Temporal (Dynamic) Networks

Lecture 20
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CSCI 5352, Network Analysis and Models

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given network \( G = (V, E) \)

- centrality measures (degree-based, geometric, etc.)
- assortativity, transitivity, reciprocity
- distributions (degrees, distances, etc.)
- random walks on networks
- differences relative to configuration model
- community structure
- generative models
- etc.
static network analysis

$G$

empirical network

$f(G)$

network summary statistics

$\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  \vdots \\
  x_k
\end{bmatrix}$
temporal network analysis

idea 1:

empirical network sequence

\[ G_{t=1} \rightarrow G_2 \rightarrow G_3 \rightarrow G_4 \rightarrow G_5 \]

time-stamped interactions: \( e = (i, j, t) \)
temporal network analysis

idea 1:

empirical network sequence

\[ G_{t=1} \rightarrow G_2 \rightarrow G_3 \rightarrow G_4 \rightarrow G_5 \]

time-stamped interactions: \( e = (i, j, t) \)

\[
\begin{align*}
  f(G_1) &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}_{t=1}, \\
  f(G_2) &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}_2, \\
  f(G_3) &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}_3, \\
  f(G_4) &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}_4, \\
  f(G_5) &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}_5.
\end{align*}
\]
idea 1:
given network sequence $G_t = (V, E_t)$

- compute statistics for each “snapshot” in sequence
- makes time series of scalar or vector values

$$\bar{x} = x_1, x_2, x_3, \ldots, x_T$$

- apply standard time series analysis tools
  - autocorrelation (periodicities)
  - change-point detection, non-stationarity
  - covariance of features
  - etc.
temporal network analysis

idea 2:

edges have durations $e = (i, j, t_s, \Delta t)$

- durations of telephone calls
- time spent together
- etc.
temporal network analysis

idea 2:

edges have durations $e = (i, j, t_s, \Delta t)$

- durations of telephone calls
- time spent together
- etc.

discretize time and reduce to idea 1
temporal network analysis

idea 2:

edges have durations $e = (i, j, t_s, \Delta t)$
dynamic proximity network

• MIT Reality Mining Project
• 100 mobile phones, 2 groups
• scan area with bluetooth
• every 5 minutes for 12 months (~100,000 minutes of data)
• record proximate devices (range: 5m)
• convert to dynamic proximity network (assume phone = person

paper reference:

Persistence and periodicity in a dynamic proximity network
Aaron Clauset and Nathan Eagle

http://realitycommons.media.mit.edu
[ x, y, 15:45:23 ]
[ x, z, 15:45:23 ]
[ z, x, 15:46:02 ]
[ u, w, 15:46:12 ]
proximity inference rule

- proximities are time-stamped \((i, j, t)\)
- we want to infer durations \((i, j, t_s, \Delta t)\)
- proximities are noisy [some edges unobserved]
- high-resolution temporal sampling [every 5 mins]

- rule:
  - define tolerance \(\tau\); if gap less than \(\tau\), assume continuous proximity
single day of proximities
Tuesday, 19 Oct 2004

very few connections
Tuesday, 19 Oct 2004

single day of proximities

more connections
Tuesday, 19 Oct 2004

single day of proximities

peak connections, two communities
Tuesday, 19 Oct 2004

single day of proximities

fewer connections
single day of proximities

Tuesday, 19 Oct 2004

very few connections
timing is everything?

• how long do edges last?
• how does structure vary over time?
• how stable is a local neighborhood?
• how does discrete time impact measures?
edge persistence

how long do edges last?

measure durations $\text{Pr}(\Delta t)$
edge persistence

how long do edges last?
measure durations $\Pr(\Delta t)$

- month of October
- broad distribution
- changes at many time scales
- consistent up to $\Delta t < 400$ minutes
network dynamics

how does structure vary over time?

vary aggregation window for snapshots

compute mean degree over time
network dynamics

how does structure vary over time?

vary aggregation window for snapshots
compute mean degree over time

- one week of October
- highly periodic
- aggregation time matters
network dynamics

how does structure vary over time?

- vary aggregation window for snapshots
- compute mean degree over time

- one week of October
- highly periodic
- aggregation time matters

![Graph showing network dynamics over time with different aggregation windows (5 mins, 15 mins, 4 hrs, 8 hrs, 24 hrs).]
network dynamics

how stable are local neighborhoods?
vary aggregation window for snapshots
compute **adjacency correlation** over time

\[
\gamma_j = \frac{\sum_{i \in N(j)} A_{ij}^{(x)} A_{ij}^{(y)}}{\sqrt{\sum_{i \in N(j)} A_{ij}^{(x)}} \sum_{i \in N(j)} A_{ij}^{(y)} }
\]

for two adjacency matrices \(A^{(x)}, A^{(y)}\)
measures similarity among neighbors observed in either network
average overlap = mean value \(\langle \gamma \rangle\)
network dynamics

how stable are local neighborhoods?

vary aggregation window for snapshots
compute adjacency correlation over time

- one week of October
- highly consistent neighborhoods
- daily / weekly periodicity
- aggregation time matters
network dynamics

**how does discrete time impact measures?**

vary aggregation window for snapshots

compute **summary statistics**
network dynamics

how does discrete time impact measures?

vary aggregation window for snapshots
compute summary statistics

• all statistics depend on aggregation duration
• choose a time scale = choose a statistical value

![Graph showing mean degree, mean clustering coeff., and 1 - mean adjacency corr. vs. sampling frequency.](image)
network dynamics

how to choose aggregation time?
recall highly periodic dynamics
compute autocorrelation function on network measures
network dynamics

how to choose aggregation time?
recall highly periodic dynamics
compute autocorrelation function on network measures

![Graph showing correlation over separation time](image)

- mean degree
- clustering coefficient
- adjacency correlation

separation time, $\tau$ (hours)
network dynamics

how to choose aggregation time?

recall highly periodic dynamics
use frequency spectrum to choose sampling rate

- periodicity at 1, 2, 3 samples per day
- Nyquist rate
  \[ \Delta_{nat} \approx 4 \text{ hours} \]

degree \( \langle k \rangle_{nat} = 2.24 \)
triangles \( \langle C' \rangle_{nat} = 0.084 \)
adj. corr. \( \langle \gamma \rangle_{nat} = 0.88 \)
other ideas

• temporal “reachability” and continuous-time methods
• different parts evolving at different rates
• generative models?
• densification dynamics?
• temporal anomalies
• etc.