to the amount contributed. (By “incentive” we mean an intervention intended to influence an individual’s behavior by altering the economic costs or benefits of some targeted activity).

To model the crowding problem, we need to distinguish between an individual’s social preferences in the absence of incentives and the preferences that will account for his behavior. The two will of course be identical in the absence of incentives, but unless separability holds will be different otherwise. The former, social preferences in the absence of incentives, is a latent motivation which is here taken as exogenous and termed baseline social preferences (or just social preferences where no ambiguity will result.) The latter are realized preferences that vary with the presence and level of the incentives and that determine the citizen’s behavior and we term these values.

We implement this distinction by assuming that depending on the incentives in force each citizen has “values” $v$ that motivate prosocial behaviors. (The relevant values include ethnical norms, a positive valuation on the well-being of others, intrinsic pleasures of cooperation _per se_, and other motives. But taking account of the multidimensional nature of the relevant values would not illuminate the question we address here.) Thus citizen $i$’s utility is

$$u(a_i) = \phi \left( \sum_j a_j \right) - g(a_i) + s a_i + v(s) a_i,$$  \hspace{1cm} (1)

which makes it clear that the total effect of the subsidy on the citizen’s action is a direct effect operating via the net costs of the targeted action (the penultimate term on the right-hand side) plus a possible indirect effect operating via the influence of the subsidy on citizen’s values (the final term on the right-hand side).

To characterize the effects of categorical crowding and marginal crowding in terms of parameters, we adopt the following functional forms for the value function:

$$v(s) = \lambda_0 (1 + 1_{\{s > 0\}} \lambda_c + s \lambda_m),$$  \hspace{1cm} (2)

where $\lambda_0 \geq 0$ is the individual’s baseline social preference motives to contribute to the public good in the absence of an incentive, the indicator variable $1_{\{s > 0\}}$ takes the value of 1 when $s > 0$ and zero otherwise, and the “crowding parameters” $\lambda_c$ and $\lambda_m$ are the categorical and marginal effects of incentives representing the state-dependent nature of preferences (preferences would be endogenous if variations in $s$ affected these or other parameters of the value function, a case we do not address here).
From (1) and (2), we obtain citizen $i$’s best response $a^{BR}_i := a^{BR}_i (a_{-i}, s)$, which is implicitly given by

$$g'(a^{BR}_i) = \phi' \left( a^{BR}_i + \sum_{j \neq i} a_j \right) + s + \lambda_0 (1 + \mathbf{1}_{s > 0} \lambda_c + s \lambda_m). \quad (3)$$

The last two terms in the right-hand side of (3) show that the introduction of a subsidy may increase contributions by raising the marginal benefits of contributing which we denote

$$\beta := s + \lambda_0 (1 + \mathbf{1}_{s > 0} \lambda_c + s \lambda_m).$$

Considering the case in which there initially is no incentive, the effect of the introduction of an incentive on the net benefits of contributing (expressed in discrete terms so as to be able to account for the discontinuity in the value function at $s = 0$) is

$$\left. \frac{\Delta \beta}{\Delta s} \right|_{s = 0} = 1 + \lambda_0 \left( \frac{\lambda_c}{\Delta s} + \lambda_m \right) \quad (4)$$

and is composed (as expected) of a direct effect (the first term on the right-hand side) and the indirect effect which will be negative in the case of crowding out, and larger in absolute value the greater are the baseline social preferences of the individual ($\lambda_0$). When the subsidy level is positive, we find

$$\left. \frac{\Delta \beta}{\Delta s} \right|_{s > 0} = 1 + \lambda_0 \lambda_m, \quad (5)$$

We likewise see that

$$\left. \frac{\Delta \beta}{\Delta \lambda_0} \right|_{s > 0} = 1 + \mathbf{1}_{s > 0} \lambda_c + s \lambda_m, \quad (6)$$

which in the case of crowding out is declining in $s$. Equations (4), (5), and (6) make it clear that, when $\lambda_c$ and $\lambda_m$ are negative, incentives and baseline social preferences are substitutes in providing the motivation to contribute to the public good: the marginal effect of each varies inversely with the level of the other. Thus in the presence of crowding out, the adverse effects of incentives will be greater for individuals with a high level of baseline social preferences (for entirely self-interested individuals there is nothing to crowd out), which is what observed in experiments (Bohnet and Baytelman 2007; Kessler 2008). If instead the second term on the right-hand side of (4) or (5) is positive incentives and social preferences are complements.

Using (4) and (5), we say that categorical crowding out obtains if $\frac{\Delta \beta}{\Delta s} < 1$ for $s = 0$ and for sufficiently small $\Delta s$ marginal crowding out obtains if $\frac{\partial \beta}{\partial s} < 1$ for $s > 0$. If the signs are reversed, we have categorical and marginal crowding in. Note that when $\Delta \beta / \Delta s < 1$, the total effect of the incentive is less than the direct effect (and conversely for the case of crowding in). Crowding out
will not occur if \( \lambda_c \) and \( \lambda_m \) or \( \lambda_0 \) are zero (values are not state-dependent, or they are absent). Strong crowding out holds if \( \Delta \beta / \Delta s < 0 \) which can occur if categorical crowding out is large relative to the size and marginal effect of the subsidy, or if the marginal effect is negative. Note that crowding out does not require that the effect of the incentive be the opposite of that intended, only that it be less than would be the case were \( \lambda_c \) and \( \lambda_m \) or \( \lambda_0 \) zero.

Because citizens are identical and the subsidy nondiscriminatory, there will be a symmetric Nash equilibrium in which everyone contributes the same amount to the public project. We discuss the detailed conditions for the existence of such equilibrium in the Appendix. We thus can drop the individual subscript and using (3) note that in order to be a mutual best response, the Nash contribution \( a^* \) must satisfy

\[
\phi'(na^*) - g'(a^*) + s + \lambda_0(1 + 1_{s>0}\lambda_c + s\lambda_m) = 0. \tag{7}
\]

To study the stability of the Nash equilibrium, we consider a myopic best response dynamic whereby citizens maximize their utility, conditional on the contributions of others in the previous period (see the Appendix). We find that the condition for the asymptotic stability of the Nash equilibrium is given by

\[
n \left| \phi''(na^*) \right| - g''(a^*) < 0, \tag{8}
\]
a condition that ensures that the series of reciprocal effects of the citizens’ actions converges (given the convex cost function, (8) is satisfied if the benefit function is concave or “not too convex” given the number of group members).

The public project’s net benefit function, defined as \( \phi(na) - g(a) \), plays an important role in what follows. We make the following assumptions on the net benefit function.

**ASSUMPTION 1 (A1):** In the absence of a subsidy, citizens undercontribute to the project: the net benefit function is increasing in the level of contribution (i.e., \( n\phi'(na) - g'(a) > 0 \), where \( a \) denotes the contribution level without subsidy.

**ASSUMPTION 2 (A2):** The Nash equilibrium \( a^* \) is asymptotically stable so that (8) holds: \( n |\phi''(na^*)| - g''(a^*) < 0 \).

The sophisticated planner affects citizens’ contributions by selecting \( s \) to implement a Nash equilibrium given by (7), implying the “implementation technologies” for alternative values of the crowding parameters illustrated in Figure 2: \( a^*(s, \lambda_m, \lambda_c) \).

When the level of subsidy \( s \) is positive, using (7) we find expressions of the derivatives of \( a^* \) with respect to \( s, \lambda_c, \lambda_m \) as follows:

\[
\frac{\partial a^*}{\partial s} = \frac{1 + \lambda_0 \lambda_m}{g'' - n\phi''}, \quad \frac{\partial a^*}{\partial \lambda_c} = \frac{\lambda_0}{g'' - n\phi''}, \quad \frac{\partial a^*}{\partial \lambda_m} = \frac{s \lambda_0}{g'' - n\phi''}. \tag{9}
\]
A2 ensures that the signs of $\partial a^*/\partial s$, $\partial a^*/\partial \lambda_c$, and $\partial a^*/\partial \lambda_m$ are all positive. Note that the effect of variations in the subsidy on the Nash contribution level depends not only on the sum of the direct incentive effect and its crowding effect $(1 + \lambda_0 \lambda_m)$ but also (inversely) on the rate at which the individual’s marginal net benefits diminish with increases in the contribution level $(g''(a) - \phi''(na))$. Thus strongly increasing marginal costs of contribution or diminishing marginal returns to the project dampen the effect of the subsidy. The same is true of the effects of variations in the two crowding parameters, as the two other equations in (9) make clear.

Note from the definition of crowding in and out, Equation (3) (the individual best response) and Figure 2 that crowding out may be present even in the absence of a negative effect of the subsidy on the individual’s action or on the Nash equilibrium $(\partial a^*/\partial s)$. In the case of marginal crowding out, the effect of the incentive on the individual’s action will be positive unless the marginal crowding out effect on values more than offsets the effect of incentives on the citizen’s payoffs. In the case of categorical crowding out the effect on individual and Nash equilibrium contributions will be positive for a sufficiently large subsidy (provided that strong crowding out does not hold).

4. The Planner’s Problem when Preferences Are State-Dependent

We now turn to the problem of a social planner who chooses a subsidy level to maximize the net benefits of the public project, taking account not only of
the direct effect of the incentive on the individual’s private marginal benefits of contributing but also the effect of the subsidy on the citizen’s preferences and thereby indirectly on the Nash equilibrium contribution levels. We suppose that there is a cost per citizen of administering the subsidy \(c(s)\), where \(c'(0) = 0 = c(0)\), \(c'(s) > 0\), and \(c''(s) > 0\) for \(s > 0\). Since the sophisticated planner is aware of nonseparability, she correctly expects the citizens’ Nash equilibrium in response to the subsidy to be \(a^* = a^*(s, \lambda_c, \lambda_m)\) and selects \(s^*\). Her naive counterpart is a planner who ignores nonseparability assumes that no crowding out obtains \((\lambda_c = \lambda_m = 0)\); so to him the expected Nash equilibrium is simply \(a^N = a^*(0, 0, 0)\), that is, the top line in Figure 2. Except where crowding is absent, the two planners will select different levels of subsidy. We say that the subsidy is overused by the naive planner if the subsidy he selects, \(s^N\) exceeds that selected by the sophisticated planner, \(s^*\).

Because citizens are identical, the sophisticated social planner’s optimizing problem can be reduced to maximizing the net benefits of the public project for a single citizen, and written as

\[
\max_{a \in [0,1], s \in [0,1]} \omega(a, s) := \phi(na) - g(a) - c(s) \tag{10}
\]

subject to \(a = a^*(s, \lambda_c, \lambda_m)\).

To exclude uninteresting cases, we assume that \(c(s)\) is sufficiently convex so that there is a unique interior solution of (10) (see the Appendix). Notice we assume that the social planner treats the citizens’ values as a component of individual motivation, but does not include them in her objective function (10). Thus, from a normative point of view we exclude from the planner’s welfare function the citizen’s subjective pleasure of contributing, and restrict the effect of the project on social welfare to its conventionally defined benefits and costs. See Bowles and Hwang (2008) for the alternative case in which the citizens’ values are included not only in the citizens’ best response functions but also in the planner’ objective function. In this latter case, if crowding out obtains, incentives reduce the citizen’s preexisting pleasure of contributing, and the sophisticated planner takes account of this not only in anticipating the citizen’s behavior (as we do here) but also in evaluating the outcome.

To investigate underuse and overuse of incentives by the naive planner, we define the marginal rate of substitution \((\sigma)\) and the marginal rate of transformation \((\tau)\):

\[
\tau(a, \lambda_m) := \frac{1 + \lambda_0 \lambda_m}{g''(a) - n\phi''(na)}, \quad \sigma(a, s) := \frac{c'(s)}{n\phi'(na) - g'(a)}. \tag{11}
\]

The marginal rate of transformation, evaluated at the citizen’s Nash response \(a^*(s)\), is just the effectiveness of the subsidy in altering Nash contributions (from (9), the slope of the implementation functions). The marginal rate of substitution is the ratio of the planner’s marginal costs of varying the
subsidy to the marginal benefits to the planner of variations in the contribution level (the slope of planner’s indifference loci). Then, the first-order condition for an interior equilibrium of the sophisticated planner’s problem equates the marginal rate of transformation of the subsidy into the provision of public good with the marginal rate of substitution between the public good and the subsidy:

$$\tau(a(s, \lambda_c, \lambda_m), \lambda_m) - \sigma(a^*(s, \lambda_c, \lambda_m), s) = 0. \quad (12)$$

5. The Effect of Crowding Out on Optimal Incentives

Crowding out will affect both the marginal rate of substitution and the marginal rate of transformation in (12). Consider first categorical crowding out and its effect on the optimal subsidy, taking the naive planner’s subsidy $s^N$ as the status quo.

This shifts downward the planners’ implementation technology, reducing the provision of the public good and thereby increasing or decreasing the marginal net benefit of public good provision (if the benefit function is, respectively, concave or convex). The result is that at the given subsidy level ($s^N$) and resulting citizen’s contribution level the planner’s indifference locus will be flatter if the net benefit function is concave as shown in Panel A of Figure 3 (and steeper if it is convex). We will see below that categorical crowding may (but need not) also affect the marginal rate of transformation, but in Figure 3 we show a case where it does not. In the case illustrated the net benefit function is concave so the reduced public goods provision raises the marginal net benefit of the public good, flattening the isobenefit locus. This flattening of the planner’s indifference loci, shown in Figure 3, Panel A, displaces the optimal subsidy to the right, so that the presence of crowding out increases the optimal subsidy (from $s^N$ to $s^*$.)

Marginal crowding out has two effects. The first is the reduction in the marginal effectiveness of the subsidy, which reduces the marginal rate of transformation (“flattens” the planner’s implementation function, as shown in Panel B, Figure 3). Second, as in the case of categorical crowding out, the reduced provision of the public good alters the marginal rate of substitution, flattening the planner’s indifference locus if the net benefit function is concave. Which of these effects dominates determines the effect of crowding out on the optimal subsidy. In Panel B of Figure 3, we illustrate the case in which marginal crowding out increases the optimal subsidy because the heightened marginal net benefit of the public good more than offsets the reduced effectiveness of the subsidy. (If the net benefit function is convex, marginal crowding out unambiguously induces a reduction in the sophisticated planner’s $s^*$ because both effects work in the same direction: the incentive is less effective and the marginal net benefits of the project are reduced by the lesser level of provision.)
Figure 3: The naive planner’s underuse of incentives when crowding out occurs: illustration of Proposition 1. To illustrate the intuition behind categorical crowding out leading to underuse of the subsidy by the naive planner, the case shown assumes $\phi'' = 0 = g''$ so that the reduction in $a$ occasioned by categorical crowding out does not alter the marginal rate of transformation. In the case of marginal crowding out, either underuse or overuse may occur. Panel B shows the joint effect of the reduction in the marginal rate of substitution between $a$ and $s$ (the latter indifference locus shifting $s^*$ to the right) and the reduced marginal rate of transformation between $a$ and $s$ (the latter implementation function) partially offsetting the first effect.

We can now formalize these intuitions. First, we find the effect of categorical crowding on the optimal subsidy ($ds^*/d\lambda_c$). To do this, we totally differentiate (12) with respect to $s$ and $\lambda_c$ and find that

$$\frac{ds^*}{d\lambda_c} = -\frac{1}{\left(\frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a}\right)\frac{\partial a^*}{\partial s} - \frac{\partial \sigma}{\partial s}} \left[\frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a}\right] \frac{\partial a^*}{\partial \lambda_c}. \quad (13)$$

The second-order condition for the social planner’s maximization problem requires that $c(s)$ is sufficiently convex and thus the denominator of (13) (which is just the derivative of the left-hand side of (12) with respect to $s$) is negative. Note that when categorical crowding out occurs (i.e., $\lambda_c$ decreases from 0 to a negative value), the contribution level will be reduced ($\partial a^*/\partial \lambda_c > 0$), so the sign of $ds^*/d\lambda_c$ depends on $\partial \tau/\partial a - \partial \sigma/\partial a$.

Concerning marginal crowding, similarly we find

$$\frac{ds^*}{d\lambda_m} = -\frac{1}{\left(\frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a}\right)\frac{\partial a^*}{\partial s} - \frac{\partial \sigma}{\partial s}} \left[\frac{\partial \tau}{\partial \lambda_m} + \left(\frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a}\right)\frac{\partial a^*}{\partial \lambda_m}\right]. \quad (14)$$
which using (9) and (11) can be rewritten

\[
\frac{ds^*}{d\lambda_m} = -\frac{1}{\left(\frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a}\right) \frac{\partial a^*}{\partial s} - \frac{\partial \sigma}{\partial s}} \left[ 1 + \left(\frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a}\right) s^* \right] \frac{\lambda_0}{g'' - n\phi''}. \tag{15}
\]

Since the denominator of this expression is negative and the final term positive (by the condition for the stability of the Nash equilibrium), \(ds^*/d\lambda_m\) will have the same sign as the terms in the square bracket in (15). Using (13) and (15) we find:

**PROPOSITION 1:** (Underuse of Incentives by the Naive Planner When Incentives Crowd Out Social Preferences) Suppose that A1 and A1 hold and \(s^* > 0\). Then

\[
\frac{ds^*}{d\lambda_c} < 0 \quad \text{if and only if} \quad \frac{\partial \sigma}{\partial a}(a^N, \lambda_m) - \frac{\partial \tau}{\partial a}(a^N, s^N) > 0,
\]

\[
\frac{ds^*}{d\lambda_m} < 0 \quad \text{if and only if} \quad \frac{\partial \sigma}{\partial a}(a^N, \lambda_m) - \frac{\partial \tau}{\partial a}(a^N, s^N) > \frac{1}{s^N}.
\]

Notice that Proposition 1 provides the characterization of marginal and categorical crowding “locally”; it asserts that whenever the conditions on \(\tau\) and \(\sigma\) hold at the naive planner’s choice, a small decrease in \(\lambda_c\) (or \(\lambda_m\)) would raise the subsidy level selected by the sophisticated planner. Therefore, if the conditions in Proposition 1 hold at \(\lambda_c = \lambda_m = 0\), we may conclude that the underuse of incentives would occur if a small degree of crowding out obtains (see also Figure 4).

The proposition shows that, if categorical crowding out reduces the marginal rate of substitution (at \(s^N\)) by more than it reduces the marginal rate of transformation, it will induce the sophisticated planner to make greater use of incentives than her naive counterpart. In the case of marginal crowding out, the former effect must exceed the latter by \(1/s^N\) because if \(s^N\) is very small the change in the slope of the implementation function will dominate the flattening of the planner’s indifference loci. This is a result of the fact that for a small subsidy the reduction in contributions will be small (because with marginal crowding the implementation function does not shift down, it rotates clockwise), and hence the increase in the marginal net benefit of the public good will be insufficient to offset the decreased effectiveness of the subsidy.

Using (13) and (15), we also find

\[
\frac{ds^*}{d\lambda_m} = -\frac{1}{\left(\frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a}\right) \frac{\partial a^*}{\partial s} - \frac{\partial \sigma}{\partial s}} \frac{\partial a^*}{\partial \lambda_c} + s^* \frac{ds^*}{d\lambda_c} > 0.
\]

**PROPOSITION 2:** (Categorical Crowding Out vs. Marginal Crowding out) Suppose that A1 and A2 hold and \(s^* > 0\). The condition under which categorical
crowding out leads to the underuse by the naive planner is less stringent than the condition under which marginal crowding out leads to the underuse, i.e.,

\[
\frac{ds^*}{d\lambda_m} < 0 \text{ implies } \frac{ds^*}{d\lambda_c} < 0.
\]

Proposition 2 shows that the conditions for the parameters under which categorical crowding out leads to the underuse of incentives by the naive planner are more general than those for marginal crowding out and this suggests that underuse phenomena occurs more widely under categorical crowding out than marginal crowding out. This is because compared to the case of the categorical crowding, the marginal rate of transformation is directly affected by \(\lambda_m\) (the effect captured by \(\partial \tau / \partial \lambda_m\)). Therefore, for \(ds^*/d\lambda_m\) to be negative (leading to underuse of incentives by the naive planner), the flattening effect of crowding out on the indifference locus must be greater than the case of categorical crowding if it is to offset the direct effect of flattening of the best responses (see Figure 3, Panel B).

In Figure 4, to illustrate the economic intuitions underlying our results, we show categorical and marginal crowding out, adopting simple functional forms. In Panels A and B, moderate crowding out induces the sophisticated planner to adopt a larger subsidy than her naive counterpart, indicating underuse of the incentive by the naive planner. But there is a critical level of crowding out which if exceeded induces her either to abandon the incentive altogether (Panel A) or to adopt a lesser incentive than her naive counterpart. Notice that (as in Panel C of Figure 4) the sophisticated planner may adopt a lower subsidy than the naive planner (the naive planner overuses the incentive) irrespective of whether nonseparability takes the form of crowding in or crowding out. This, and the nonmonotonicity of \(s^*\) in \(\lambda_m\) in Panel B, occurs because the sign of the terms in brackets in (15) changes due to variations in contribution levels resulting from the variation in the crowding parameter. In panel C, we have chosen parameters such that the expression changes sign at \(\lambda_m = 0\).

6. Extensions and Modifications: Taxes, Additional Instruments, and the Utilitarian Planner

A natural question is to ask whether the same analysis applies to taxes. The simple answer is yes: the model is entirely unaffected by considering a tax on the citizen’s shortfall from the socially optimal contribution level \(a = 1\) rather than a subsidy on her contribution level. As an empirical matter, taxes might have a greater crowding out effect than subsidies if the mechanism by which crowding occurs is that the incentive conveys adverse information about the individual designing the incentive—penalties being more likely to convey this kind of “bad news” than rewards. (Evidence for this “bad news”
**Optimal Incentives with State-Dependent Preferences**

Figure 4: Sophisticated planner’s optimal subsidy illustrating underuse of incentives by the naive planner under crowding out. Negative (positive) values of $\lambda_c$ and $\lambda_m$ imply crowding out (in). In the case of the categorical crowding out (Panel A), the optimal subsidy increases as $\lambda_c$ decreases (meaning stronger categorical crowding out) until at $\lambda_c \approx -0.57$, the social planner will set $s^* = 0$. Panels B and C show the choices of the sophisticated planners in the case of marginal crowding, depending on different values of the parameters. In Panel B, for the case of crowding out (negative values of $\lambda_m$) for any level of marginal crowding out greater than $-4$, the sophisticated planner implements a larger subsidy than the naive planner. In Panels B and C, when $\lambda_m = -5$, the sophisticated planner abandons use of the subsidy entirely while larger negative values of $\lambda_m$ imply what we have defined as strong crowding out (the effect is opposite of that intended). For three panels: $\phi(\sum a_i) = 0.11 \sum a_i$, $\lambda_0 = 0.2$, $g(a_i) = \frac{k}{2}a_i^2$, $c(s) = \frac{\gamma}{2}s^2$. Panel A: $\gamma = 3$, $\kappa = 1$, $\lambda_m = -0.2$. Panel B: $\gamma = 1$, $\kappa = 0.2$, $\lambda_c = 0$. Panel C: $\gamma = 1$, $\kappa = 1$, $\lambda_c = 0$. See the Appendix for the definitions of parameters and analysis.

Interpretation of the crowding out phenomenon is surveyed in Bowles and Polanía-Reyes 2012.

How robust is the result that the sophisticated planner may make greater use of incentives than the naive planner? The underuse of incentives by the naive planner arises because the net benefit function is concave (so that shortfalls from the optimal provision of the public good are increasingly costly in welfare terms), and crowding out leads to a larger shortfall than would occur in its absence. If the only instrument available to the planner is the subsidy, then a sufficiently concave net benefits function will induce her to adopt a larger subsidy. But suppose the planner were to have instruments for the promotion of contributions to the public good other than incentives. She might, for example, promote publicity making the citizen’s duty to contribute more salient, as in Frey (1999). In this case, marginal crowding out would induce the sophisticated planner not only to devote more resources to enhancing contributions to the public good but also to substitute the alternative instrument for the subsidy. Thus, the conditions for underuse of the incentive by the sophisticated planner (Proposition 1) are more stringent.
when there exists an alternative instrument that is not subject to crowding out.

To consider a final extension, recall that our planner maximizes the conventionally measured net benefits of the public project, taking account of the behavioral responses of the citizens induced by their social preferences but disregarding the possibly negative effect of incentives on the citizen’s ethical or intrinsic pleasure of contributing. But ought the planner instead to consider the citizen’s ethical values hedonistically, treating them as simply another form of “tastes,” the satisfaction of which is pleasurable to the citizen and which the planner should consider relevant in her evaluation of the outcomes of her policies? Like others (for example, Diamond 2006 and Bergstrom 2006) we question whether the preferences that explain individual behavior should necessarily constitute the planner’s objective function.

The problem is not specific to the case of ethical and other social preferences. It arises because individual utility functions play both a positive and a normative role in public economics—explaining individual behaviors and how they are effected by alternative public policies, and at the same time providing a basis for the evaluation of the outcomes of the policies under study. It is far from obvious that the same concept can perform both tasks. Diamond (2006), after having presented the citizen’s response to incentives based on the individual’s utility function (including warm glow altruism), remarks (correctly, we think): “That behavior is describable in this way does not necessarily imply that social welfare should be defined in the same way” (p. 909). The two uses of utility often coincide: we think that the citizen’s enjoyment in eating fruit is part of what the planner should maximize. But what about the obese citizen’s pleasure in consuming sweets? The utility functions used to predict behaviors may require taking account of addictions, hyperbolic discounting, weakness of will, and other empirically observed aspects of motivation. Does that also require that we value the satisfaction of these often self-destructive preferences when considering what the planner should optimize? We do not think so.

In cases where these motives lead to self-destructive behavior, the case for not treating their satisfaction as part of the normative standard for policy evaluation is convincing. If some treatment for drug addiction, for example, were to reduce the addict’s exhilaration when shooting up, we would not count this reduced exhilaration as a cost of the program. A thoroughgoing utilitarian planner, however, might count these subjective effects as part of her objective function when evaluating the drug policy.

Addressing this philosophically complex question adequately in this brief note is not possible, and without resolving the issue we think that our “materialist” planner’s objective function is a plausible formulation. But one wonders if similar results would obtain under what we will term a “utilitarian” planner’s objective function. Fortunately, the use of either of the two planners’ objective function does not alter our qualitative results.
One’s intuition is that the effects of nonseparability would be exaggerated in the case of the utilitarian planner. In the case of marginal crowding out, for example, the sophisticated planner would have a second reason for minimizing the use of incentives: not only would the incentive be less effective in inducing the citizen to contribute (the reason studied above) but it would also reduce the citizen’s utility derived from the value of giving. For the case of marginal crowding out studied in our 2008 paper, this intuition is confirmed. But nonetheless we find that for a sufficiently concave net benefits function the sophisticated utilitarian planner will make greater use of the subsidy than her naive counterpart.

To see that this result holds also for the utilitarian planner in the case of categorical crowding out, recall that in the case of the materialist planner studied in this paper the conditions under which categorical crowding out will lead to underuse of the incentive by the naive planner are less stringent than the case of marginal crowding (Proposition 2). The reason is that while both forms of crowding reduce public goods provision and raise the marginal net benefits of provision, unlike marginal crowding out, under categorical crowding out there is nothing to offset the reason for the sophisticated planner’s greater use of the incentive by reducing the marginal effectiveness of the subsidy. In the Appendix, we show that the same holds for the utilitarian planner, with the result that the fact (from our 2008 paper) that the utilitarian planner may underuse the incentive in the presence of marginal crowding out holds \textit{a fortiori} in the case of categorical crowding out.

7. Conclusion

The sophisticated planner now knows that incentives and social preferences need not be separable but may be either complements or (more likely) substitutes, that both categorical and marginal crowding effects may occur, that she may be able to estimate their magnitude on the basis of experiments, and that taking account of crowding out effects may induce her to adopt either greater or lesser incentives than would have been the case had she remained unaware of the nonseparability problem. The curious cases in which her naive counterpart would underuse incentives in the presence of crowding out will occur (unsurprisingly, she now realizes) because there are diminishing net returns to the public project. She wonders why she did not learn any of this in school.

There were two things missing from the standard model, she recalls. First, because preferences were assumed to depend only on one’s own material payoffs, there were no social preferences to crowd in or out. And second (a more subtle point): if there were social preferences relevant to the problem under analysis they were assumed (implicitly) to be just additive with any payoff-based incentives that the planner might provide. This separability
assumption appeared natural because in the standard model preferences were evaluations of states, consumption bundles, for example, defined simply as vectors of commodities that one might consume. The reason why some states were feasible and others not was a matter of the budget constraint, and had no effect on preferences (social or otherwise). Other than its effect on the budget constraint, a vector of goods acquired by purchase was, in this framework, no different from one acquired directly by one’s own labor, from a charity, or from state as a citizen’s right.

There is something wrong with this picture, the planner now realizes: people care about the processes by which states come to be in their feasible set. The most plausible psychological foundation for what we will call process-dependent preferences is that when people take an action it may be to acquire something (as in many economic actions) but often it is also because the person wants to be or to become a certain kind of person in one’s own eyes or in the eyes of others (Cooley 1902; Leung and Martin 2003; Akerlof and Kranton 2010). For example, the desire not to be (and be seen as) a chump may explain why experimental subjects respond very differently to being offered a disadvantageous share of a pie in an Ultimatum Game depending on whether the share was determined by another subject or by a computer (Blout 1995). A disadvantageous offer from a computer is just bad luck and tends to be accepted as preferable to no offer at all; but the same offer from another subject signals unfairness of the proposer and tends to be rejected. In economic situations preferences appear to be process-dependent for two reasons; the process by which a state comes to be in one’s feasible set often reveals important information about the intentions of others, and also provides cues concerning socially appropriate behaviors. Responses to incentives indicating nonseparability are simply a well-documented case of this more general class.

Were we to index states by the kinds of incentives associated with their being feasible, the planner muses, we might have a model of economic behavior in which nonseparability of material incentives and social preferences could occur naturally. This would entail treating states not simply as vectors of things to have, but as activities, that is, something one does. She then remembers that Kelvin Lancaster had long ago proposed a “new approach to consumer theory” along just these lines. The paper is still in her filing cabinet. She reads: “The good, per se, does not give utility to the consumer...”; then, “Consumption is an activity in which goods are inputs and in which the output is a collection of characteristics” (Lancaster 1966, pp. 133–134). Like Lancaster, the planner might reconstruct the fundamentals of economic behavior by representing consumption as just another form of production, in which the object of production would include the actor’s self-esteem, standing in the community, and other nonmaterial objectives. But doing this would take the planner away from her job, and take the authors of this paper beyond the limits of this brief note.
Appendix

A.1. Maximization Problems in Section 2 and Section 3

Here we determine the conditions under which a symmetric Nash equilibrium exists. We suppose that $s > 0$ for simplicity. Recall that from the citizen’s maximization problem, we have the following first order condition:

$$
\phi'(a_i^{BR} + \sum_{j \neq i} a_j) - g'(a_i^{BR}) + s + \lambda_0(1 + \lambda_c + \lambda_m s) = 0.
$$

Since we are looking for a symmetric equilibrium, we denote the equilibrium by $(a^*, a^*, \ldots, a^*)$ so the first order condition can be rewritten as

$$
\phi'(na^*) - g'(a^*) + s + \lambda_0(1 + \lambda_c + \lambda_m s) = 0.
$$

For such an $a^*$ to exist and lie in the unit interval, we need

$$
\phi'(0) - g'(0) + s + \lambda_0(1 + \lambda_c + \lambda_m s) > 0 \quad (A1)
$$

$$
\phi'(n) - g'(1) + s + \lambda_0(1 + \lambda_c + \lambda_m s) < 0. \quad (A2)
$$

Then when $\lambda_0 \lambda_m > -1$ and $\lambda_c > -1$ hold, (A1) is satisfied. When $g'(A1)$ is sufficiently high, (A2) is satisfied. The Nash equilibrium $a^*$ will be unique if

$$
\text{for all } a, \quad n\phi''(na) - g''(a) < 0. \quad (A3)
$$

When $g$ is sufficiently convex, (A3) is satisfied. In particular, when the stability condition $|n\phi''(na)| - g''(a) < 0$ in the text (8) is satisfied, (A3) is satisfied. Thus under condition (A1) – (A3), there exists a unique symmetric Nash equilibrium.

For the social planner’s maximization problem to yield a unique interior solution, it suffices that (i) the marginal rate of transformation is greater than the marginal rate of substitution at $s = 0$ ($\tau(a(0, \lambda_c, \lambda_m), \lambda_m) > \sigma(a(0, \lambda_c, \lambda_m), 0)$) (ii) the marginal rate of substitution is greater than the marginal rate of transformation at $s = 1$ ($\sigma(a(1, \lambda_c, \lambda_m), 1) > \tau(a(1, \lambda_c, \lambda_m), \lambda_m)$), and (iii) for a given $\lambda_c$ and $\lambda_m$ the marginal rate of substitution as a function of $s$ ($\sigma(a(s, \lambda_c, \lambda_m), s)$) increases faster than the marginal rate of transformation as a function of $s$ ($\tau(a(s, \lambda_c, \lambda_m), \lambda_m)$) as the subsidy increases. For (i), since we know that $\tau > 0$, $c'(0) = 0$ ensures this holds. For (ii) and (iii), we require that $c'(1)$ is sufficiently large and $c$ is sufficiently convex.
A.2. Partial and Total Effect of the Subsidy on the Citizen’s Contribution

The partial effect of the subsidy on the citizen’s contribution in (A3) in the text (that is conditional on a given level of contributions by others) is thus (for $s > 0$)

$$\frac{\partial a_i^{BR}}{\partial s} = \frac{1 + \lambda_0 \lambda_m}{g'' - \phi''}. \quad (B1)$$

But the planner is interested in the effect of the subsidy on the Nash equilibrium level of contribution, and unless the benefit function is linear ($\phi'' = 0$), the citizen’s contribution level will depend on the contribution levels of others, as these will affect the marginal benefit of contributing. From (A3) in the text we know that

$$\frac{\partial a_i^{BR}}{\partial a_j} = \phi'' g'' - \phi'' \frac{\partial a_i^{BR}}{\partial s}. \quad (B2)$$

Thus when the benefit function is convex the members’ individual contributions are complements and where the benefit function is concave they are substitutes.

To see that $\frac{\partial a^*}{\partial s}$ is the sum of a direct effect on the individual’s best response function and indirect effects operating via other individuals contributions, we note that

$$\frac{\partial a^*}{\partial s} = \sum_{j \neq i} \frac{\partial a_i^{BR}}{\partial a_j} \frac{\partial a^*}{\partial s} + \frac{\partial a_i^{BR}}{\partial s},$$

where the first term on the right hand side is the reciprocal effects of the actions of the citizens on each other’s actions, positive or negative depending on whether contributing is a strategic complement or substitute. From (B1) and (B2), we find

$$\frac{\partial a^*}{\partial s} = (n - 1) \frac{\phi''}{g'' - \phi''} \frac{\partial a^*}{\partial s} + \frac{1 + \lambda_0 \lambda_m}{g'' - \phi''} \frac{\partial a_i^{BR}}{\partial s},$$

and rearranging gives the same expression for $\frac{\partial a^*}{\partial s}$ in (9) in the text. Thus the ratio of the Nash effect to the partial effect is

$$\frac{\partial a^*}{\partial s} = \left(1 + \frac{(n - 1)\phi''}{g'' - n\phi''}\right) \frac{\partial a_i^{BR}}{\partial s} = (1 + m) \frac{\partial a_i^{BR}}{\partial s},$$

which shows that when $\phi$ is convex, there is a positive social multiplier ($m$) reflecting the fact that, as we have seen, increasing returns in the benefit function makes the citizens’ contributions strategic complements. Thus the partial effect of the subsidy on the individual’s contribution (holding others’ contributions constant) will be less than the total effect which includes the reciprocal indirect effects of each citizen’s contributions on the marginal
A.3. Stability Condition for the Nash Equilibrium

We consider the following continuous best reply dynamics (See for example Young 1998, p. 35):

$$\frac{da_i}{dt} = a_i^{BR}(a_{-i}) - a_i, \quad i = 1, \ldots, n,$$

where \(a_{-i} := (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)\). Then it can be shown that

$$\sum_{j \neq i} \left| \frac{\partial a_j^{BR}}{\partial a_i} \right| < 1 \text{ for all } i$$

implies the asymptotic stability of \((a^*, a^*, \ldots, a^*)\) (See for example the Appendix in Carpenter et al. 2009). Then from (B2) condition (C1) becomes

$$(n - 1) \left| \frac{\phi''(na^*)}{g''(a^*) - \phi''(na^*)} \right| < 1 \text{ for all } i$$

and it is easy to see that condition (D1) in the text implies (C2).

A.4. Numerical Example

We adopt the following simple functional forms:

$$\phi \left( \sum a_j \right) = \tilde{\phi} \sum a_j, \quad g(a_i) = \frac{\kappa}{2} a_i^2, \quad c(s) = \frac{\gamma}{2} s^2,$$

where \(\frac{n}{n-1} < \tilde{\phi} < \frac{\kappa}{2} - \tilde{\lambda}, \quad \kappa > 0, \quad \gamma > 0\).

Then by the choice of \(\tilde{\phi}\), Assumption A1 is satisfied and since \(\phi'' = 0\), A2 is satisfied. In this case we find

$$\tau(a, \lambda_m) = \frac{1 + \lambda_0 \lambda_m}{\kappa}, \quad \sigma(a, s) = \frac{\gamma s}{n\phi - \kappa a}$$

and

$$\frac{\partial a^*}{\partial s} = \frac{1 + \lambda_0 \lambda_m}{\kappa}, \quad \frac{\partial \sigma}{\partial a} = \frac{\gamma s}{(n\phi - \kappa a)^2}, \quad \frac{\partial \sigma}{\partial s} = \frac{\gamma}{n\phi - \kappa a}, \quad \frac{\partial \tau}{\partial a} = 0.$$
From A2 we have $n\phi - \kappa a^* > 0$ and thus the second order condition for the social planner’s problem \((\frac{\partial^2 \tau}{\partial a^2} - \frac{\partial^2 \sigma}{\partial s^2} < 0)\) is satisfied. For pure categorical crowding out \((\lambda_m = 0)\), there exists $\tilde{\lambda}_c$ such that

$$\frac{ds^*}{d\lambda_c} < 0 \text{ for } \lambda_c > \tilde{\lambda}_c \text{ and } \frac{ds^*}{d\lambda_c} = 0 \text{ for } \lambda_c < \tilde{\lambda}_c \quad \text{(Figure 4 Panel A)}.\]$$

In the case of the marginal crowding out, the underuse condition in Proposition 1 becomes

$$\frac{\partial \sigma}{\partial a}(a^*, s^*) - \frac{\partial \tau}{\partial a}(a^*, \lambda_m) = \frac{\gamma s^*}{(n\phi - \kappa a^*)^2} \kappa > \frac{1}{s^*}. \quad \text{(D2)}$$

Then notice that at an equilibrium, we have

$$\frac{1 + \lambda_0 \lambda_m}{\kappa} = \tau(a^*, \lambda_m) = \sigma(a^*, s^*) = \frac{\gamma s^*}{n\phi - \kappa a^*},$$

and from (D2), we find

$$\left(\frac{1 + \lambda_0 \lambda_m}{\kappa}\right)^2 = \left(\frac{\gamma s^*}{n\phi - \kappa a^*}\right)^2 > \frac{\gamma}{\kappa}.$$

Thus we see that the condition (D2) holds if and only if $1 + \lambda_0 \lambda_m > \sqrt{\kappa \gamma}$. Thus for $\tilde{\lambda}_m := (\lambda_0 (\sqrt{\kappa \gamma} - 1))^{-1}$,

$$\frac{ds^*}{d\lambda_m} < 0 \text{ if and only if } \lambda_m > \tilde{\lambda}_m \quad \text{(Figure 4 Panels B, C)}.$$

### A.5. Problem of Utilitarian Planner

In this appendix we show that the similar characterization of Propositions 1 and 2 holds for the case of utilitarian planner; i.e., we show that the underuse of incentives under marginal crowding out always implies the underuse of incentive under categorical crowding out, hence that the conditions under which categorical crowding out leads to underuse of the incentive by the sophisticated planner is less stringent than the case of marginal crowding out. We consider the following utilitarian planner’s maximization problem:

$$\max_{a \in [0, 1], s \in [0, 1]} \omega(a, s) := \phi(na) - g(a) + v(s)a - c(s)$$

s.t. $a = a^*(s, \lambda_c, \lambda_m)$,

where $v(s) := \lambda_0 (1 + 1[s > 0] \lambda_c + s \lambda_m)$. Then for $s > 0$, similarly to the text we define

$$\tau(a, \lambda_m) = \frac{1 + \lambda_0 \lambda_m}{g''(a) - n\phi''(na)}$$

$$\sigma(a, s, \lambda_c, \lambda_m) = \frac{c'(s) - a_0 \lambda_m}{n\phi'(na) - g'(a) + \lambda_0 (1 + \lambda_c + s \lambda_m)}.$$
Then the interior equilibrium satisfies
\[ \tau(a^*(s^*, \lambda_c, \lambda_m), \lambda_m) - \sigma(a^*(s^*, \lambda_c, \lambda_m), s^*, \lambda_c, \lambda_m) = 0. \]

From this we find
\[
\frac{ds^*}{d\lambda_c} = -\frac{1}{\left( \frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a} \right)} \left[ \left( \frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a} \right) \frac{\partial a^*}{\partial \lambda_c} - \frac{\partial \sigma}{\partial \lambda_c} \right]_{(i)}
\]
\[
\frac{ds^*}{d\lambda_m} = -\frac{1}{\left( \frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a} \right)} \left[ \left( \frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a} \right) \frac{\partial a^*}{\partial \lambda_m} + \frac{\partial \tau}{\partial \lambda_m} - \frac{\partial \sigma}{\partial \lambda_m} \right]_{(ii)}
\]

Note that the terms (i) and (ii) are additional terms accounting for the nature of the utilitarian planner, compared to the materialist planner in the text. And we recall that
\[
\frac{\partial a^*}{\partial \lambda_m} / \frac{\partial a^*}{\partial \lambda_c} = s^*, \quad \frac{\partial \tau}{\partial \lambda_m} = \frac{1}{s^*} \frac{\partial a^*}{\partial \lambda_m}. \quad \text{(E1)}
\]

So, using
\[
\frac{\partial \sigma}{\partial \lambda_c} = -\sigma(a, s, \lambda_c, \lambda_m) \frac{\lambda_0}{\eta^\prime - g + \lambda_0(1 + \lambda_c + s \lambda_m)}
\]
\[
\frac{\partial \sigma}{\partial \lambda_m} = -\sigma(a, s, \lambda_c, \lambda_m) \frac{s \lambda_0}{\eta^\prime - g + \lambda_0(1 + \lambda_c + s \lambda_m)}
\]
\[
+ \frac{-a \lambda_0}{\eta^\prime - g + \lambda_0(1 + \lambda_c + s \lambda_m)},
\]

we find
\[
\frac{\partial \sigma}{\partial \lambda_c} s - \frac{\partial \sigma}{\partial \lambda_m} = \frac{a \lambda_0}{\eta^\prime - g + \lambda_0(1 + \lambda_c + s \lambda_m)}. \quad \text{(E2)}
\]

We define
\[
mco := \left( \frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a} \right) \frac{\partial a^*}{\partial \lambda_m} + \frac{\partial \tau}{\partial \lambda_m} - \frac{\partial \sigma}{\partial \lambda_m}
\]
\[
cco := \left( \frac{\partial \tau}{\partial a} - \frac{\partial \sigma}{\partial a} \right) \frac{\partial a^*}{\partial \lambda_c} - \frac{\partial \sigma}{\partial \lambda_c}
\]

and find that
\[
mco = \left( cco + \frac{\partial \sigma}{\partial \lambda_c} \right) \frac{\partial a^*/\partial \lambda_m}{\partial a^*/\partial \lambda_c} + \frac{1}{s^*} \frac{\partial a^*}{\partial \lambda_m} - \frac{\partial \sigma}{\partial \lambda_m}
\]
\[
= cco s^* + \frac{1}{s^*} \frac{\partial a^*}{\partial \lambda_m} + \frac{a \lambda_0}{\eta^\prime - g + \lambda_0(1 + \lambda_c + s \lambda_m)}
\]
using (E1) and (E2). Since
\[
\frac{1}{s^*} \frac{\partial a^*}{\partial \lambda_m} + \frac{a^*\lambda_0}{n\phi' - g' + \lambda_0(1 + \lambda_c + s\lambda_m)} > 0,
\]
we find that \(mco < 0\) implies \(cco < 0\), which is the desired result.

References


