The developmental dynamics of terrorist organizations

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The authors identify robust patterns in the frequency and severity of violent attacks by terrorist organizations, and examine their relationship to organizational size and experience. Using group-level static and dynamic analyses of terrorist attacks worldwide from 1968–2008 and a simulation model of organizational growth, the authors show that the typical frequency of violent events by an organization accelerates according to a power-law function with increasing size and experience; in contrast, these events' severity follows Richardson's Law—a power-law distribution—independent of such covariates. These patterns are explained by event production rates that are fundamentally constrained by labor availability. Thus larger, more experienced organizations are more deadly because they attack more frequently, not because their attacks are more deadly. The implications of these robust patterns are discussed with respect to counter-terrorism policies, the underlying sociopolitical processes that generate terrorist attacks and their relationship with other forms of violent conflict, such as civil wars.

I. INTRODUCTION

There is a long tradition of research on terrorism. Much of the academic interest in it has been inspired by particular historic events and "waves" of specific forms of terrorist attacks [48, 52]. Just as the rise in international aircraft hijackings in the 1970s led to a resurgence of studies of terrorism, the 11 September 2001 attacks renewed interest in why groups resort to terrorism, the specific choice of attack targets, and the relative effectiveness of particular counterterrorism measures. Many researchers have developed typologies of specific forms of terrorism and highlighted the distinctiveness of different terrorist groups. By contrast, in this manuscript we examine whether there are fundamental constraints on the frequency and severity of attacks by terrorist organizations, and if so, the possible mechanisms that may generate such constraints.

One traditional theoretical approach to studying terrorist attacks uses a rational-actor framework [53], in which outcomes result from strategic calculations over and tradeoffs among costs, benefits and preferences. Much evidence supports the utility of this approach, for example, in understanding the particular choice of targets [20, 54] or the specific choice of tactics for individual attacks [19, 45].

Our approach is different and complementary, focusing on global trends and patterns rather than the particulars of attacks. At the global scale, the importance of individual strategic calculations is lessened while the impacts of generic non-strategic processes are enhanced. Explanations using this approach often focus on physical constraints, network effects and endogenous population dynamics, which are well suited to explain the behavior of strategically weak or uncöordinated populations of actors. By shedding new light on the dynamics, patterns and trends in terrorism, and on how such patterns emerge from the local-level decisions of individual terrorists, organizations and conflicts, this approach can help supplement the contributions from formal models of individual interactions, more broadly inform counter-terrorism policy and clarify our general ability to forecast or anticipate future terrorist events.

A. Trends and Patterns in Global Terrorism

The research approach of focusing on global trends and patterns in terrorism owes much to the seminal work on Lewis Fry Richardson. Richardson — a physicist and meterologist perhaps best known to social scientists for collecting data on conflicts (or deadly quarrels) and modelling arms races using differential equations — also made early contributions to modelling the frequency-severity distributions of wars. More specifically, Richardson [49, 50] showed that the frequency and severity of wars robustly follows a power-law relationship, where severity is inversely proportional to frequency.[1] Two implications of this empirical fact are notable. First, there is no fundamental statistical difference between rare but

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[1] Power-law distributions can indicate unusual underlying or endogenous processes, e.g., feedback loops, network effects, self-organization or optimization. From a purely statistical perspective, power-law distributions generate large events orders of magnitude more often than we would expect under a Normal assumption. Recently, power-law distributions have been identified in a wide range of social and biological systems [12]. See Kleiber and Kotz [35], Mitzenmacher [40] and Newman [44] for reviews, or Appendix A of Clauset and Wiegel [13] for a gentle introduction.

catastrophic events and more common but less severe events—the likelihoods of both are compactly described by a single mathematical function:

$$\Pr(\text{event with severity } x) \propto x^{-\alpha} \ ,$$

where x counts the number of fatalities (severity) and α is the "scaling exponent," which controls how quickly the frequency decreases as severity increases. Second, the generative processes for both large and small events may be fundamentally the same, and large but rare events may simply be "scaled up" versions of small but common events. Thus, studying the statistically more common small events may shed light on certain aspects of extremely rare events.[2]

Recently, Clauset et al. [12, 14] showed that this same pattern—a power-law, "Richardson's Law"—also holds for the frequency of severe terrorist attacks (reported fatalities), while Bohorquez et al. [7] suggest a similar pattern for events within insurgencies. For terrorism, Clauset et al. demonstrated that the power-law pattern is highly robust: it persists over the past 40 years despite large structural and political changes in the international system and is independent of the type of weapon used (explosives, firearms, arson, knives, etc.), the emergence and increasing popularity of suicide attacks, the demise of many individual terrorist organizations, and the economic development of the target country.

Thus, fundamental regularities in terrorism can and do emerge at the global level despite the highly contingent and context-specific nature of the individual attacks, conflicts and decisions. Insights into these patterns' origin should shed considerable new light on the underlying social or political processes that drive and constrain global trends and on the policies aimed at responding to or managing those processes. Such insights may also advance the provocative possibility of principled long-term statistical forecasting for terrorism, in a manner similar to modern seismology, which routinely makes fairly good long-term forecasts for the frequency of earthquakes worldwide.[3]

B. An Organizational Perspective on Terrorism

Here, we turn to the question of statistical patterns in the frequency and severity of attacks over time by individual terrorist organizations. We argue that organization size (number of personnel) plays a fundamental role in constraining or governing the frequency and severity of violent events (attacks) over the group's lifetime. Our inspiration comes partly from observing that "size" plays a fundamental role in system dynamics in many other complex systems, for example, species body mass in ecology [8] and evolution [9], city populations in urban growth [6] and firm valuation in finance [55]. In these systems, size correlates strongly with a wide range of system covariates and often fundamentally drives or constrains the overall dynamics. In characterizing terrorist organizations, the role of size has been comparatively understudied (although see Asal and Rethemeyer [4]).

We set forward four hypotheses on the importance of organizational size in governing the behavior of terrorist organizations, which we argue are most significant in the early stages of an organization's development. We will develop the hypotheses more formally later, but state our key hypotheses here to explicitly anticipate the argument.

- H1: *Labor-constraint hypothesis*: the production rate of violent events by a terrorist organization is fundamentally constrained by its available labor force (organizational size) and thus the time between subsequent attacks is roughly inversely proportional to organizational size.
- H2: *Event-recruitment hypothesis*: increased organizational size is driven by recruitment associated with the production of new attacks such that additional attacks lead to recruitment which leads to organizational growth.
- H3: *Frequency-acceleration hypothesis*: H1 and H2 jointly imply that the time between subsequent attacks decreases as an organization carries out more attacks.
- H4: *Severity-increase hypothesis*: the severity of new attacks depends on available labor and thus increases with organizational size. When combined with H2, the severity of attacks will also increase as more attacks are carried out.

To test these "developmental dynamics" hypotheses, we present novel statistical analyses of organizational size and the frequency and severity of their attacks for nearly 400 terrorist organizations worldwide over the period 1968–2008. We find strong support for the first three hypotheses, indicating that the available labor is a fundamental constraint on the frequency of attacks by an organization and that the dynamics of the frequency of attacks over an organization's lifetime—but particularly when they are young—can be explained by a feedback

[2] Seismologists study large earthquakes in this way: the frequencies of both large and small quakes follow a power-law distribution, called the Gutenberg-Richter Law, and the physical processes that generate both small and large quakes are fundamentally the same.

[3] Statistical *forecasting* should not be confused with the more difficult task of event *prediction*, which seeks to specify ahead of time the when, where, how and why of the next big attack. In forecasting, the goal is instead to estimate a probability distribution over some region of space or window of time. For discussion of these issues in the context of earthquake prediction and forecasting, see Hough [26].

loop in which attacks lead to recruitment which leads to growth which leads to increased production rate of new attacks. We further show that the rate at which an organization cycles through this feedback loop can depend on covariates like its political ideology, with religiously-motivated organizations accelerating the frequency of their attacks the fastest. This finding contradicts the recent claims by Johnson et al. [29] that an acceleration in the frequency of events observed within individual Iraqi and Afghani provinces during their recent insurgent conflicts is caused by gains in production efficiency ("learning") rather than by increases in the number of fighters.

In contrast, we find no support for the severity-increase hypothesis: rather, the distribution of attack severities associated with an organization universally follows a rough form of Richardson's Law, regardless of organizational size, experience or political motivation. This suggests that there may be no generic relationship between an organization's covariates and the severity of its attacks. This finding contradicts a significant prediction of the mathematical model of terrorist group internal dynamics, proposed by Bohorquez et al. [7] and generalized by Clauset and Wiegel [13], that has been advanced as an explanation for Richardson's Law in the frequency of severe terrorist attacks.

Taken together, these results imply that very large events are equally likely to be generated by small groups as by large groups, and that larger organizations are indeed more deadly [4], not because their individual attacks are systematically more spectacular but because they typically carry out many more attacks. That is, the size of the beast directly determines the overall level of terror activity (frequency) but not the quality (severity) of those actions. These results may be interpreted as support for viewing terrorist organizations as a kind of labor-constrained manufacturing firm, whose principal product is political violence.

C. Impact of Size on Frequency

How does organizational size impact the frequency of its attacks? If size plays a dominant role, then changes in the frequency of attacks by the organization as a whole should track increases and decreases in organizational size. This implies that the number of personnel required to plan and execute a single attack remains roughly constant over time.

H1. *Labor-constraints*: the production rate of violent events by a terrorist organization depends on its available labor pool, and thus the time between subsequent attacks Δt is roughly inversely proportional to the size s of the organization. Mathematically, $s \propto 1/\Delta t$.

H1 argues that violent event production, e.g., terrorist bombings, transportation hijacking, hostage taking, etc., cannot be fundamentally automated, which would allow an organization to produce potentially arbitrary numbers of events with roughly a constant number of individuals.[4] Some aspects of event production, however, may benefit from technology or efficiency improvements, e.g., organizational learning [1, 18, 28, 58], which would moderate but not eliminate the constraint of size on production rate. Some group-level covariates may also influence the precise relationship between size and frequency, e.g., the size of the organization's militant wing, the centrality of violence to its political goals, etc. However, such factors likely play a secondary role to that of labor in determining the overall production rate of new events. That is, in general, size sets the tempo of an organization's activities.

This labor constraint should be strongest for small organizations, which may have less access to efficiency-improving resources like specialized personnel, training facilities or factories and which may reap the largest benefit from maximizing event production, e.g., by maximizing media visibility. In contrast, large organizations may be less inclined toward maximizing their production of violence, e.g., because of their more complex internal and external constituencies or strategic goals, and may allocate significant personnel or recruits to non-violent activities, e.g., the provision of social services.

A spatial corollary of H1 is that if an "organization" is defined as those militants within some geographic locale, e.g., a province or district, then the labor-constraint hypothesis predicts that the frequency of events within that locale will be roughly inversely proportional to the number of militants within that locale. That is, the $s \propto 1/\Delta t$ relationship should hold when both s and Δt are defined by a geographic boundary.

D. Events, Recruitment and Growth

For most organizations, size varies over the organization's lifetime: most begin, and remain, as small groups of violence-inclined individuals and few ever grow to include more than a hundred members. What role do attacks play in increasing organizational size? If an event gains the organization wider visibility among potential members or sympathizers, the organization may grow in size as a result of that event.[5] Let k denote the cumulative number of events an organization has carried out, which measures the organization's "experience".

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[4] In this light, *cyber terrorism* is an interesting case: it remains unclear to what degree the planning and execution of cyber terrorist attacks can be done automatically, by computers. Our current belief is that cyber terrorism is not fundamentally mass produceable and thus some labor constraint will persist.

[5] Decreases in size are likely driven by distinct social processes (see Cronin [17]). Although we omit them from further consideration, they remain an important and interesting area of study.

H2. *Event-recruitment*: organizational growth (increased s) is driven partly by recruitment associated with the production of new events (increased k), i.e., events lead to recruitment which leads to organizational growth. Mathematically, $\mathrm{d}s/\mathrm{d}k > 0$.

We note that recruitment efforts, and therefore organizational growth, need not be fully driven by the production of violent events; they could also be driven by nonviolent efforts like the provision of social services or political communication. However, so long as recruitment is not fully independent of violent events, a roughly positive relationship between size and events will hold. As above, we expect such recruitment effects to be most significant for small or young organizations.

E. Frequency Acceleration

We observe that organizational size s appears in both H1, which relates size to production rates, and H2, which relates size to cumulative production. Taken jointly, these two hypotheses imply a positive feedback loop in which attacks lead to recruitment which leads to organizational growth and thus an increased production rate of new attacks. So long as an organization allocates some portion of its growth to the production of new events, i.e., so long as the militant wing grows proportionally to the overall organization, H1 and H2 imply an acceleration over time in the production rate of attacks.

H3. *Frequency-acceleration*: as an organization carries out more attacks (increased k), the time between subsequent attacks Δt decreases. Mathematically, $\mathrm{d}\Delta t/\mathrm{d}k < 0$.

That is, H1 predicts $s \propto 1/\Delta t$ while H2 predicts $\mathrm{d}s/\mathrm{d}k > 0$. Eliminating the common factor of s yields the mathematical prediction that $\mathrm{d}\Delta t/\mathrm{d}k < 0$, in which the continued production of violent events leads to decreased production time per event. We note that this dynamical relationship produces a similar pattern to that observed in "learning" or "progress curves," in which continued production covaries with lowered production costs or time [2, 18, 58].

F. Impact of Size on Severity

What role does organizational size play in the severity of attacks? If increased size brings greater access to capital and skilled labor, e.g., experienced professionals, advanced arms, sensitive intelligence, etc., then larger size will facilitate more spectacular attacks, yielding increased event severity.

H4. *Severity-increase*: the severity of new attacks x depends on available labor and thus increases with organizational size s and, via H2, with the number of attacks k. Mathematically, $\mathrm{d}x/\mathrm{d}s > 0$ and $\mathrm{d}x/\mathrm{d}k > 0$, respectively.

When combined with H2, in which the size of an organization increases as a result of additional attacks, H4 predicts that the severity should also increase as additional attacks are carried out, implying that individual attacks by experienced, larger groups are consistently and significantly more deadly than those by less experienced, smaller groups.

We note that the severity-increase hypothesis assumes some tangible benefit for maximizing the severity of attacks, e.g., to gain wider visibility for the organization's cause or to demonstrate power or resolve to both the target actors and the organization's constituents. Such incentives are not foregone conclusions: severe attacks may also attract harsh attention from state-level actors, leading to repression, police action or the destruction of physical or financial resources. They may also induce counterproductive effects on potential sympathizers, e.g., due to the shockingness of spectacular events. As a result, the theoretical argument supporting the severity-increase hypothesis is the most marginal of the four.

II. A MODEL OF TERRORIST ORGANIZATIONS

To illustrate the predictions of these four hypotheses for the interactions between an organization's size and the frequency and severity of attacks over its lifetime, we construct a toy model of a terrorist organization's development. Figure 1 shows a schematic of this model.

Historically, terrorist organizations typically begin as a small collection of highly motivated, terrorism-inclined individuals [24]. Call this initial group a "cell" and let it be composed of roughly η individuals, which denotes the typical or characteristic size of a terrorist cell. The particular value of η is not important, but likely depends political ideology, socio-economic context [36], the attack's target, etc. The cell plans and conducts its first attack, which gains it some visibility with the wider public, via either traditional media coverage or informal channels. Recruitment following the attack yields a characteristic number of additional members (H2), denoted ν, and now the organization is larger. As with η, the particular value of ν is not important, but likely depends on factors related to the target, political messaging associated with the attack, etc.

This first cell continues planning and carrying out new attacks, roughly once every τ days, which denotes the typical or characteristic production time for a new attack (H1). Again, the particular value of τ is not important, but likely depends on the target, personnel training, resource acquisition, etc. Newly recruited members form new cells, each of size roughly η (H1). These new cells then plan and carry out their own attacks. Note that at the organizational level, any attack by any cell leads to organizational growth via recruitment (H2), which in turn increases the organization's overall production rate of attacks by adding new cells (H3). Furthermore, as

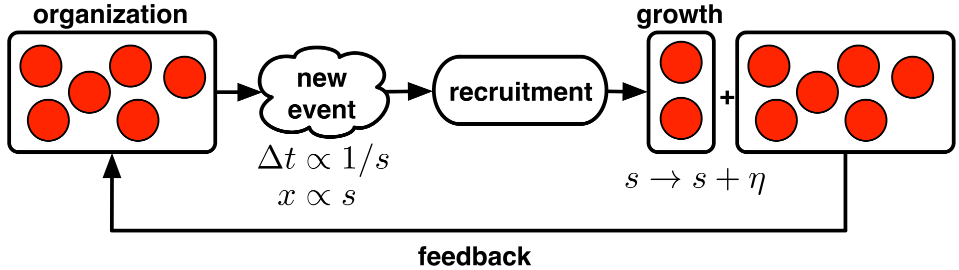

FIG. 1: A schematic illustrating the feedback loop relationship between size s and the frequency and severity of attacks: the delay between subsequent attacks Δt is inversely related to an organization's size s while the severity of subsequent attacks x grows with s; new events lead to recruitment which leads to growth, which increases the size variable s.

the group grows, the increased manpower, access to resources, etc. also increases its ability to carry out more severe events (H4), e.g., because more supporting roles allow better surveillance, access to better equipment, etc.

As the organization grows, some older members move into administrative positions, which reduces the overall production rate by reducing the size of the militant wing. However, so long as recruitment efforts more than make up for this loss of active militants, the positive feedback loop remains. As the organization continues to grow, existing members may be reallocated or new members may be increasingly allocated to non-violent activities, thereby diversifying the organization's overall capabilities.

This toy model is intentionally simplistic and omits many factors likely to have some impact on the overall behavior of the organization, including organizational structure, political motivation, geography, etc. It is this simplicity that allows us to focus on the generic way that organizational size governs the frequency (and severity) of attacks and allows us to quantify the predicted developmental dynamics via a direct numerical simulation. Specification details and computer code for this simulation are given in Appendix B.

Because the model is stochastic, with each cell's delay τ being drawn from a stationary distribution $\Pr(\tau)$, each simulated terrorist organization generates a unique sequence of events representing the collective behavior of its cells over time. We extract the model's central tendencies by generating a full life-history for each of many organizations and then computing quantiles over variables of interest. In this case, we are interested in how the delay between subsequent attacks Δt varies with cumulative production k, and how the size of the organization, measured by the number of cells s/η varies with calendar time t from the first event. Under this model, event severity is correlated with organizational size and thus no additional information is gained by explicitly simulating event severities.

Figure 2 shows the results for 10,000 simulated organizations, for three choices of the ratio ν/η, which controls the number of new cells created after each event. In the $\nu/\eta < 1$ regime, organizational growth is slowed because multiple events are required to populate a new terrorist cell. In contrast, when $\nu/\eta > 1$, organizational growth is fast because each event produces at least one new cell.

The generic behavior of our model is clear: (i) organizational size grows exponentially with time, with the ratio ν/η controlling the exponential growth rate, and (ii) the feedback between a size and event production rate induces a strong correlation between experience k, size s and the frequency of attacks (inverse delay), as predicted by H1–H3. Further, the model produces a universal functional relationship linking delay Δt and cumulative production k, which as the form $\Delta t \propto k^{-1}$ and which is independent of the rate which new events lead to new cells ν/η. This latter point is worth reiterating: so long as new events lead in some way to marginal increases in the overall production capacity, e.g., via recruitment (H2), a positive feedback loop between size and event production will exist. Further, if the number of events required to produce a new cell ν/η is largely independent of experience k, the feedback process will be precisely linear, i.e., inter-event delays will decrease like k^{-1}. However, if ν/η increases with time, the feedback will be sub-linear, i.e., $k^{-\beta}$ with $\beta < 1$, yielding slow frequency acceleration. On the other hand, if it decreases with time, the feedback process will be super-linear, i.e., $k^{-\beta}$ with $\beta > 1$, and the frequency acceleration will be explosive.

III. EMPIRICAL DATA

To test our hypotheses empirically, organizational size data were drawn from the Big Allied And Dangerous (BAAD) data set [4], which offers the currently best available size estimates for terrorist organizations worldwide.[6] These data were generated by a survey of domain

[6] Other sources of size data lack the breadth or temporal resolution for accurate analysis. For instance, the START program and the MIPT database previously held a small number of estimates of uncertain accuracy, generated by Detica, Inc., a British defense

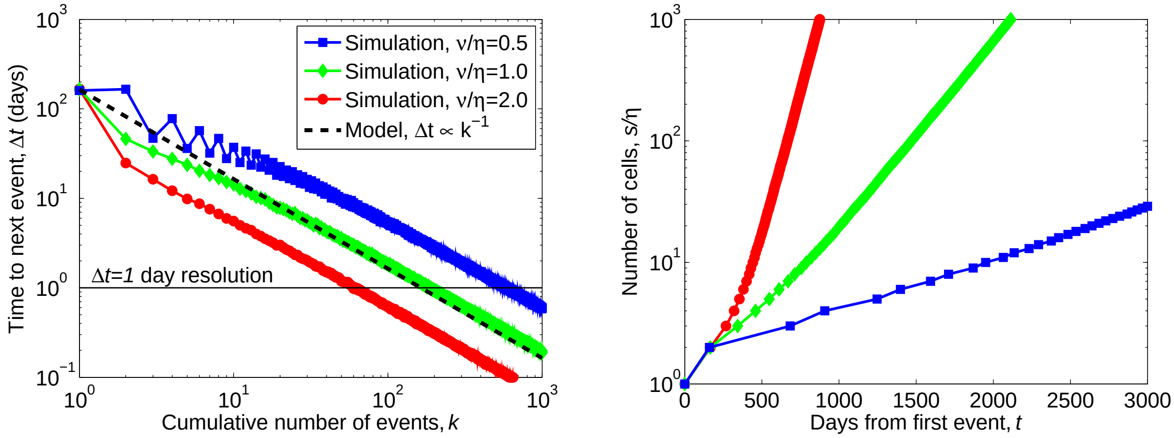

FIG. 2: (a) Median event delay Δt vs. cumulative number of events k, for 10,000 simulated terrorist organizations and three choices of the number of cells ν/η added per event. Dashed line shows the function $\Delta t \propto k^{-1}$, from Eq. (1). (b) Median size (number of terrorist "cells" s/ν) vs. calendar time from the first event, showing exponential growth with rate set by ν/η.

experts at the Monterey Institute of International Studies (MIIS) who estimated the rough order of magnitude (1–100, 100–1000, 1000–10,000 and >10,000 personnel) of the maximum size achieved by each of 381 groups, between 1998 and 2005, identified in the MIPT [39] event database. Of these, 161 organizations conducted at least one deadly attack, and 80 conducted at least two in that period.

To ensure good compatibility with this organization list, event data were drawn from the MIPT Terrorism Knowledge Base [39],[7] which contained 35,668 terrorism events, of which 13,274 resulted in at least one fatality, as of 29 January 2008. For the period 1968–1997, the MIPT database includes mainly international events involving actors from at least two countries, while for 1998–2008 it includes both domestic and international events from much of the world.[8] Each event is defined as an attack on a single target in a single location (city) on a single day. For example, the Al Qaeda attacks in the United States on 11 September 2001 appear as three events in the database, one for each of the locations: New York City, Washington D.C. and Shanksville, Pennsylvania. Each record includes the date, target, city (if applicable), country, type of weapon used, terrorist group(s) responsible (if known), number of deaths (if known), number of injuries (if known), a brief description of the attack and the source of the information.

The organizations identified in the MIPT database are a superset of those contained in the BAAD data set, and we will use these additional data in some of our analyses. For each of these organizations, we extracted the full sequence of events attributed to or claimed by that group, yielding 10,335 events worldwide from 1968–2008 associated with 910 identifiable organizations. For each of the 1,204 events worldwide with unknown severity, we assign a severity of $x = 0$ to preserve timing information. Further, because of the day-level temporal resolution of events in the database, multiple events on the same day by the same group have ambiguous "delay" (inverse frequency). We eliminate this ambiguity by aggregating such events into a single "event day" with severity equal to the sum of the component severities. This step slightly reduces the number of events and impacts mainly the most active organizations late in their life history. As a consequence, the smallest delay between two events by the same organization is $\Delta t = 1$ day, the minimum resolvable delay in the database.

IV. REGRESSION MODELS

Before analyzing the evolution of attacks by individual organizations we first conduct static or cross-sectional regression analysis at the level of the individual organizations. We examine the relationship between group size and attack patterns, in particular the delay between attacks, the "experience" of a group in terms of number of

contractor, and Jones and Libicki [32] compiled a database of information on 649 terrorist groups that included only estimates of the maximum size over a group's entire lifetime.

[7] Other sources of event data include the Global Terrorism Database [56], the Worldwide Incident Tracking System [42] and the ITERATE data [37]. We note that neither these nor the MIPT database provide complete and consistent worldwide coverage.

[8] The MIPT data were originally drawn from the RAND Terrorism Chronology 1968–1997, the RAND-MIPT Terrorism Incident database (1998–Present), the Terrorism Indictment database (University of Arkansas & University of Oklahoma), and DFI International's research on terrorist organizations. In 2008, however, the U.S. Department of Homeland Security discontinued its funding for the maintenance of the database in favor of the University of Maryland's START center's Global Terrorism Database [56].

TABLE I: Ordered logit regression of group size, by fatal attack patterns

Variable	$\hat{\beta}$	SE($\hat{\beta}$)
Delay: ln min(Δt)	-0.351	0.119
Experience: ln max(k)	0.707	0.193
Severity: ln max(x)	0.150	0.159
$\hat{\alpha}_{0\|1}$	-0.163	0.840
$\hat{\alpha}_{1\|2}$	2.652	0.895
$\hat{\alpha}_{2\|3}$	5.039	1.056

N = 80, LR χ^2 = 41.42, df = 3, 58.75% correctly classified

events, and the severity of attacks.

To recap, we expect larger groups to generate a larger number of attacks, have shorter delays between attacks (H1), and generate more severe attacks even accounting for other attack patterns (H4). We can evaluate H1 by comparing maximum group size s data from the BAAD data and the minimum delay between attacks Δt. We can assess H4 by comparing size and the maximum maximum severity x of attacks. Finally, H2 implies that larger groups should have we higher maximum experience k or cumulative number of events. (H3, postulating a declining delay with subsequent attack, cannot be evaluated with static data, and we return to this later.)

Although we that group size should predict attack patterns, an individual measure such as maximum severity will be at least in part a function of the total number of attacks. That is, for any distribution of severities, if a group draws more events from that distribution, the increased production rate (sampling intensity) will naturally inflate the maximum severity over a fixed time period, even if the distribution is stationary. Thus, in order to examine the partial relationship between size and the attack related variables—or their independent predictive value on size once we take into account the other attack pattern characteristics—it is more convenient to consider to what extent we can account for size as function of all the attack pattern measures jointly rather than how size predicts individual attack measures.

We use an ordered logit regression model of size since the BAAD data give order-of-magnitude estimates of maximum size. As the BAAD data pertain to the time period 1998–2005, we restrict our attack pattern measures to attacks during this same time period. Since the distributions of minimum delay, maximum experience, and maximum severity are all highly skewed we take the natural logarithm, adding 1 to severity to prevent taking the log of 0 in the case of non-fatal events. We report the empirical estimates in Table I.

The results display a significant negative relationship between the delay between fatal attacks and group size, consistent with our claim that larger groups will have shorter delays between attacks (H1). We furthermore find a positive relationship between group size and experience, which in turn is consistent with the claim that larger groups generate a higher number of attacks (H2). Finally, the maximum severity of the attacks is not significantly related to group size, once we have taken into account delay and experience. This contradicts our claim that larger groups are systematically more likely to generate severe attacks than smaller groups (H4), after accounting for their tendency to generate more attacks and to attack more frequently. Overall, the model places 58.75% of all the groups in the correct bins for group size.[9]

Since the BAAD data covers only about half of the identifiable organizations in the MIPT database over a restricted time span (1998–2005), we conduct a supplementary analysis with the full MIPT dataset, where we consider how a group's total experience can be accounted for by differences in minimum delay and maximum attack severity.[10] Table II report the results for a linear regression with logged values for all the terms for fatal (F) and all attack (A, including non-fatal attacks) experience respectively. The results clearly show that minimum attack delay is a significant predictor of group experience. They also mildly support our claim about severity, as the positive coefficient for severity actually is significantly different from 0. However, comparing the change in the R^2 for estimating the model with and without the severity and delay terms respectively clearly indicates that dropping the former leads to a relatively small decline, while the impact of omitting the latter is substantial. Hence, it is clear that variation in delay between attacks can account for much more of the variation in experience than severity.

These static analyses provide substantial preliminary evidence in support of H1 and H2 and there is little evidence to support H4. However, our hypotheses make concrete predictions about the relationship of frequency and severity with cumulative production. In the following sections, we go beyond static analyses and test our predictions for all organizations in the MIPT database using a novel dynamical analysis tool called a "development curve."

V. DEVELOPMENTAL DYNAMICS

A development curve is a statistical tool that measures the evolution of organization behavioral variables along a common quantitative timeline [10].[11] This analysis fa-

[9] We considered a number of alternative specifications. Severity remains an insignificant predictor of group size when we consider combinations of delay and experience for both deadly and non-deadly attacks. Using a linear regression model rather than ordered logit does not change our substantive conclusions.

[10] We limit the analysis to MIPT organization that generated at least two events (frequency) and one deadly event (severity). Only 167 of the 910 organizations in the MIPT data satisfy these criteria.

[11] A "development curve" is similar in structure and use to the "experience", "learning" and "progress curves" sometimes used in management science [18, 58] to quantify the relationship between

TABLE II: Linear regression of experience, by attack delay and severity

	Fatal attacks (F)		All attacks (A)	
Variable	$\hat{\beta}$	SE($\hat{\beta}$)	$\hat{\beta}$	SE($\hat{\beta}$)
Delay$_F$: ln min(Δt)	-0.119	0.042	-0.110	0.040
Delay$_A$: ln min(Δt)	-0.778	0.110	-0.795	0.105
Delay$_F\times$ Delay$_A$	0.074	0.017	0.073	0.016
Severity: ln max(x)	0.190	0.059	0.150	0.056
$\hat{\alpha}$	3.115	0.236	3.336	0.225
	N = 167, R^2 = 0.545		N = 167, R^2 = 0.565	
	R^2 ($\neg$severity) = 0.515		R^2 ($\neg$severity) = 0.546	
	R^2 ($\neg$delay) = 0.222		R^2 ($\neg$delay) = 0.182	

cilitates direct comparisons of the behaviors of different groups at similar points in their life histories, which is useful for testing developmental hypotheses.

We instrument the common timeline using organizational experience k, defined as the cumulative number of events produced by or associated with a particular organization, and we compare the delay Δt between the kth and $(k+1)$th events, or the severity x of the kth attack, across all organizations in our sample. For each of the 910 organizations, we extract from the MIPT event data an ordered sequence of coordinates $\{(1, z_1), (2, z_2), \dots\}$, which represent the group's behavioral trajectory on the variable z over its lifetime. The visualization of such trajectory is typically made using double-logarithmic axes, as illustrated in our simulation results in Figure 2. Although the curve construction itself ignores details such as the date of an organization's first attack, its location, ideology, etc., these variables can be used for subsequent analysis, e.g., comparing the trajectories across covariates. We revisit this point below.

As an example of development curve analysis, Figure 3 shows the frequency and severity development curves for the four organizations with the greatest number of attributed event-days in our dataset, including both deadly and non-deadly events: the Revolutionary Armed Forces of Colombia (FARC; 520 events), the Taliban (349 events), Basque Fatherland and Freedom (ETA; 311 events), and Hamas (308 events). Non-deadly events ($x = 0$) increment the counter k for the severity curve but do not appear on the severity curve figures; hence, ETA, which carried out 261 (84%) non-deadly events, has relatively few points in its severity curve.

For these organizations, the median delay between the $k = 1$ and $k = 2$ events is $\Delta t = 433$ days. In contrast, the median delay between the most recent pair of events by these groups is only $\Delta t = 4$ days, a 100-fold increase in frequency. In each case, the frequency curve begins in the upper-left corner of the figure, representing very long delays between subsequent events, and as k increases, the curve moves consistently, albeit stochastically, toward the bottom-right corner, representing a convergence on very short delays between events.

This progression from slow to fast event production appears to happen quite quickly: all four groups achieve delays of $\Delta t \leq 10$ days by their $k = 12$th event. However, the median calendar time required to achieve this high rate of production is 8.5 years; thus, although these first dozen events account for a small fraction of the lifetime production of these organizations (less than 4% each), they account for a large fraction of the organizations' overall lifetimes. To put it more bluntly, these first few events play a critical role in shaping the long-term trajectory of an organization's production curve and they illustrate a dramatic acceleration in the production of events as the organizations mature. This important developmental effect is obscured by high production rates later in life.

In contrast, the pattern for the severity development curve could not be more different: we observe no clear trend, either up or down, between event severity x and experience k for these organizations, and the median first and last severities are $x = 0$ and $x = 1$ deaths, respectively. If anything, the only visual pattern we can discern is a possible increase in the variance of x as k increases. This preliminary analysis thus already indicates weak support for the severity-increase hypothesis (H4) but strong support for the frequency-acceleration hypothesis (H3). In combination with our static analysis above, this provides additional evidence supporting labor constraints and event-driven recruitment (H1 and H2).

As a general analytic tool, development curves for individual organizations (Fig. 3) are a novel way to investigate the specific behavioral dynamics of individual groups over their lifetimes, e.g., the Taliban's behavior around $k = 10$ or Hamas's behavior around $k = 250$. To test our more general claims, however, we must examine the generic trajectory structure. This can be accomplished by combining the individual organization curves into a composite curve that describes the range of delays, given some level of experience k, that is, we tabulate $\Pr(\Delta t \,|\, k)$. Thus, an organization that has carried out k^* events contributes to each of the $k \leq k^*$ conditional distributions. This probabilistic model provides a strong

per-item production cost (or time) and "experience" (cumulative item production). Because we study behavioral variables rather than the costs of production, and to explicitly avoid implying learning-based mechanisms, we use a distinct term.

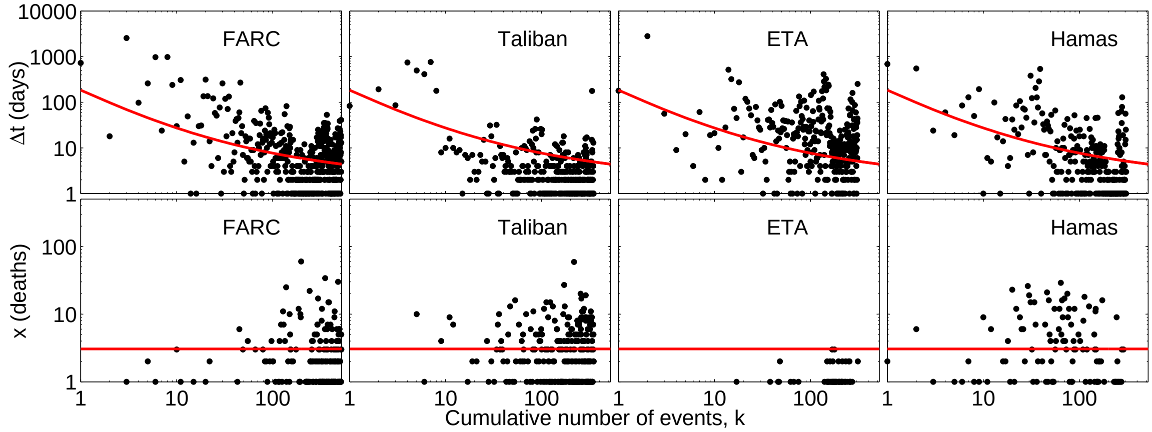

FIG. 3: Frequency (delay Δt) and severity (deaths x) development curves for the Revolutionary Armed Forces of Colombia (FARC), Taliban, Basque Fatherland and Freedom (ETA), and Hamas, with generic trajectories estimated for all groups. Similar results hold for less experienced groups.

quantitative test of the frequency-acceleration (H3) and attack-severity hypotheses (H4) predictions. The precise quantitative form of this composite curve may also serve applications in statistical forecasting or formal modeling.

A. Frequency of Attacks Over Time

Figure 4a shows the composite frequency curve for all organizations in our study. To reduce the overprinting effects of showing the trajectories for so many organizations, we bin the values of k on a logarithmic scale and plot summary statistics of the empirical distribution of delays within each bin. Remarkably, the observed empirical pattern agrees very closely with our simulation model of terrorist organization developmental dynamics (Figure 2).

The steady and parallel decrease of each of these summary statistics with increasing k indicates a generic tendency toward faster production with increased experience for all types of organizations, in strong agreement with the frequency-acceleration hypothesis (H3). But, the relationship between delay and experience is not deterministic: not every event occurs more quickly than the last but the statistical tendency toward shorter delays is clear. In general, delays can be understood as being drawn from an underlying distribution whose central tendency steadily moves toward smaller delays, i.e., $\langle \mathrm{d}\Delta t/\mathrm{d}k \rangle < 0$, where $\langle \cdot \rangle$ indicates an averaging operation.

Empirically, terrorist organizations typically begin their campaigns in the low-frequency domain (large Δt) and move in fits and starts toward the high-frequency domain (small Δt) as they mature and gain experience. To illustrate this central trend, among all groups that have produced k or more events, the median delay after the $k = 1$st event is $\Delta t = 124$ days, while by the $k = 12$th event, it has dropped to $\Delta t = 35$ days; by the $k = 25$th event, the next event typically comes only $\Delta t = 21$ days later. As we saw above, this transition to fast production takes considerable calendar time: for groups that achieve $k = 12$ events, the median calendar time between the first and twelfth event is 4.4 years. Similar results hold for the timing between deadly attacks.

None of the sampled organizations progressively slowed their attack rate over time, moving from high-frequency to low-frequency. A few unusual groups, such as Al-Qaeda in the Land of Two Rivers, begin and remain in the high-frequency domain. But, Al-Qaeda in the Land of Two Rivers is an interesting case because it is well-known to have operated under a different name prior to 2004 [21]; thus, their initial high-frequency behavior can be interpreted as support for the labor-constraint hypothesis (H1) because their initial larger size—a hold over from their previous identify—allowed them to "begin" life ($k = 1$) at a relatively high initial production rate of attacks.

B. Statistical Model for the Frequency of Attacks

Quantifying the dynamical relationship between delays and experience allows us to go beyond the static inferences made above. To do this, we statistically model the entire distribution from which delays are drawn and how this distribution varies with experience. Mathematically, this is precisely the conditional probability distribution $\Pr(\Delta t \,|\, k)$.

For these data, a truncated log-normal distribution, with the following mathematical form

$$\Pr(\Delta t \,|\, k) \propto \exp\left[\frac{-(\log \Delta t + \beta \log k - \mu)^2}{2\sigma^2}\right] \quad , \qquad (1)$$

provides an excellent fit to the empirical delay data for all organizations (see below). Here, σ^2 is the variance

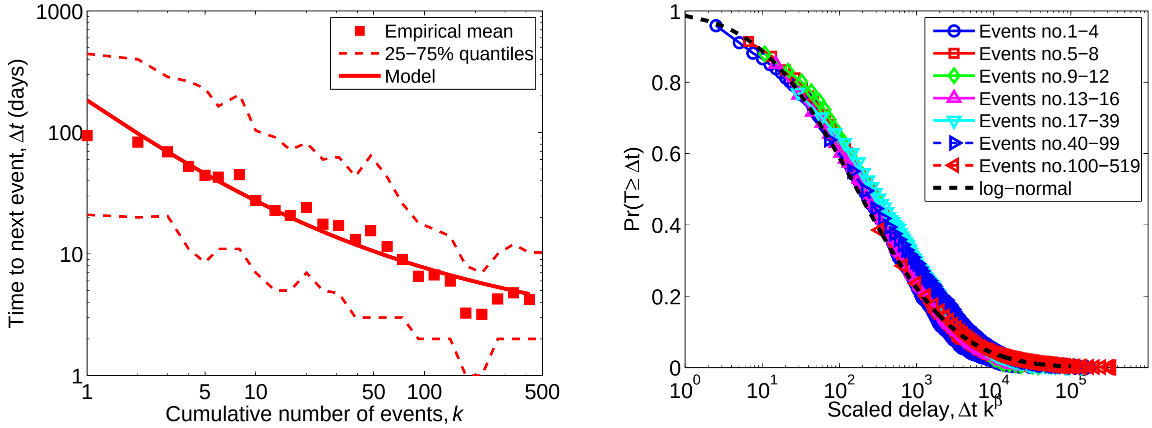

FIG. 4: (a) Mean delay $\langle \log \Delta t \rangle$ between attacks, with 25^{th} and 75^{th} quantile isoclines, vs. group experience k. The solid line shows the expected mean delay, from the statistical model described in the text. (b) A "data collapse" showing the alignment of the re-scaled conditional delay distributions $\Pr(\Delta t \cdot k^{\hat{\beta}} \,|\, k)$ with the estimated underlying log-normal distribution, as predicted by the model.

in delays at a given k, μ is related to the characteristic delay between attacks and β controls the rate at which that delay decreases with increased experience k. That is, β governs the intensity of the acceleration effect of the feedback loop between organizational experience and the production of new events: larger β indicates faster acceleration, and $\beta > 0$ indicates support for H3. To capture the minimum delay imposed by the minimum timing resolution in the empirical data data, we set $\Pr(\Delta t \,|\, k) = 0$ for $\Delta t < 1$ day.

This model's mathematical structure implies several notable facts about the overall dynamics of event production. First, the trajectory of the typical delay between attacks decreases according to a power-law function with increasing experience

$$\Delta t \approx \mathrm{e}^{\mu} \mathrm{k}^{-\beta} \quad . \tag{2}$$

Thus, if $\beta > 0$, we will observe a transition toward increasingly fast event production. In contrast, if $\beta = 0$, production rates are constant and do not vary with organizational experience, while if $\beta < 0$, production rates will decrease with increasing experience. In the $\beta > 0$ regime predicted by H3, we note that the acceleration effect is dampened as the mean delay asymptotes to the minimum timing resolution at $\Delta t = 1$; this produces slight upward curvature for large values of k (see Appendix C).

Within the $\beta > 0$ regime, the particular value of β has a strong effect on the material dynamics of the feedback loop between increasing experience and increasing production. If $\beta = 1$, then the feedback loop is linear, as we saw in our formal model above, with increases in organizational experience leading to proportional increases in event production (or, by H1 and H2, increases in organizational size). Linearity implies that the marginal growth induced by an additional event is relatively constant over the organization's lifetime and a roughly constant fraction of new recruits are allocated to increase the per-unit-time production of new events (militant activities).

In contrast, $\beta \neq 1$ implies a non-linear feedback process.[12] When $\beta > 1$, the feedback is super-linear, and one or both of these effects also increases with increased k. In this regime, either marginal growth per event increases over time or an ever increasing fraction of recruits are allocated to militant activities. On the other hand, when $\beta < 1$, the feedback is sub-linear and the marginal recruitment benefits of new events (growth) decrease over time or they are constant but recruits are increasingly allocated toward non-militant activities.

Fitting this model directly to the empirical data on all events, we find that the maximum likelihood estimate is $\hat{\beta} = 1.0 \pm 0.1$ (stderr), precisely in the linear feedback regime. Using a Monte Carlo simulation against a null model with fixed $\beta = 0$ (no acceleration over time) and with μ,σ estimated using maximum likelihood given the fixed β value, we find that the value of $\hat{\beta}$ is highly statistically significant, suggesting that the acceleration effect is quite real. (Fitting to deadly attacks alone yields a highly statistically significant $\hat{\beta} = 1.1 \pm 0.2$, slightly in the super-linear regime, but not statistically distinguishable from the linear $\beta = 1$ regime.)

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[12] Non-linear feedback processes are not common models of social processes (but see the literature on arms-races, particularly Richardson [51] and Wallace and Wilson [59]). Traditional models often focus on proportional effects in which increases in one variable cause proportional changes in other variables. In nonlinear feedback processes, small increases in one variable can produce dramatic and continuing swings in other variables, leading to highly unpredictable dynamics [57].

TABLE III: Frequency curve parameters for organizations with similar political motivations. Note: statistical significance estimated via Monte Carlo simulation of a two-tail test against a null model with $\beta = 0$ (no frequency acceleration), using the sum-of-squared errors (SSE). Values in parentheses indicate bootstrap standard uncertainty in the last digit.

political motivation	groups	events	μ	σ	β	significance
nationalist-separatist	55	2959	5.1(5)	2.2(1)	0.9(2)	$p < 0.001$
reactionary	5	143	3.2(6)	1.8(2)	0.1(3)	$p < 0.001$
religious	17	999	5(1)	2.4(5)	1.7(5)	$p < 0.001$
revolutionary	53	2527	5.7(4)	2.3(2)	1.1(2)	$p < 0.001$
all secular	883	6232	5.2(2)	2.25(9)	0.9(1)	$p < 0.001$
all groups	910	7231	5.1(2)	2.32(9)	1.0(1)	$p < 0.001$

The linear feedback implies that the marginal growth from event-driven recruitment does not vary much with organizational size or experience. Furthermore, it implies that organizational learning in terrorist groups [28, 29], in which the production rate increases due to improved efficiency of a fixed number of individuals, must play a small role in explaining the overall acceleration of event production. Instead, the frequency acceleration be compactly explained as a side-effect of increasing organizational size.

A strong test of the statistical model's plausibility is its prediction that each of the k conditional delay distributions $\Pr(\Delta t \,|\, k)$ is simply a scaled version of the underlying log-normal distribution $\mathrm{LN}(\mu, \sigma^2)$. To test this prediction, we re-scale the empirical distributions by the predicted factor, i.e., $\Pr(\Delta t \cdot k^{\hat{\beta}} \,|\, k)$ in which we multiply each delay variable Δt_i by $k_i^{\hat{\beta}}$ and then plot them against the estimated underlying log-normal distribution. A close alignment of these re-scaled conditional distributions, also called a "data collapse" [5], may be interpreted as a strong evidence for a plausible generative model. Figure 4b shows the results of this test, illustrating an excellent data collapse, with each of the re-scaled conditional distributions closely aligning with the predicted underlying log-normal form. Thus, Eq. (1) is an excellent statistical model of the generic frequency-acceleration pattern.

C. Severity of Attacks Over Time

In contrast to the universal acceleration in the frequency of attacks over time, but in agreement with our static analyses above, we find no statistically significant relationship between the severity of attacks and increased experience (Pearson's $r = -0.024$, t-test, $p = 0.17$). That is, we find no support for the severity-increase hypothesis (H4). Across all organizations in our sample, the average severity the first deadly attack is $\langle x \rangle = 6.7 \pm 0.9$, which is only slightly larger than the average severity of all deadly attacks by highly experienced groups (those with $k > 100$) $\langle x \rangle = 5.1 \pm 0.6$. Figure 5a shows the composite severity curve for all organizations in our study.

As with the frequency curves, we find that the conditional severity distributions $\Pr(x \,|\, k)$ roughly collapse onto a single, underlying form (Figure 5b), and that this form is remarkably similar to the power law observed for all deadly terrorist attacks worldwide from 1968–2008 [12, 14]. That is, Richardson's Law for terrorism appears to hold for both inexperienced and highly experienced organizations. Combined with our static analysis of organizational size, this pattern implies a highly counter-intuitive fact: the severity of attacks by larger, more experienced organizations, who presumably have significantly more resources, access to specialized facilities and information, is not significantly greater than the severity of attacks by small, inexperienced organizations who would likely lack such resources. That is, inexperienced organizations are just as likely to produce extremely severe events as highly experienced organizations, which runs counter to the common assumption that only experienced groups are capable of such mass destruction [33].

Clauset, Young and Gleditsch's [14] analysis of global terrorism showed that Richardson's Law holds independently of the type of weapon used (explosives, firearms, arson, etc.), the historical time period analyzed since 1968 and the economic development of the target country, albeit with differences in the estimated power-law exponent in each case. To this list we may now add organizational size and experience. This implies that a successful explanation of the frequency of severe terrorist events cannot depend on or predict a systematic relationship between event severity and organizational maturity or size, as do the aggregation-disintegration models of Johnson et al. [30, 31], Bohorquez et al. [7] and Clauset and Wiegel [13]. We note that a model recently proposed by Clauset, Young and Gleditsch [15] makes no such assumptions. To briefly summarize, the proposed model combines physical constraints, in the form of heterogenous population density distributions, with strategic decisions by terrorists, in the form of attacks preferring targets with greater population density, to produce a heavy tail in the observed severity of events. It remains to be seen if this explanation will be successful.

Thus, although more experienced organizations are not systematically more lethal at the individual-event level, the frequency-acceleration pattern observed above implies that more experienced groups are significantly more lethal overall. This same pattern was recently observed by Asal and Rethemeyer [4] in their analysis of the

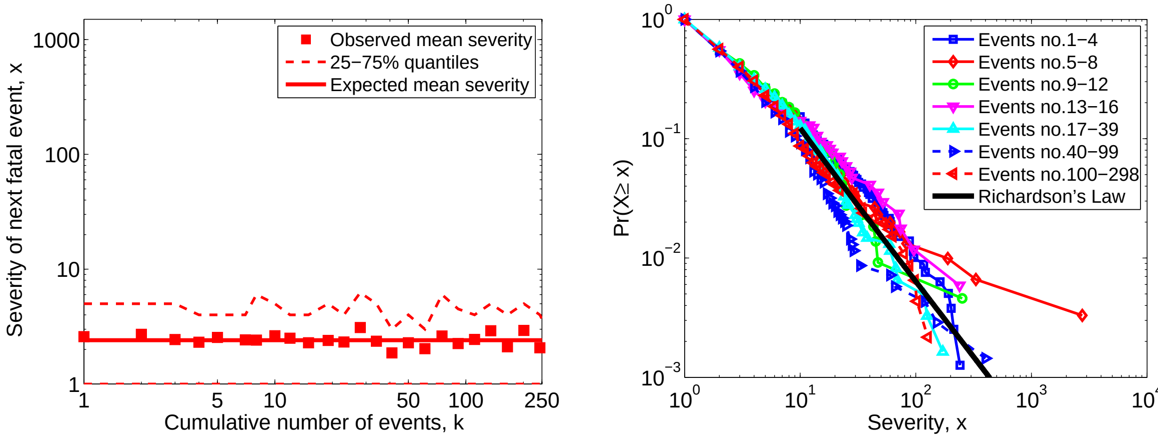

FIG. 5: (a) Mean severity $\langle \log x \rangle$ of deadly attacks, with 25^{th} and 75^{th} percentile isoclines, vs. group experience k. Solid line (with slope zero) shows the expected delay, from a simple regression model. (b) Conditional severity distributions $\Pr(x \mid k)$, showing a data collapse onto a heavy-tailed distribution, with the maximum likelihood power-law model for all severities (Richardson's Law).

BAAD organizations. Our results thus clarify their static models, showing that the observed correlation between greater lethality (total deaths attributed to an organization) and greater organizational size can be explained by the fact that larger, more experienced organizations produce events more quickly than smaller, less experienced organizations. Thus, it is the cumulative effect of the many small events that explains the increased lethality, not a systematic increase in the lethality of individual events.

VI. IMPACT OF POLITICAL IDEOLOGY ON EVENT FREQUENCY AND SEVERITY

Our results for the developmental dynamics of event frequency and severity are good descriptions of the generic behavior of terrorist organizations. However, our analyses have so far omitted any role for organizational covariates, many of which are believed to have important impacts on organizational behavior and decisions (see Asal and Rethemeyer [4], Pape [45] and Clauset et al. [11], among others). The robustness of our results above suggest that these covariates should play secondary roles to organizational size in governing the overall event production dynamics. But, what impact do such covariates have on the frequency or severity trajectories? Could different types of organizations exhibit different trends within the more general patterns? We investigate this question by studying the impact, if any, political or ideological motivation may have on the frequency curve's structure; we leave the investigation of other covariates for future work.

Miller [38] divides the political motivations for terrorism or group ideologies into four conventional categories: nationalist-separatist, reactionary, religious and revolutionary. We coded according to Miller's criteria the 131 groups in our sample with $k \geq 10$ deadly events (who together account for 85% of events) and then fitted Eq. (1) to the data within each ideological category. Organizations with multiple political motivations were placed in multiple categories, which would only lessen any differences between estimated parameters for different categories. Fig. 6a shows the corresponding central tendencies, as described by Eq. (2). Table III summarizes the estimated parameters for each ideological category and groups overall.

We again test the statistical significance of the acceleration effect within each ideological model using a two-tail test against a null model with fixed $\beta = 0$ (no acceleration over time) and μ,σ estimated using maximum likelihood given the fixed β value. In all cases, the estimated β parameter is highly statistically significant (at the $p < 0.001$ level), suggesting that the acceleration effect is real within each category, even for reactionary groups, who show the weakest acceleration.

Among the four ideological categories, we observe wide variation in the estimated values of β and thus in the strength of the endogenous feedback loop governing in the frequency of attacks. Religious groups have the largest value at $\hat{\beta} = 1.7 \pm 0.5$, placing them firmly in the super-linear feedback regime and implying very strong acceleration in the frequency of attacks over time. In contrast reactionary organizations have the smallest at $\hat{\beta} = 0.1 \pm 0.3$, placing them strongly in the sub-linear regime. Revolutionary and nationalist-separatist categories are statistically indistinguishable from the linear-feedback regime of $\beta = 1$.

The typical religious group, i.e., one accelerating along the generic production trajectory identified above, with $k = 10$ deadly attacks, attacks as frequently as the typical revolutionary group with $k = 51$ deadly attacks or the typical nationalist-separatist group with $k = 129$ attacks.

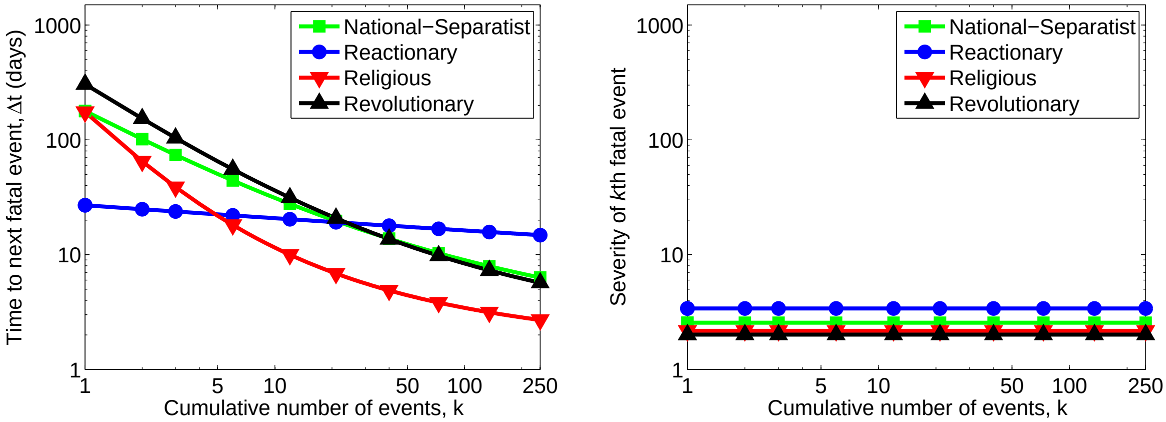

FIG. 6: (a) Estimated frequency curves for four ideological categories, showing that religious groups develop extremely quickly relative to other types. (b) Estimated severity curves for the same categories, showing the same pattern of independence as Fig. 5a.

When viewed in terms of calendar time, this difference is even more striking: it takes the typical religious terrorist organization only 400 days (1.1 years) to generate its first 10 attacks and at this point its production rate is approximately one attack every 5 days. In contrast, the typical revolutionary organization takes 1666 days (4.6 years), more than four times as long, and a typical nationalist-separatist organization takes 2103 days (5.8 years), to achieve an equal production rate. Combining this insight with the results of our static analysis on the role of size, the explosive acceleration by religious groups implies that they grow in size extremely quickly, which is the ultimate cause of their dramatic production rates.

But religious organizations are not universally more dangerous. Comparing the $\hat{\mu}$ parameters, which governs the characteristic delay between subsequent attacks, we observe a more complicated story: reactionary groups are initially the most dangerous, with the fitted model estimating typically $\Delta t = 47$ days between their first and second attacks, while all other groups take substantially longer ($\Delta t > 100$ days). However, this difference in initial production rates is quickly eliminated by the explosive acceleration of religious groups as well as the more measured development of revolutionary and nationalist-separatist organizations, whose attack rates overtake that of reactionary groups after between 5 and 25 events.

Much previous work on religious terrorism has argued, largely on theoretical grounds, that such organizations are fundamentally more dangerous than secular groups [23, 34, 38, 47] because they have fewer social restrictions on their activities and are thus more free to produce and target violence than secular organizations, whose victims may be potential sympathizers. Our results provide strong empirical support for this argument, in the sense that religious organizations exhibit explosive acceleration in the production of violence while secular organizations exhibit more moderate acceleration.

Past arguments that religious organizations are universally more dangerous may have over-simplified organizational behavior by ignoring how organizations may change their behavior over their lifespan and how these changes compare to those exhibited by other organizational types. Our results show that initially, i.e., very early in their life histories, religious groups are in fact less dangerous than reactionary groups, and only slightly more dangerous than national-separatist or revolutionary groups. It is only over the long term that the explosive acceleration experienced by religiously-motivated organizations allows them to cumulatively produce so many more events than other types of organizations. That is, only if a religious organization succeeds in reaching a more mature state does it pose a greater overall risk than groups with secular motivations. We note that historically speaking, most organizations do not live so long [17]: fully 55% of organizations in the MIPT database are associated with only a single event.

Turning briefly to the question of how event severity varies with organizational ideology, we repeat the same severity-curve analysis on the deadly events produced by the 131 highly prolific organizations. Figure 5b shows the resulting ideology-specific severity curves and Table IV summarizes the estimated model parameters, where the model now is a simple linear regression of severity x against experience k. As with the more general analysis, we find no systematic dependence of severity of attacks on organizational experience within any of the ideological categories. That is, none of the model coefficients are significant, and the average severity of events within each category vary only a little. Political ideology has no systematic impact on the severity of events or the trajectory that event severities take over the lifespan of an organization.

TABLE IV: Severity curve parameters for organizations with similar political motivations. Note: statistical significance calculated using a t-test on Pearson's correlation coefficient.

political motivation	groups	events	$\langle x \rangle$	r	significance
nationalist-separatist	51	1003	6.1	0.0071	$p = 0.75$
reactionary	5	77	7.1	0.1194	$p = 0.27$
religious	17	753	5.2	-0.0062	$p = 0.49$
revolutionary	41	725	5.1	-0.0109	$p = 0.38$
all groups	381	3143	7.3	-0.0240	$p = 0.17$

VII. DISCUSSION

Although operational, organizational, and political circumstances vary widely across terrorist groups, the generic nature of our results suggests several general conclusions. First, we find strong evidence for a positive feedback loop among organizational size (number of personnel), organizational experience (cumulative number of events) and the frequency of events by that organization (delay between days with events) such that small organizations tend to be inexperienced and exhibit long delays between events, while larger organizations are more experienced and exhibit delays orders of magnitude smaller.

Within this feedback loop, new attacks generally lead to organizational growth through recruitment and the corresponding increase in size leads to the increased production rate of events because some of the growth is reinvested in additional production capacity (militant activities). As a result, the feedback produces a generic *developmental* relationship, in the dynamics of an organization over its lifespan: the more events an organization has produced, the faster it produces its next event. In short, we find strong empirical support for hypotheses H1 (labor constraints), H2 (event recruitment) and H3 (frequency increase).

The typical frequency-increase relationship in particular can be mathematically modeled by a power-law function, in which the delay Δt between consecutive events decreases roughly like $\Delta t \propto k^{-\beta}$ where k counts the cumulative number of events and β is a model parameter that can be interpreted as the strength and direction of the feedback loop. For all events by organizations in our sample, we estimate $\beta = 1.0 \pm 0.1$, implying a simple linear feedback loop, in which the share of marginal increases in size allocated to increasing the event production rate remains relatively constant over the lifespan of an organization. This dynamic appears strongest for small or inexperienced organizations, e.g., those with $k \leq 10$ events, which covers 87% of the 910 organizations in our sample. In contrast, highly experienced organizations seem to saturate their event production rates at the daily or weekly level, which may be indicative of a tendency of large organizations to engage in multiple types of activities, e.g., the provision of social services, criminal activities, etc., rather that simply maximizing the production of new events.

The mathematical precision of this relationship, however, is striking, as is the ability of a simple simulation to reproduce it. Except for Richardson's Law for the frequency and severity of wars, few statistical relationships in the study of political violence exhibit such regularity. This fact supports the provocative goal of developing reliable statistical forecasting tools for terrorism. In this case, such a tool would take as input knowledge of an organization's past activities, and relevant covariates such as its size and political motivation, and produce probabilistic estimates of the delay until the next attack. Because these forecasts would be based on a constraint not fundamentally under the organization's control, and so long as the forecasts are not too precise, it is unlikely that they could be foiled by simple contrarian responses.

The power-law relation between organizational experience and production rate is both conceptually and mathematically similar to the relationship between cost and cumulative production observed in manufacturing [18] or organizational learning [1, 58], where decreases per-item production costs or time can be described by a power law in the cumulative number of items produced. That a similar patterns appears in the production of terrorist events is surprising. Thus, it may not be a superficial analogy to describe terrorist organizations as a special type of manufacturing firm whose principal product is political violence and whose production is fundamentally constrained by its employed labor.

The existence of strong labor constraints on the production rates of events suggests that terrorism is inherently non-amenable to mass-production, perhaps because each event must be planned according to the particular target, tactic, environment, etc. In the language of economics, we find that historically capital and labor are non-substitutable for the production of new events. If terrorism is also an inherently low-skill "job," as argued by Krueger [36], then the potential pool of labor for any particular conflict is likely extremely large and event production rates at the whole-conflict level should ultimately be responsive to policy and counter-terrorism efforts that impact this pool's mobilization. Thus, these findings suggest that successful "hearts and minds" strategies [27] for counter-terrorism are likely to lead directly to lower incident rates by both restricting the growth and reducing the size of terrorist organizations. They may not, however, eliminate the possibility of spectacular attacks.

In classic economic theory [3], whenever capital and labor are not substitutable, labor plays a fundamental role in shaping overall production. Thus, studies of non-terrorist organizations who exhibit similar labor constraints on their activity levels may provide insight into the developmental and organizational dynamics of terrorist groups. Such results would have strong implications for effective counter-terrorism efforts, e.g., those that specifically target organizational dynamics like recruitment, growth, personnel turnover and internal coordination. The difficulty in acquiring high-quality empirical data on the internal dynamics of such groups increases the attractiveness of studying them indirectly via related phenomena in non-terrorist organizations. Prime examples of potentially relevant organizations would be ones that grow mainly by promoting their activities through the media, e.g., certain types of political activist groups [28].

Recently, Johnson et al. [29] observed a similar empirical frequency acceleration in deadly events within individual provinces of Iraq and Afghanistan, which they argued is caused by efficiency gains related to learning. They hypothesize an arms-race between Taliban insurgents and coalition military forces, in which improvements in efficiency on one side drive similar improvements on the other, and vice versa. Although this model produces frequency acceleration, it also assumes that dramatic efficiency gains can, in principle, be made by an arbitrarily small number of personnel on either side. Furthermore, Johnson et al. observe that the frequency of attacks in a number of provinces increased by a factor of nearly 100. Empirical studies of learning [2, 58], however, find that learning can increase production rates typically by a factor of 2 or so, and in special circumstances a factor of 10. Greater improvements are only observed in highly mechanized production, e.g., automobiles and electronics [18]. Thus, Johnson et al.'s learning hypothesis can reasonably only account for about 10% of the observed 100-fold frequency increase.

In contrast, organizational size can compactly explain the entire phenomenon: the frequency of attacks in a province increases over time because the number of resident Taliban fighters also increases over time. These additional fighters could be recruited from the local population or emigrate from other provinces; either case leads to growth via "recruitment" and thus increased frequency of attacks due to the increased pool of insurgents. Although learning may play some role in the increased frequency, e.g., in the assembly and placement of IEDs or in the gathering of intelligence, it seems unlikely to explain a 100-fold increase in the frequency of deadly attacks. In contrast, a comparable increase in the number of fighters is highly plausible.

Although the acceleration toward faster production of events (and larger size) is remarkably strong, the vast majority of organizations in our sample did not achieve a high level of experience (only 23% of groups are associated with $k > 10$ events). The leakiness of the developmental pipeline could be caused both by high rates of organizational death, e.g., from counter-terrorism activities or internal conflicts [16, 32], or shifts away from violence. Further, the one-day resolution limit in our timing data prevents us from testing whether the feedback loop, which seems strongest for small organizations, continues to hold just as strongly for highly experienced groups, although we suspect it does not. From a theoretical perspective, once an organization is sufficiently large to produce new events each day, e.g., the Taliban, Hamas and FARC, it may function more like a stable or mature social institution, with fundamentally different constraints and incentives than still developing organizations. The large size and stability of such organizations may pose special risks, e.g., leading to larger or longer conflicts and arguably the conflicts involving those three particular organizations better resemble civil wars than terrorist campaigns. On the other hand, non-violent activities, e.g., engagement with political processes, may also become more attractive with increased size. The exploration of these possibilities is an interesting avenue for future work.

In contrast to the strong support for organizational size governing the overall frequency of terrorist events, we find no evidence supporting a similar relationship with the severity of attacks (H4). Rather, Richardson's Law—a power-law distribution in the frequency of severe events—appears to govern the severity of events at all levels of organizational size or experience, and independent of the organization's political ideology.

The reason severity does not systematically covary with size or experience may be a mixture of different effects at different stages in an organization's lifespan. Larger or more experienced organizations may run a greater risk of alienating important constituents by generating extremely severe attacks. For instance, when the Tamil Tigers' used chemical weapons in their assault on East Kiran, one factor cited in their discontinuance of this tactic was the potential loss of local and international support for the Tigers' cause [25]. A similar logic naturally applies for deadly attacks of all kinds. In contrast, although smaller or less experienced organizations likely have fewer constituents to lose by this means, they may not have the resources or expertise necessary to generate such spectacular events [46]. Thus, as organizations mature, the increasing weight of the former factor may cancel the decreasing weight of the latter one, yielding no overall trend.

That the severity of terrorist events is independent of organizational covariates like size and experience clarifies ongoing efforts to identify the underlying social, political or physical mechanism that generates Richardson's Law in terrorism. Several existing explanations assume or predict such a relationship, e.g., the aggregation-disintegration model of Johnson et al. [30, 31], Bohorquez et al. [7] and Clauset and Wiegel [13], but these seem increasingly unlikely given our results here. The explanation proposed by Clauset, Young and Gleditsch [14], which posits a coevolutionary competition between states and terrorists in which event planning time and severity are strongly related, and the one proposed by Clauset, Young and Gledistch [15], in which population densities are broad-scaled and terrorists preferentially target high-density locations, are not ruled out by our results.

Turning now to the joint consequences of our results for both frequency and severity curves, we see that the total lethality of larger and more mature groups recently observed by Asal and Rethemeyer [4] is probably best explained as a natural consequence of their much more frequent activities, rather than by a systematic increase in the deadliness of their individual events. If event severities by the same organization are somewhat independent, organizations that produce the most events—i.e., large organizations—are probabilistically more likely to generate large severity events. Thus, policies that limit the ability of organizations to grow or reduce the effective size of the militant wing will lower the long-term probability of a severe event by that organization. Such growth-limiting policies could be described as "starving the beast" of the labor necessary to produce rare but highly severe events.

The most likely and productive targets of such growth-limiting policies will be large, established organizations with long histories of producing terrorist attacks. By virtue of their size, these organizations are likely to be well-known players in their particular conflicts and thus easy targets for specific policies. However, an notable corollary of the failure of the severity-increase hypothesis (H4) is that such policies are unlikely to eliminate the risk of severe events from all sources. Because the distribution of event severities is independent of organizational experience and size, small organizations are equally likely to produce severe events, and policies aimed mainly at large, well-known organizations may not have the same impact on small, unknown organizations. For those organizations, the most effective policies may be those aimed at preventing their formation in the first place, i.e., policies that curtail the acquisition of the means and resort to violence. Lacking this, once such a terrorist cell carries out its first attack and begins its developmental trajectory, the best action by a government may be an "overwhelming response" to encourage through various means the dissolution of the nascent organization and the truncation of its growth trajectory. This policy is not without risk to the state, however, as countermeasures may actually be counterproductive, to the extent that they could instead serve to further the terrorist's goals [22, 41].

In closing, we point out that the acceleration in the frequency of terrorist events is independent of many commonly studied factors associated with terrorism, including geographic location, time period, international vs. domestic targets, ideological motivations (religious, national-separatist, reactionary, etc.), and political context. Our results thus demonstrate that some aspects of terrorism are not nearly as contingent or unpredictable as is often assumed, in both the academic literature and in public policy, and the actions of terrorists may be constrained by processes unrelated to strategic tradeoffs among costs, benefits and preferences. Identifying and understanding these processes offers a complementary approach to the traditional rational-actor framework, and a new way to understand what regularities exist, why they exist, what they imply for long-term social and political stability, and how they might relate to large-scale violent conflicts such as civil and interstate wars.

Acknowledgments

The authors thank Lars-Erik Cederman, Konstantinos Drakos, Brian Karrer, James McNerney, John Miller, Mark Newman, Andrea Ruggeri, Didier Sornette, Cosma Shalizi, Brian Tivnan, Valerie Wilson and Maxwell Young for helpful conversations. This work was supported in part by the Santa Fe Institute, the Economic and Social Research Council (RES-062-23-0259) and the Research Council of Norway (180441/V10).

Appendix A: Domestic vs. Transnational Events

From 1968–1997, the MIPT event database was maintained by RAND as part of its project on transnational terrorism. As a result, almost no domestic terrorist attacks are included before 1998, after which the scope of the database was significantly expanded (in part due to the Oklahoma City bombing in 1995) to include purely domestic events worldwide. Although organizations and events are not coded as being transnational or domestic, the inconsistency in database scope provides an opportunity to test whether the frequency dynamics of domestic terrorism organizations differs from those of transnational organizations.

By dividing events into those generated by organizations whose first event occurred 1968–1997 and those generated by organizations whose first event occurred in 1998–2008, and then repeating the frequency-curve analysis from the main text, we may test whether the frequency-acceleration phenomena appears only in one time period or the other. Further, because events in the 1998-2008 period are mainly domestic events, while those in the 1968–1997 period are only transnational events, the two time periods serve as proxies for transnational-only and domestic-only terrorism. This division does not control for non-stationary effects.

Figure A1 shows that the development curve phenomenon is robust to this division, indicating that the frequency-acceleration appears to hold for both transnational and domestic terrorism. One difference between these time periods does emerge: the rate of acceleration for the 1968–1997 data (transnational only) is $\hat{\beta}_{t_1 \leq 1997} = 1.0 \pm 0.2$ (stderr), statistically indistinguishable from the analysis of all organizations in the main text, while the estimated acceleration for the 1998–2008 data (mainly domestic) is slightly faster, with $\hat{\beta}_{t_1 > 1997} = 1.3 \pm 0.2$. The origin of this difference may be related to the increasing

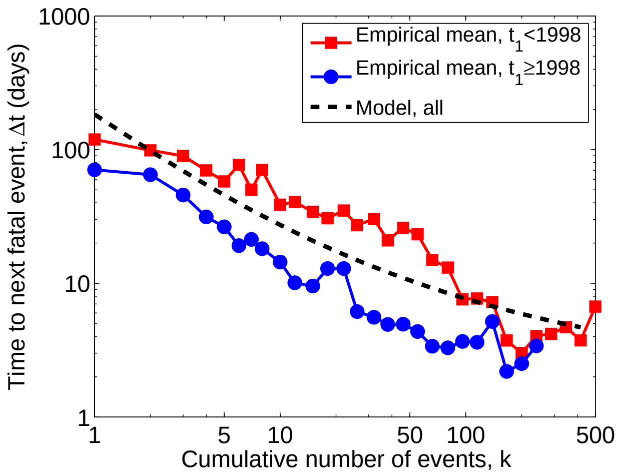

FIG. A1: The attack frequency development curves, plotted as the average delay versus experience, for groups whose first attack was in 1968–1997 versus those whose first attack was in 1998–2008, along with the model estimated for all events from the main text.

frequency of religiously-motivated terrorism in the 1990s and beyond [48, 52], who collectively exhibit a lower value of $\hat{\beta}$ than other types of terrorism. An interesting alternative explanation, however, is that some non-stationary process is having a consistent upward pressure on β over time, for all organizations. One candidate process is the development and spread of modern communications and digital technology, which may enable more widespread or effective recruiting efforts and thus enabling faster organizational growth.

Appendix B: Terrorist Organization Simulation

The toy model described in the main text can be formalized and simulated explicitly. Below is computer code that implements the simulation in Matlab. In words, the simulation works as follows.

Let η be a constant that denotes the number of individuals that make up a terrorist "cell" within the organization, and let ν be the number of individuals the organization as a whole gains via recruitment after each event. Thus, η/ν events are required to produce a single new cell; the particular values of η and ν serve only to change the scale of the dynamics, not their fundamental character. Each cell is assigned a "clock" that measures the number of days remaining before that cell generates an event. We denote this delay τ and draw it from a log-normal distribution with parameters μ and σ, i.e., $\Pr(\tau) \sim \mathrm{LN}(\mu, \sigma)$. This is the only stochastic element of the simulation. When a cell generates an event, it then draws a new delay from the same distribution.

As described in the main text, each organization begins as a single cell, which has generated a single event at $t = 0$. Thus, initially $s_1 = \eta$. We then choose a delay τ for its next event. The simulation will generate a specified number of events, specified by the parameter `nok`. For the kth event, the simulation then checks which cell has the smallest remaining delay and advances all cells' clocks by that much. It then generates the kth event, records its time as an ordered pair (k, t_k), and draws a new clock value for the generating cell. Additionally, it increments the organization's size by ν individuals, i.e., $s_k = s_{k-1} + \nu$, and adds $\lfloor s_k/\eta \rfloor$ new cells, each with a clock drawn from $\Pr(\tau)$.

A number of variations of this model generate equivalent results. For instance, the distribution $\Pr(\tau)$ can generate very small delays, e.g., less than 1 day, which may be considered unrealistic. Imposing a minimum value on the $\Pr(\tau)$ does not change the fundamental feedback between size and event production and thus leaves the k^{-1} trend unchanged. And, the ratio η/ν only re-scales the underlying k^{-1} behavior, as seen in Figure 2. Finally, changing the parameters of $\Pr(\tau)$ has no impact on the fundamental behavior: the μ parameter sets the delay between the first and second events, which appears as the expected y-intercept on the resulting development curve, and varying σ simply changes the scatter around the underlying trend. In fact, the particular functional form of $\Pr(\tau)$ we have chosen is not important, and other choices lead to similar results; here, we choose the log-normal distribution due to its similarity to the empirical data (Fig. 4).

```
% --- Terrorist organization simulation
% --- by Aaron Clauset ( aaron.clauset@colorado.edu )

% --- set up simulation parameters
[mu  sigma] = deal(5.1,2.32); % parameters for Pr(tau) = LN(mu,sigma)
[eta nu]    = deal(5,5);      % size of cell,  marginal growth after an attack
nok         = 1000;           % number of events to generate

% --- set up simulation data structures
s   = zeros(nok+1,1); % organization size over time
c   = s;              % number of cells over time
[s(1) c(1)] = deal(eta,1);
fk  = zeros(nok+1,2);
fk(:,1) = (1:size(fk,1))';    % assign ids to events
gr      = zeros(nok+1,2);     % holds event clocks for each cell
gr(:,1) = (1:size(gr,1))';    % assign ids to cells

% --- initialize simulation: create the first cell
t   = 0;                      % global clock
k   = 1;                      % number of attacks to date (first attack at t=0)
tau = exp(sigma*randn(1)+mu); % choose delay from Pr(tau)
gr(1,:) = [1 tau];            % make first cell

% --- generate exactly nok events
while k<size(fk,1)

    % -- advance time to next attack
    [dt i] = min(gr(1:c(k),2)); % find cell with next attack
    t      = t + dt;            % advance all clocks by that much time
    gr(1:c(k),2) = gr(1:c(k),2) - dt;

    % -- generate the kth event
    k        = k + 1;           % increment attack number
    fk(k,2)  = t;               % record time of this event
    tau = exp(sigma*randn(1)+mu);
    gr(i,2) = tau;              % choose new delay for this cell

    % -- recruitment / growth
    s(k) = s(k-1) + nu;         % grow total personnel
    c(k) = floor(s(k)/eta);     % count no. cells
    dc = c(k) - c(k-1);         % calculate cell growth
    if dc>0                     % create the new cells and choose their delays
        tau = exp(sigma*randn(dc,1)+mu);
        gr(c(k-1)+1:c(k),2) = tau;
    end;
end;
% --- done generating events; extract results
[dt k] = deal(diff(fk(:,2)),(1:size(fk,1)-1));

% --- plot resulting development curve
figure(1); clf;
loglog(k,dt,'r-','LineWidth',2); hold on;
loglog([1 nok],exp(mu).*([1 nok]).^(-1),'k--','LineWidth',3); hold off;
xlabel('Cumulative number of events, \it{k}','FontSize',16);
ylabel('Time to next event, \Delta\it{t} \rm{(days)}','FontSize',16);
set(gca,'FontSize',16,'YTick',10.^(-6:4));
h1=legend(strcat('Simulation, \nu/\eta=',num2str(nu/eta,'%3.1f')), ...
     'Model, \Deltat\propto k^-^1',1); set(h1,'FontSize',16);
```

Appendix C: Statistical Model for the Frequency of Attacks

The probabilistic model for event delays used in the main text, given by Eq. (1), has the precise form of

$$\Pr(\Delta t \,|\, k) = \left(\frac{\sqrt{2/\pi}}{\sigma \left(1 - \mathrm{Erf}\left[\frac{\beta \log k - \mu}{\sigma\sqrt{2}}\right]\right)} \right) \exp\left[\frac{-(\log \Delta t + \beta \log k - \mu)^2}{2\sigma^2} \right] \tag{C1}$$

where the leading term is the normalization constant and $\mathrm{Erf}(\cdot)$ is the error function. In words, this model asserts that the logarithm of the delay Δt is a random variable distributed according to a Normal distribution $\mathcal{N}(\nu, \omega)$ (or equivalently, the delay is log-normally distributed) with a lower cutoff at $\Delta t = 1$ day (to reflect the timing resolution of the event data), constant variance ω and a distributional mean ν that decreases systematically with increasing experience k. In Eq. (C1), the parameter μ denotes the characteristic delay between attacks, and in particular the delay between the first and second attacks, while σ^2 denotes the variance in the expected delay.

The equation given in the main text for the expected delay as a function of experience—the central tendency of the conditional distribution of delay as a function of experience—can be derived in the usual way. Doing so yields

$$\mathrm{E}[\log \Delta t] = \mu - \beta \log k + \left(\frac{\exp\left[\frac{-(\beta \log k - \mu)^2}{2\sigma^2}\right] \sqrt{2/\pi}}{\sigma^{-1} \left(1 - \mathrm{Erf}\left[\frac{\beta \log k - \mu}{\sigma\sqrt{2}}\right]\right)} \right) , \tag{C2}$$

which has a simple leading form and a complicated trailing term. For small values of k, the expected delay is dominated by the leading two terms, i.e., the trailing term is small in relative magnitude, and thus the trend is well-approximated by a power-law function $\Delta t \approx \mathrm{e}^{\mu} \mathrm{k}^{-\beta}$, where e^{μ} represents the initial rate of attack of a group. At larger values of k, the expected delay is dominated by the trailing term, which makes the expected delay to approach $\Delta t = 1$ more slowly than a power law.[13]

When fitting this model to the empirical data, we estimate its parameters using standard numerical procedures to maximize the likelihood of the data (in this case, the Nelder-Mead method [43]). Standard error estimates for the uncertainty in the parameters are then estimated using a bootstrap procedure on the organizations in the sample.

The striking "data collapse" shown in Figure 4b illustrates that the conditional probability distributions do indeed align closely with the estimated log-normal model for delays. Why delays should be log-normally distributed remains a mystery.

Finally, we point out that very few groups (e.g., Hamas, Fatah, LTTE, FARC, etc.) manage to become highly experienced ($k \gtrsim 100$). This means that the fit of the model for large-k is primarily controlled by the delays at much smaller values of k, where the vast majority of the data lay. This fact explains the slight misfit of the model to the delays for highly experienced groups. However, it also highlights the fact that the behavior of inexperienced groups early in their lifetime is highly predictive of the behavior of mature organizations.

[13] Although a power law is a good approximation for small-k behavior, caution should be taken when applying it to data with a minimum-delay resolution, e.g., events timestamped by only the date of the attack. In this case, the induced upward curvature for large-k, represented by the complicated trailing term in Eq. (C2), causes non-trivial deviations from the simple power-law behavior, which can lead to incorrect inferences, as in Johnson et al. [29], if ignored.

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