

Santa Fe Institute
2007 Complex Systems Summer School

Week I: Introduction to Nonlinear Dynamics

Instructor:

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Syllabus:

1. Introduction; Dynamics of Maps chs 1 & 10 of [50]
 - a brief tour of nonlinear dynamics [32] (in [17])
 - an extended example: the logistic map
 - how to plot its behavior
 - initial conditions, transients, and fixed points
 - bifurcations and attractors
 - chaos: sensitive dependence on initial conditions, λ , and all that
 - pitchforks, Feigenbaum, and universality [22] (in [17])
 - the connection between chaos and fractals [23], ch 11 of [50]
 - period-3, chaos, and the u-sequence [31, 34] (latter is in [17])
 - *maybe*: unstable periodic orbits [2, 25, 49]

2. Dynamics of Flows [50], sections 2.0-2.3, 2.8, 5, and 6 (except 6.6 and 6.8)
 - maps vs. flows
 - time: discrete vs. continuous
 - axes: state/phase space [9]
 - an example: the simple harmonic oscillator
 - some math & physics review [8]
 - portraying & visualizing the dynamics [9]
 - trajectories, attractors, basins, and boundaries [9]
 - dissipation and attractors [42]
 - bifurcations

- how sensitive dependence and the Lyapunov exponent manifest in flows
 - anatomy of a chaotic attractor: [23]
 - stretching/folding and the un/stable manifolds
 - fractal structure and the fractal dimension ch 11 of [50]
 - unstable periodic orbits [2, 25, 49]
 - shadowing
 - *maybe*: symbol dynamics [26] (*in* [13]); [28]
3. Tools [1, 9, 37, 40]
- ODE solvers and their dynamics [8, 33, 35, 44]
 - *maybe*: PDE solvers [8, 44]
 - Poincaré sections [27]
 - stability, eigenstuff, un/stable manifolds and a bit of control theory
 - embedology [29, 30, 39, 46, 47, 45, 52] ([39] is in [37] and [45] is in [53];)
 - *maybe*: calculating Lyapunov exponents and fractal dimensions [1, 9, 37, 40]
4. Applications [13, 37, 38]
- prediction [3, 4, 5, 14, 15, 53]
 - filtering [20, 21, 24]
 - control [7, 6, 11, 36, 48] ([36] is in [37])
 - communication [16, 41]
 - classical mechanics [10, 43, 51, 54, 55]
 - music, dance, and image [12, 18, 19]

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References [1, 3, 4, 13, 15, 17, 28, 37, 50, 53] are in the CSSS library.

More Resources:

www.cs.colorado.edu/~lizb

amath.colorado.edu/faculty/jdm/faq.html

www.mpipks-dresden.mpg.de/~tisean/Tisean_3.0.1/index.html