Lecture 1
Discounting Climate Change
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The articles has a large set of references.
Population is taken to be constant. Assume time to be discrete: \( t = 0, 1, 2, \ldots \) Let \( U \) be (social) felicity (i.e., flow of social well-being). Imagine that \( U \) depends solely on a scalar, aggregate consumption, which we denote by \( C \). \( U \) at \( t \) is \( U(C_t) \). The infinite stream \( \{C_t\} \) is the forecast.

We now conduct a thought experiment on that forecast by asking how much additional consumption society should demand on behalf of tomorrow's people in payment for a reduction in today's consumption by one unit.
We say that the "social rate of discount", or the consumption rate of interest, between today's and tomorrow's consumptions is that additional consumption demanded, less unity. If $\rho$ is that rate, society would demand $(1+\rho)$ units of additional consumption tomorrow as a price for giving up one unit of consumption today; meaning that society regards an additional unit of consumption tomorrow to be worth $1/(1+\rho)$ units of additional consumption today.
Question: What is the justification for supposing, as we almost always do, that $\rho$ should be positive?

Response: There are two reasons. (A) An additional unit of consumption tomorrow would be of less value than an additional unit of consumption today if society is impatient to enjoy that additional unit now. (B) Considerations of equality demand that consumption should be evenly spread across the generations, other things being equal. So, if future generations are likely to be richer than us, there is a case for valuing an extra unit of their consumption less than an extra unit of our consumption, other things being equal.
A formal model: We consider only intergenerational tradeoffs. So let $U(C)$ be the flow of social well-being if $C$ is consumption. We take intergenerational welfare at $t = 0$ to be

$$V_0 = U(C(0)) + U(C(1))/(1+\delta) + ... + U(C(t))/(1+\delta)^t + ... = \sum_{t=0}^{\infty}[U(C(t))/(1+\delta)^t].$$

(1)

$U$ is unique up to positive affine transformations. $\delta$ is the time discount rate $(\geq 0)$

NB: Expression (1) amalgamates the Ramsey-Koopmans formulation. $V$ is a Utilitarian Social Welfare Function.
How should the social evaluator identify $U$?

(1) Infer $U$ from the choices people make as they go about their lives ("revealed preference"). (2) Choice behind the veil of ignorance (Harsanyi-Rawls). (3) Altruistic parents concerned with dynastic welfare. (4) $V$ is a numerical representation of a set of ethical requirements on orderings over consumption sequences (Koopmans). (5) Philosophical introspection (Mill-Ramsey)!
Central case:

\[ U(C) = C^{(1-\eta)/(1-\eta)}, \quad \text{for } \eta > 0 \text{ and } \eta \neq 1, \]

and \[ U(C) = \ln C, \quad \text{corresponding to } \eta = 1. \] (2)

NB: The larger is \( \eta \), the greater is the curvature of \( U(C) \). \( U(C) \) is bounded above but unbounded below if \( \eta > 1 \), but \( U(C) \) is bounded below but unbounded above if \( \eta < 1 \). \( \eta \) is the elasticity of marginal felicity. We show below that it is a measure of *inequality* (and *risk*; see below) aversion.
Let $g(C(t))$ be the % rate of change in $C(t)$ along the forecast. Use (2) in (1) to obtain an expression for $\rho$. Show that

$$1 + \rho_t = (1+\delta)(1+g(C(t)))^n. \quad (3)$$

Suppose $\delta$ and $g(C(t))$ are both small. Then (3) becomes

$$\rho(t) \approx \delta + \eta g(C(t)). \quad (4)$$

If the interval between dates was to be made smaller and smaller, (4) would be a better and better approximation. (If time is continuous, (4) is an equality.)
δ, η, and the forecast, g(C(t)), together determine ρ(t). Observe that ρ(t) increases with δ and g(C(t)), respectively, and increases with η if and only if g(C(t)) > 0. So (3) (and (4)) reflects reasons (A) and (B) for the sign of ρ. δ is "impatience" and η is the index of inequality aversion.

Proposition 1. η is the index of the aversion society ought to display toward consumption inequality among people - be they people in the same period or in different periods.
(3) says that when $g(C(t)) > 0$, $\delta$ and $\eta$ play similar roles in the determination of $\rho(t)$: a higher value of either parameter would reflect a greater aversion toward consumption inequality. Which may explain why it hasn't been uncommon to suppose that higher values of $\delta$ reflect a greater ethical concern for consumption equality. But if $g(C(t)) < 0$, $\delta$ and $\eta$ assume diametrically opposite features: in contrast to $\eta$, higher values of $\delta$ raise $\rho(t)$, implying an ethical preference for even greater inequality in consumption across the generations.
Observations on equation (4):
(a) $\rho(t)$ is not a primary ethical object, it has to be derived from an overall conception of intergenerational well-being and the consumption forecast: consumption discount rates cannot be plucked from air.
(b) Just as growing consumption provides a reason why discount rates in use in social cost-benefit analysis should be positive, declining consumption would be a reason why they could be negative. Example: Suppose $\delta = 0$, $\eta = 2$, and $g(C(t)) = -1\%$ per year. Then $\rho(t) = -2\%$ per year. Such reasoning assumes importance when we come to discuss that people in the tropics, who are in any case very poor, will very likely suffer greatly from climate change under business as usual. The reasoning takes on an interesting application when we come to consider uncertainty in future consumption.
(c) If intertemporal external diseconomies are substantial, as is the case with climate change under business as usual, both \( \rho(t) \) and private rates of return on investment could be positive for a period of time, even while the social rate of return on investment is negative.
(d) Only in a fully optimizing economy is it appropriate to discount future consumption costs and benefits at the rate that reflects the direct opportunity cost of capital. In imperfect economies $\rho(t)$ should be used to discount consumption costs and benefits, but the capital deployed in projects ought to be revalued so as to take account of the differences between $\rho(t)$ and the various rates of return on investment.
(e) Unless consumption is forecast to remain constant, social discount rates depend on the numeraire: $\rho(t) = \delta$ if and only if $g(C(t)) = 0$. (f) If $g(C(t))$ varies with time, so does $\rho(t)$. For example, suppose it is forecast that long-run consumption growth is not sustainable but will decline at a constant rate of 1% a year - from the current figure of 2% a year to zero. Suppose $\delta = 0$ and $\eta = 2$. In that case $\rho(t)$ will decline over time at 1% a year, from a current-high 4% a year, to zero.
Examples from the Economics of Climate Change:
Cline (1992): $\delta = 0; \eta = 1.5$
Nordhaus (1994): $\delta = 3\%$ a year; $\eta = 1$
Stern (2006): $\delta = 0.1\%$ a year; $\eta = 1$

NB: In the context of (4), the authors are close in their choice of $\eta$. Notice also how close Cline and Stern are in their specifications of $\delta$. 
The point estimate of consumption growth under business as usual in Stern (2006) is \( g(C(t)) = 1.3\% \) a year. Use this in equation (4) to obtain:

\[
\rho(t) = 2.05\% \text{ a year for Cline}
\]

\[
\rho(t) = 4.30\% \text{ a year for Nordhaus}
\]

\[
\rho(t) = 1.40\% \text{ a year for Stern}
\]

That is why Cline and Stern arrive at similar conclusions and why they differ in their recommendation from Nordhaus.
The Fully Optimum Economy

Suppose \( \eta \geq 1 \). Let \( K(t) \) be the economy's wealth at \( t \) and let the economy's accumulation process be

\[
K(t+1) = [K(t) - C(t)](1+r), \quad K(0) (> 0) \text{ is given.} \tag{5}
\]

Assume \( r > \delta \). In a fully optimum economy, the \{\( C(t) \)\} that society chooses maximizes expression (1), subject to the accumulation equation (5). NB: Under our hypotheses an optimum exists and satisfies the condition:

\[
\rho(t) = r, \quad \text{all } t \geq 0. \tag{6}
\]
It is only in a fully optimum economy that the direct opportunity cost of capital should be used for discounting future benefits and costs.

What does an optimum \{C(t)\} look like? Using (5) and (6), it can be shown that \(C(t)\) grows at the compound rate, \(g\), where

\[
\frac{C(t+1)}{C(t)} - 1 = g = \left[\frac{1+r}{1+\delta}\right]^{1/\eta} - 1. \tag{7}
\]

If \(r\) and \(\delta\) are small, then \(g\) is small, and (7) becomes the approximation

\[
g \approx \frac{r-\delta}{\eta}. \tag{8}
\]

(Equation (8) is exact in continuous time.)
Let the optimum saving rate, \([K(t)-C(t)]/K(t)\), be \(s\). Then

\[
s = (1+r)^{-\eta/\eta}(1+\delta)^{-1/\eta}.
\]

(9)

**Proposition 2.** The optimum saving rate is a decreasing function of \(\eta\) and \(\delta\). If, holding \(\delta\) and \(r\) constant, larger and larger values of \(\eta\) are admitted, \(s\) declines to \((1+r)^{-1}\).

NB: The "Rawlsian" case is \(\eta = \infty\).
Note: Net saving is zero if $s = 1/(1+r)$. Normalise round that figure. Moreover, the maximum possible rate of saving is 1, implying that the range of non-negative saving rates is $[(1+r)^{-1}, 1]$. Since the saving-wealth ratio is $[K(t)-C(t)]/K(t)$, its normalised value is $[(K(t)-C(t))/K(t)-(1+r)^{-1}]/[1-(1+r)^{-1}]$. Now, output at $t+1$ is $rK(t)$. Confirm that the normalised saving-wealth ratio is none other than the more familiar saving-output ratio.
Let $s^*$ be the optimum saving-output ratio. If $r$ and $\delta$ are both small, then (9) becomes
\[s^* \approx (r-\delta)/\eta r.\] (10)

Example: Let $r = 4\%$ a year. (10) says that at $\delta = 0.1\%$ a year, the optimum saving-output ratio is 97%. This is an absurdly high rate of saving out of income, suggesting that $\eta = 1$ (the log case) is misleading.
Uncertain Production Economy

Suppose at each date, \( \ln(1+r) \) in equation (5) is distributed independently, identically, and normally, with mean \( \mu \) and variance \( \sigma^2 \). Let \( \bar{r} \) be the expected value of \( \tilde{r} \). Assume \( \bar{r} > \delta \). Obviously, \( \bar{r} \) is a function of \( \mu \) and \( \sigma \); as is the variance of \( \tilde{r} \). Assume that \( \eta \geq 1 \). Let \( s^{**} \) be the optimum saving-output ratio and \( \bar{r} \) and \( \delta \) are both small. Then

\[
\tilde{s}^{**} \approx (\bar{r}-\delta)/\eta \bar{r} + (\eta-1)\sigma^2/2 \bar{r}.
\]
Proposition 3. $\eta$ is not only an index of inequality aversion, it is also an index of risk aversion. At the saving rate $s^{**}$, future generations can be expected to be richer than the present generation. Because of the growth effect, larger values of $\eta$ recommend earlier generations to save less for the future (the equity motive). However, as future productivity is uncertain, larger values of $\eta$ recommend earlier generations to save more (the precautionary motive). The combined effect depends on the parameters $\eta$, $\delta$, $\bar{r}$, and $\sigma$. 
Large Uncertainties: Equation (11) says that $s^{**} \geq 1$ if

$$\frac{\sigma^2}{2} \geq \ln(1+\delta)/\eta(\eta-1) + \ln(1+r)/\eta.$$  \hfill (12)

As $s^{**} \geq 1$ is nonsensical, we can summarise the finding as

**Proposition 4.** If $\sigma$ satisfies inequality (14), no optimum policy exists.

Discuss ways out of Proposition 4.