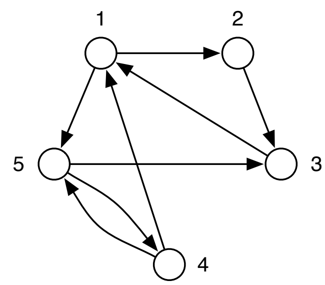
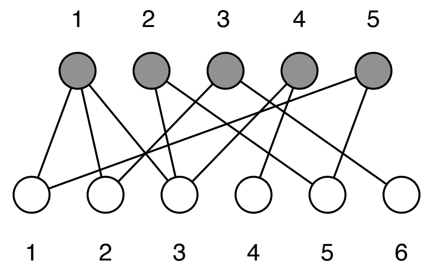
**Network Analysis and Modeling**
**CSCI 5352, Fall 2013**
**Prof. Aaron Clauset**
**Problem Set 1, due 9/9**

1. (12 pts) Consider the following two networks:




(A)



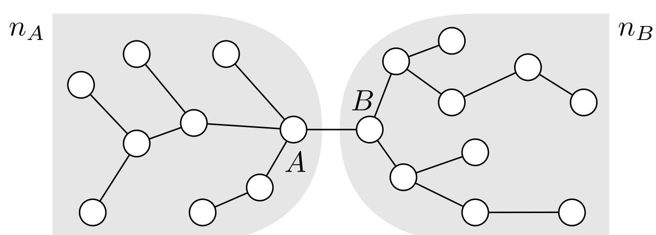

(B)

   (a) (3 pts) Give the adjacency matrix for network (A).
   (b) (3 pts) Give adjacency list for network (A).
   (c) (6 pts) Give the adjacency matrices for both one-mode projections of network (B).

2. (10 pts) Consider a bipartite network, with its two types of vertices, and suppose there are $n_1$ vertices of type 1 and $n_2$ vertices of type 2. Show that the mean degrees $c_1$ and $c_2$ of the two types are given by

$$c_2 = \frac{n_1}{n_2} c_1 \ .$$

3. (15 pts) Consider an undirected, unweighted network of $n$ vertices that contains exactly two subnetworks of size $n_A$ and $n_B$, which are connected by a single edge $(A, B)$, as sketched here:



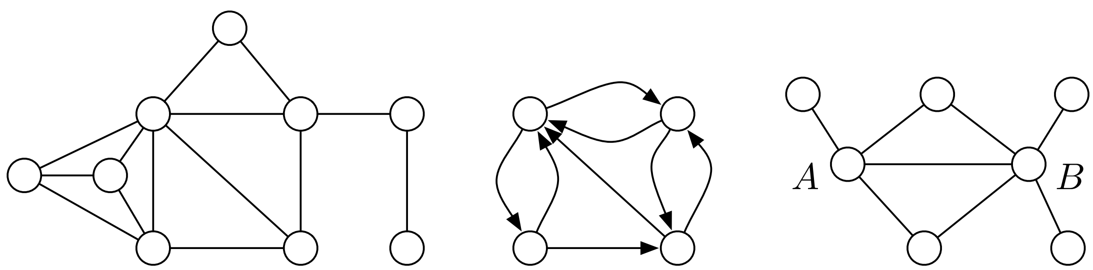

Show that the closeness centralities $C_A$ and $C_B$ of vertices $A$ and $B$, as defined by Eq. (7.29) in *Networks*, are related by

$$\frac{1}{C_A} + \frac{n_A}{n} = \frac{1}{C_B} + \frac{n_B}{n} \ .$$

4. (15 pts) Consider an undirected (connected) tree of $n$ vertices. Suppose that a particular vertex in the tree has degree $k$, so that its removal would divide the tree into $k$ disjoint regions, and suppose that the sizes of those regions are $n_1, \ldots, n_k$. Show that the betweenness centrality $b$ of the vertex is

$$b = n^2 - \sum_{i=1}^{k} n_i^2 \ .$$

5. (12 pts) Consider these three networks:




(A) (B) (C)

- (4 pts) Find a 3-core in network (A).
- (4 pts) What is the reciprocity of network (B)?
- (4 pts) What is the cosine similarity of vertices $A$ and $B$ in network (C)?

6. (36 pts) Using the PS1 network file on the class webpage (also on the next page), construct a table listing the following centrality scores for each vertex: (i) degree centrality, (ii) closeness (harmonic) centrality (Eq. 7.30 in Networks), (iii) eigenvector centrality (Eq. 7.6 in Networks), (iv) betweenness centrality (Eq. 7.38 in Networks). Within each centrality, sort the (family,value) pairs in decreasing order of importance; three decimal places is sufficient detail. Comment on the degree to which these scores agree that the Medici family was the most "central" in the network.

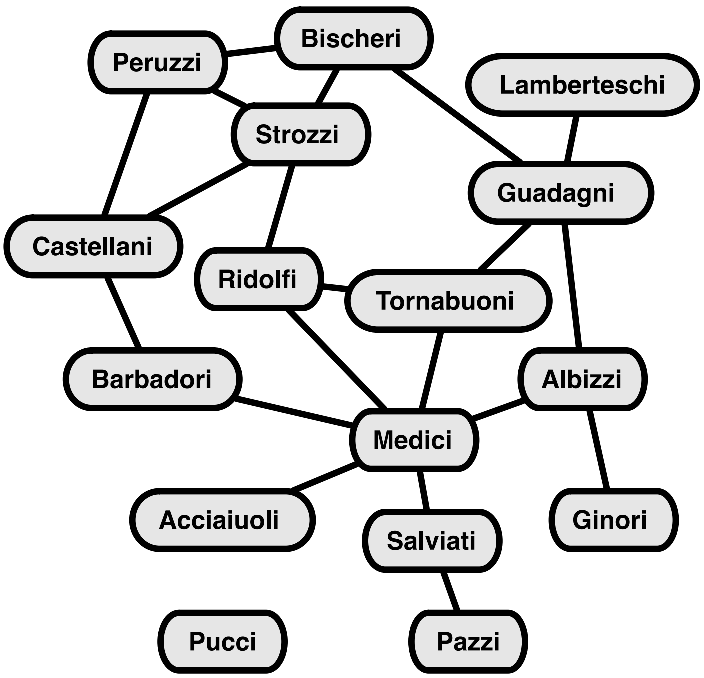



Figure 1: The Medici network, from Padgett and Ansell (1993).