Energy, Growth, Sustainability and the Underlying Threat of Urbanization

SIZE MATTERS

GEOFFREY WEST
SOME CHARACTERISTICS OF COMPLEX SYSTEMS

• MANY COMPONENTS
• MANY INDIVIDUAL ACTORS / AGENTS
• MULTI SPATIAL AND TEMPORAL SCALES
• STRONGLY COUPLED / INTERACTING
• NON-LINEAR
• SENSITIVITY TO BOUNDARY CONDITIONS (CHAOS)
• EMERGENT PHENOMENA / MULTIPLE PHASES
• UNINTENDED CONSEQUENCES
• ADAPTIVE / EVOLVING
• HISTORICALLY CONTINGENT / PATH DEPENDENT
• ROBUST / RESILIENT
• NON-EQUILIBRIUM
• UNDERLYING SIMPLICITY
• COMPLICATED vs COMPLEX
• COARSE-GRAINED DESCRIPTION
• KINETIC THEORY
• QUARK MODEL
• LONGEVITY
Scaling of economics with energy use
• LIVING/MAINTENANCE
• GROWTH
• REPRODUCTION
• AGING/DEATH
• EVOLUTION
• SLEEP/REPAIR

• DISEASE/CANCER

• ENERGY & RESOURCES vs. INFORMATION

• THE SEARCH FOR UNDERLYING LAWS AND PRINCIPLES LEADING TO A QUANTITATIVE PREDICTIVE CONCEPTUAL FRAMEWORK
ARE BUSINESSES, CORPORATIONS AND CITIES JUST VERY LARGE ORGANISMS SATISFYING THE LAWS OF BIOLOGY?
Relation between number and diameter of trees in a forest recapitulates the branches of the largest trees

\[ N \propto D^{-2} \propto M^{-3/4} \]
Mammals vary in size by 8 orders of magnitude

Shrew
2g

Elephant
2,000,000g

Blue Whale
200,000,000g
Kleiber’s law: metabolic rate scales as $3/4$ power of body mass.
Whole-organism metabolic rate ($B$) scales as the $3/4$ power of body mass ($M$)

$$B \propto M^{3/4}$$

Hemmingson 1960
METABOLIC RATE INCREASES NON-LINEARLY WITH SIZE

EXAMPLE
NAIVELY EXPECT THAT IF MASS (SIZE) INCREASES BY A FACTOR OF 10,000 ($10^4$) THEN METABOLIC RATE WOULD INCREASE BY A FACTOR OF 10,000 ($10^4$)

BUT
IN FACT,

METABOLIC RATE INCREASES BY A FACTOR OF ONLY 1,000 \((10^3)\)

\[ B \sim M^{3/4} \]
SPECIFIC METABOLIC RATE (PER UNIT MASS)

\[
\frac{B}{M} \propto M^{-1/4}
\]

SO METABOLIC RATE OF AVERAGE CELL

\[
B_{cell} \propto M^{-1/4}
\]
LIFE EXHIBITS A SYSTEMATIC ECONOMY OF SCALE
TO SUSTAIN 1 gm MOUSE REQUIRE 3 TIMES THE POWER FOR 1 gm of DOG AND 9 TIMES THE POWER FOR 1 gm of ELEPHANT!!

SMALL MAY BE BEAUTIFUL BUT LARGE IS MORE EFFICIENT
FIG. 4 - VARIATION IN RADIUS OF AORTA WITH BODY WEIGHT

$ R \sim M^{3/8} $

SAME SCALING FOR TREE TRUNKS
Metabolic rate sets the pace of life.
Small animals live fast and die young.

Heart rate scales as -1/4 power of body mass.
Lifespan

\[ T \sim M^{\frac{1}{4}} \]

If heart-rate (number of beats per sec.)

\[ \sim M^{-\frac{1}{4}} \]

\[ \Rightarrow \text{Total number of heart-beats in a typical life-time is independent of size!} \]

\[ \approx 1.5 \times 10^9 \]

Each animal species regardless of size has approximately the same number of heart-beats in its life-time (roughly 1 billion)
MORE FUNDAMENTALLY, ACROSS AEROBIC METABOLISM: THE NUMBER OF TURNOVERS IN A LIFETIME OF Cyto
ENZYMES (RESPIRATORY COMPLEX) IS AN APPROXIMATE IN Variant (~ $10^{16}$)
RECALL SPECIFIC METABOLIC RATE

\[ \bar{B} = \frac{B}{M} \propto M^{-\frac{1}{4}} \]

\[ \Rightarrow \text{TOTAL ENERGY NEEDED TO SUPPORT UNIT MASS OF AN ANIMAL DURING A LIFETIME IS THE SAME FOR ALL ANIMALS REGARDLESS OF SIZE:} \]

\[ E_{\text{tot}} \approx 1.2 \times 10^6 \text{ Joules/gm} \]
\[ \approx 300 \text{ kcals/gm} \]
\[ \approx 5.2 \text{ miles/deg cen.} \]
\[ \log_{10} W = (1.23 \pm 0.01) \log_{10} G - (1.47 \pm 0.04) \]

\[ r = 0.998 \]
Dependence of Prokaryotic Genome Length on Cellular Mass

- Non-photosynthetic Prokaryotes
- Cyanophyta

log (genome length, bp)

log (Cellular mass, g)

Slope = 0.24 +/- 0.02
Intercept = 9.4 +/- 0.2
LIFE IS THE MOST COMPLEX SYSTEM

SCALING LAWS ARE REMARKABLE BECAUSE

i) THEY EXIST

ii) THEY ARE VERY SIMPLE

iii) THEY ARE UNIVERSAL

\[
\text{Dominance of } \frac{1}{4} \text{ Power }
\]

iv) \( \Rightarrow \) BIGGER IS MORE EFFICIENT

v) FEW QUANTITATIVE "LAWS" IN BIOLOGY
NETWORKS!!!

(FRACTALS!!)
FUNDAMENTAL PRINCIPLES

1. AT ALL SCALES ORGANISMS ARE SUSTAINED BY THE TRANSPORT OF ENERGY AND ESSENTIAL MATERIALS THROUGH HIERARCHICAL BRANCHING NETWORK SYSTEMS IN ORDER TO SUPPLY ALL LOCAL PARTS OF THE ORGANISM

II. THESE NETWORKS ARE SPACE-FILLING

III. THE TERMINAL BRANCHES OF THE NETWORK ARE INVARIANT UNITS

IV. ORGANISMS HAVE EVOLVED BY NATURAL SELECTION SO AS TO

   1) MINIMISE ENERGY DISSIPATED IN THE NETWORKS

   AND/OR

   2) MAXIMISE THE SCALING OF THEIR AREA OF INTERFACE WITH THEIR RESOURCE ENVIRONMENT
Large vessels branch into smaller ones

Beating heart

Pulse wave propagates through elastic vessels
A slice through the cerebellum showing the progressive branching structure of the white matter is distributed throughout the cerebellar volume. The geometric complexity of these structures provides for rapid dissemination of information (or energy) via a large surface area in a compact space. This feature is a hallmark of the compartmentalized structures which maximize the surface area within a finite volume.
Relation between number and size of branches within a tree
Microcapillary tubes follow branching architecture from trunk to leaves
What is this?
Fig. 1. Mitochondrial network in a mammalian fibroblast. A COS-7 cell labeled to visualize mitochondria (green) and microtubules (red) was analyzed by indirect immunofluorescence confocal microscopy. Mitochondria were labeled with antibodies to the β subunit of the F₁-ATPase and a rhodamine-conjugated secondary antibody. Microtubules were labeled with antibody to tubulin and a fluorescein-conjugated secondary antibody. Pseudocolor was added to the digitized image. Scale: 1 cm = 10 μm.

HEAD

ARMS, CHEST

LUNG

LUNG

BRONCHI

AORTA

HEART

LIVER

SPLEEN

KIDNEYS

LEGS, FEET
Since the fluid (blood) transports oxygen, nutrients, etc. from the aorta to the capillaries

Metabolic rate $\propto$ Volume flow rate

$B \propto Q_o$

But the conservation of fluid (blood)

$\Rightarrow Q_o = N_c Q_c$

Total number of capillaries $\quad$ Volume flow rate in average capillary

Capillary is an invariant unit

($Q_c$ is same for all mammals)

$\Rightarrow$ Number of capillaries ($N_o$) must scale in same way as the metabolic rate ($B \propto Q_o$)

So, if $B \sim M^{3/4}$ then

$N_c \sim M^{3/4}$ (not $N_c \sim M$)
TOTAL NUMBER OF CELLS

\[ N_{\text{cell}} \sim M \quad \text{(LINEAR)} \]

TOTAL NUMBER OF CAPILLARIES

\[ N_c \sim M^{3/4} \]

MISMATCH!

\[ \frac{N_{\text{cell}}}{N_c} \sim M^{1/4} \]

(ANOTHER MANIFESTATION THAT EFFICIENCY INCREASES WITH SIZE)

IMPORTANT IMPLICATIONS FOR GROWTH AND DEATH!
PULSATILE TREATMENT

\[ \mathbf{\sigma}_{ij} = \lambda \mathbf{\varepsilon}_{kk} \delta_{ij} + 2\mu \mathbf{\varepsilon}_{ij} - p \delta_{ij} \]  
(NeWTONIAN)

\[ \mathbf{\varepsilon}_{ij} = \frac{1}{2} \left( \partial_i v_j + \partial_j v_i \right) \]

EOM. OF MOTION :  
\[ p \frac{Dv_i}{Dt} = \partial_j \sigma_{ij} \]  
(NA\'VIER-STOKES
EOM. OF CONTINUITY :  
\[ \frac{\partial p}{\partial t} + \partial_j (p v_j) = 0 \]
Walls:

\[ w_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2B \varepsilon_{ij} - \rho \frac{\partial \delta_{ij}}{\partial t} \quad \text{Hooke's Law} \]

\[ \varepsilon_{ij} = \frac{1}{2} (\delta_{ij} \varepsilon_i + \varepsilon_i \delta_{ij}) \]

\[ \rho \frac{Du_i}{Dt} = \partial_j \theta_{ij} \quad ; \quad u_i = \frac{\partial \delta_i}{\partial t} \quad \text{Navier Eq.} \]

\[ \varepsilon_i \delta_i = 0 \]

Neglect non-linearities:

\[ \rho \frac{\partial \varepsilon_i}{\partial t} = B \varepsilon_i \varepsilon_i - \varepsilon_i \rho \]

Solve using Fourier as with fluid, walls and fluid coupled via boundary conditions: continuity of velocity and force:

\[ u_r = \frac{\partial \delta_r}{\partial t} \]

and \[ \int \theta_{ij} \partial_s \delta_i \quad \text{continuous} \]

Can be solved: Big mess!

Simplify using thin wall approximation

\[ \frac{h}{R} \ll 1 \]
“EVERYTHING” follows from the 4 basic principles.

I. Hierarchical “fractal-like” branching networks

II. Space-filling

III. Invariant terminal units

IV. Energy dissipated is minimised

\[ \frac{L_{\text{total}}}{L} = \frac{1}{2^{4/3}} \frac{1}{n^{1/4}} \]

\[ \frac{T_{\text{A}}}{T_k} = \frac{1}{2^{4/3}} \]

\[ \text{IV} \Rightarrow \begin{cases} 
\text{iii) Area-preserving branching} \\
\text{iv) Volume of network (blood)} \\
\sim \text{Volume of whole body} \sim M 
\end{cases} \]

\[ B \propto M^{3/4} \quad \text{(and much more)} \]

More generally: \( B \propto M^{k(M)} \) with \( k(M) \propto 3/4 \) large \( < 3/4 \) for small \( M \leq M \) not a strict power law!
IN d DIMENSIONS

\[ B \propto M^{\frac{d}{d+1}} \]

WE LIVE IN 3 SPATIAL DIMENSIONS SO \( B \propto M^{3/4} \)

\( \Rightarrow \) "3" REPRESENTS DIMENSIONALITY OF SPACE

"4" INCREASE IN DIMENSIONALITY DUE TO FRAC TAL-LIKE SPACE FILLING

LIFE HAS TAKEN ADVANTAGE OF THE POSSIBILITY OF USING SPACE-FILLING FRAC TAL-LIKE SURFACES (WHERE ENERGY AND RESOURCES ARE EXCHANGED)

TO MAXIMISE ENERGY TRANSFER FROM THE ENVIRONMENT

NON-FRACTAL: \( M^{2/3} \)

BIOLOGICAL (FRAC TAL): \( M^{3/4} \)

BY ANALOGY: LIFE EFFECTIVELY OPERATES IN FOUR SPATIAL DIMENSIONS

[ FIVE IF TIME IS INCLUDED ]
<table>
<thead>
<tr>
<th>Variable</th>
<th>Exponent</th>
<th>Predicted</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aorta radius $r_0$</td>
<td>$3/8 = 0.375$</td>
<td></td>
<td>0.36</td>
</tr>
<tr>
<td>Aorta pressure $\Delta p_0$</td>
<td>0</td>
<td>0.00</td>
<td>0.032</td>
</tr>
<tr>
<td>Aorta blood velocity $u_0$</td>
<td>0</td>
<td>0.00</td>
<td>0.07</td>
</tr>
<tr>
<td>Blood volume $V_b$</td>
<td>1</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Circulation time</td>
<td>$1/4 = 0.25$</td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>Circulation distance $l$</td>
<td>$1/4 = 0.25$</td>
<td></td>
<td>ND</td>
</tr>
<tr>
<td>Cardiac stroke volume</td>
<td>1</td>
<td>1.00</td>
<td>1.03</td>
</tr>
<tr>
<td>Cardiac frequency $\omega$</td>
<td>$-1/4 = -0.25$</td>
<td></td>
<td>-0.25</td>
</tr>
<tr>
<td>Cardiac output $E$</td>
<td>$3/4 = 0.75$</td>
<td></td>
<td>0.74</td>
</tr>
<tr>
<td>Number of capillaries $N_c$</td>
<td>$3/4 = 0.75$</td>
<td></td>
<td>ND</td>
</tr>
<tr>
<td>Service volume radius</td>
<td>$1/12 = 0.083$</td>
<td></td>
<td>ND</td>
</tr>
<tr>
<td>Womersley number $\alpha$</td>
<td>$1/4 = 0.25$</td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td>Density of capillaries</td>
<td>$-1/12 = -0.083$</td>
<td></td>
<td>-0.095</td>
</tr>
<tr>
<td>$O_2$ affinity of blood $P_{50}$</td>
<td>$-1/12 = -0.083$</td>
<td></td>
<td>-0.089</td>
</tr>
<tr>
<td>Total resistance $Z$</td>
<td>$-3/4 = -0.75$</td>
<td></td>
<td>-0.76</td>
</tr>
<tr>
<td>Metabolic rate $B$</td>
<td>$3/4 = 0.75$</td>
<td></td>
<td>0.75</td>
</tr>
<tr>
<td>Variable</td>
<td>Exponent Predicted</td>
<td>Exponent Observed</td>
<td></td>
</tr>
<tr>
<td>--------------------------------</td>
<td>--------------------</td>
<td>-------------------</td>
<td></td>
</tr>
<tr>
<td>Tracheal radius</td>
<td>3/8 = 0.375</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>Interpleural pressure</td>
<td>0 = 0.00</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>Air velocity in trachea</td>
<td>0 = 0.00</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Lung volume</td>
<td>1 = 1.00</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>Volume flow to lung</td>
<td>3/4 = 0.75</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>Volume of alveolus $V_A$</td>
<td>1/4 = 0.25</td>
<td>ND</td>
<td></td>
</tr>
<tr>
<td>Tidal volume</td>
<td>1 = 1.00</td>
<td>1.041</td>
<td></td>
</tr>
<tr>
<td>Respiratory frequency</td>
<td>$-1/4 = -0.25$</td>
<td>-0.26</td>
<td></td>
</tr>
<tr>
<td>Power dissipated</td>
<td>3/4 = 0.75</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>Number of alveoli $N_A$</td>
<td>3/4 = 0.75</td>
<td>ND</td>
<td></td>
</tr>
<tr>
<td>Radius of alveolus $r_A$</td>
<td>1/12 = 0.083</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Area of alveolus $A_A$</td>
<td>1/6 = 0.083</td>
<td>ND</td>
<td></td>
</tr>
<tr>
<td>Area of lung $A_L$</td>
<td>11/12 = 0.92</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>$O_2$ diffusing capacity</td>
<td>1 = 1.00</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>Total resistance</td>
<td>$-3/4 = -0.75$</td>
<td>-0.70</td>
<td></td>
</tr>
<tr>
<td>$O_2$ consumption rate</td>
<td>3/4 = 0.75</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>Plant mass</td>
<td>Branch radius</td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td>------------</td>
<td>---------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Exponent predicted</td>
<td>Symbol</td>
<td>Exponent predicted</td>
</tr>
<tr>
<td>Number of leaves</td>
<td>$\frac{3}{4}$ (0.75)</td>
<td>$n_0$</td>
<td>$n_k$</td>
</tr>
<tr>
<td>Number of branches</td>
<td>$\frac{3}{4}$ (0.75)</td>
<td>$N_0$</td>
<td>$N_k$</td>
</tr>
<tr>
<td>Number of tubes</td>
<td>$\frac{3}{4}$ (0.75)</td>
<td>$n_0$</td>
<td>$n_k$</td>
</tr>
<tr>
<td>Branch length</td>
<td>$\frac{1}{4}$ (0.25)</td>
<td>$l_0$</td>
<td>$l_k$</td>
</tr>
<tr>
<td>Branch radius</td>
<td>$\frac{3}{8}$ (0.375)</td>
<td>$r_0$</td>
<td></td>
</tr>
<tr>
<td>Area of conductive tissue</td>
<td>$\frac{3}{4}$ (0.875)</td>
<td>$A_0^{ct}$</td>
<td>$A_k^{ct}$</td>
</tr>
<tr>
<td>Tube radius</td>
<td>$\frac{1}{16}$ (0.0625)</td>
<td>$a_0$</td>
<td>$a_k$</td>
</tr>
<tr>
<td>Conductivity</td>
<td>1 (1.00)</td>
<td>$K_0$</td>
<td>$K_k$</td>
</tr>
<tr>
<td>Leaf-specific conductivity</td>
<td>$\frac{1}{4}$ (0.25)</td>
<td>$L_0$</td>
<td>$L_k$</td>
</tr>
<tr>
<td>Fluid flow rate</td>
<td></td>
<td>$Q_k$</td>
<td></td>
</tr>
<tr>
<td>Metabolic rate</td>
<td>$\frac{3}{4}$ (0.75)</td>
<td>$Q_0$</td>
<td></td>
</tr>
<tr>
<td>Pressure gradient</td>
<td>$-\frac{1}{4}$ (-0.25)</td>
<td>$\Delta P_0/l_0$</td>
<td>$\Delta P_k/l_k$</td>
</tr>
<tr>
<td>Fluid velocity</td>
<td>$-\frac{1}{6}$ (-0.125)</td>
<td>$u_0$</td>
<td>$u_k$</td>
</tr>
<tr>
<td>Branch resistance</td>
<td>$-\frac{3}{4}$ (-0.75)</td>
<td>$Z_0$</td>
<td>$Z_k$</td>
</tr>
<tr>
<td>Tree height</td>
<td>$\frac{1}{4}$ (0.25)</td>
<td>$h$</td>
<td></td>
</tr>
<tr>
<td>Reproductive biomass</td>
<td>$\frac{3}{4}$ (0.75)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total fluid volume</td>
<td>$\frac{26}{24}$ (1.0415)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 1. Similarity of predicted scaling relations for branches within a tree [quantities denoted by uppercase symbols and subscripts $i$ (20)], and for trees within a forest (denoted by lowercase symbols and subscripts $k$)*

<table>
<thead>
<tr>
<th>Scaling quantity</th>
<th>Individual tree</th>
<th>Entire forest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area preserving</td>
<td>$\frac{R_{i+1}}{R_i} = \frac{1}{n^{1/2}}$</td>
<td>$r_{i+1} = \frac{1}{\lambda^{1/2}}$</td>
</tr>
<tr>
<td>Space filling</td>
<td>$\frac{L_{i+1}}{L_i} = \frac{1}{n^{1/3}}$</td>
<td>$l_{i+1} = \frac{1}{\lambda^{1/3}}$</td>
</tr>
<tr>
<td>Biomechanics</td>
<td>$R_i^2 = L_i^3$</td>
<td>$r_i^2 = l_i^3$</td>
</tr>
<tr>
<td>Size distribution*</td>
<td>$\Delta N_i \propto R_i^{-2} \propto M_i^{-3/4}$</td>
<td>$\Delta n_k \propto r_k^{-2} \propto m_k^{-3/4}$</td>
</tr>
<tr>
<td>Energy and material flux*</td>
<td>$B_i \propto R_i^2 \propto N_i \propto M_i^{3/4}$</td>
<td>$B_k \propto r_k^2 \propto n_k \propto m_k^{3/4}$</td>
</tr>
</tbody>
</table>

Stand property                | Predicted stem radius, $r_k$, based scaling function |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Size class neighbor separation</td>
<td>$d_k \propto r_k$</td>
</tr>
<tr>
<td>Canopy scaling</td>
<td>$r_k^{can} \propto r_k^{2/3}$</td>
</tr>
<tr>
<td>Canopy spacing</td>
<td>$d_k^{can} = c_1 r_k \left[ 1 - \left( \frac{r_k}{r_k} \right)^{1/3} \right]$</td>
</tr>
<tr>
<td>Energy Equivalence</td>
<td>$\Delta n_k B_k \propto r_k^0$</td>
</tr>
<tr>
<td>Total forest resource use</td>
<td>$B_{Tot} \propto \Sigma \Delta n_k r_k^2 \leq \dot{R}$</td>
</tr>
<tr>
<td>Mortality rate</td>
<td>$\mu_k \approx Ar_k^{-2/3}$</td>
</tr>
<tr>
<td>Size distribution</td>
<td>$N_k \approx \frac{\dot{R}}{(K + 1) b_0} r_k^{-2}$</td>
</tr>
</tbody>
</table>
HYDRODYNAMIC RESISTANCE OF THE NETWORK

\[ \sim \frac{1}{M^{3/4}} \]

TOTAL RESISTANCE DECREASES WITH SIZE !!

SMALL MAY BE BEAUTIFUL BUT LARGE IS MORE EFFICIENT !!

BLOOD PRESSURE \( \sim M^0 \) \}
AORTA BLOOD VELOCITY \( \sim M^0 \) \]
INVARIANT!

RADIUS OF A WHALE'S AORTA \( \sim 30 \text{ cm} \)
RADIUS OF A SHREW'S AORTA \( \sim \frac{1}{10} \text{ mm} \)

YET THEY HAVE THE SAME BLOOD PRESSURE
This decrease of $B_0$ with size is driven by the hegemony of the network (controls fundamental biochemical rates).

If the network were removed so cells become free (in vitro) $B_0$ should become independent of what mammal they originated in: predict.

\begin{itemize}
  \item $B_0$ \quad 6x10^{-9}$ Watts
  \item M
\end{itemize}
Data from Brown, et al.

Cultured cells

in vitro, $B \propto M^0$

Cells in vivo, $B \propto M^{-1/4}$

$M = \mu$
INTERSPECIFIC SIZE DISTRIBUTION

All species in a Malaysian Rainforest

\[ N = 62 \, D^{-2.07} \]

\[ N = 55 \, D^{-1.95} \]

Manokaran and Kochummen (1987)
Food
"fire of life"
Energy
INCOMING METABOLISED ENERGY

\[ \downarrow \]

MAINTENANCE
(of existencing cells)

+ 

GROWTH
(of new cells)
\[ B = N_{\text{cells}} B_{\text{cell}} + E_{\text{cell}} \frac{dN_{\text{cell}}}{dt} \]

**IN TERMS OF MASS AT AGE t**

\[ \Rightarrow \frac{dm}{dt} = am^{3/4} - bm \]

where

\[ a \equiv \frac{B_0 m_c}{E_c} \]

\[ b \equiv \frac{B_c}{E_c} \]

**SOLUTION:**

\[ \left( \frac{m}{M_0} \right)^{1/4} = 1 - \left[ 1 - \left( \frac{M_0}{m} \right)^{1/4} \right] e^{-at/4M_0^{3/4}} \]

**WHERE** \( M_0 = \text{MASS AT BIRTH} \) \( (m = m_0 \text{ when } t = 0) \)
'in vivo' data (patients)

- breast (ref10)
- prostate (ref11)
- Eq(2)
Tum Viable volume vs. Tum Total Volume

Slope = 0.7846 (0.7384, 0.8308)
Biology   Life

- NON-LINEAR SCALING LAWS
- UNIVERSAL QUARTER POWERS
- SUB-LINEAR EXPONENTS (< 1)
- ECONOMIES OF SCALE (~ $M^{-1/4}$)
- PACE OF LIFE DECREASES WITH SIZE:
  - TIMES ~ $M^{1/4}$
  - RATES ~ $M^{-1/4}$
- SIGMOIDAL GROWTH CURVES
- STABLE ASYMPTOTE
- SUSTAINABLE
- GOVERNED BY NETWORKS (~ FRACTAL)
Can one construct a general theory of social organizations that is quantitative and predictive?

Are there “universal” scaling laws that reveal underlying principles?

Are there average idealized social organizations?

Did they evolve under “natural selection” in a “free market” environment via competition?

What is the nature of their hierarchies and generic network structure?
Social Organizations
(Urban/Corporate Structures)

• Are there universality classes of networks?
• Is there an optimal maximum (or minimum) size?
• What drives mergers?
• Growth, mortality, aging, evolution, …
• Energy (resources) vs. information: which dominates?
Are Cities Approximate Scaled Versions of Each Other?

Do They Obey Power Law Scaling? $\beta$

Do Exponents Manifest “Universality” (analogous to quarter powers in Biology)?
\[ R \sim N^b \]

NETWORK DYNAMICS IMPLIES THAT THE PACE OF LIFE IS DETERMINED BY

\[ \text{RATES} \sim N^{b-1} \]

\[ b < 1 \quad \text{PACE OF LIFE SLOWS DOWN} \]

\[ b > 1 \quad \text{PACE OF LIFE SPEEDS UP} \]
Example of scaling relationships

a) Total wages per MSA in 2004 for the USA vs. metropolitan population.

b) Supercreative employment per MSA in 2003, for the USA vs. metropolitan population.
Innovation measured by Patents

From “Innovation in the city: Increasing returns to scale in urban patenting”
Bettencourt, Lobo and Strumsky

Data courtesy of Lee Fleming, Deborah Strumsky
Or to a disproportionate agglomeration of inventors with urban size?

Source data:
U.S. patent office
Includes all patents between 1980 – 2001

Data courtesy of Lee Fleming, Deborah Strumsky
Table 1. Scaling exponents for urban indicators vs. city size

<table>
<thead>
<tr>
<th>Y</th>
<th>β</th>
<th>95% CI</th>
<th>Adj-$R^2$</th>
<th>Observations</th>
<th>Country–year</th>
</tr>
</thead>
<tbody>
<tr>
<td>New patents</td>
<td>1.27</td>
<td>[1.25,1.29]</td>
<td>0.72</td>
<td>331</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Inventors</td>
<td>1.25</td>
<td>[1.22,1.27]</td>
<td>0.76</td>
<td>331</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Private R&amp;D employment</td>
<td>1.34</td>
<td>[1.29,1.39]</td>
<td>0.92</td>
<td>266</td>
<td>U.S. 2002</td>
</tr>
<tr>
<td>&quot;Supercreative&quot;</td>
<td>1.15</td>
<td>[1.11,1.18]</td>
<td>0.89</td>
<td>287</td>
<td>U.S. 2003</td>
</tr>
<tr>
<td>R&amp;D establishments</td>
<td>1.19</td>
<td>[1.14,1.22]</td>
<td>0.77</td>
<td>287</td>
<td>U.S. 1997</td>
</tr>
<tr>
<td>R&amp;D employment</td>
<td>1.26</td>
<td>[1.18,1.43]</td>
<td>0.93</td>
<td>295</td>
<td>China 2002</td>
</tr>
<tr>
<td>Total wages</td>
<td>1.12</td>
<td>[1.09,1.13]</td>
<td>0.96</td>
<td>361</td>
<td>U.S. 2002</td>
</tr>
<tr>
<td>Total bank deposits</td>
<td>1.08</td>
<td>[1.03,1.11]</td>
<td>0.91</td>
<td>267</td>
<td>U.S. 1996</td>
</tr>
<tr>
<td>GDP</td>
<td>1.15</td>
<td>[1.06,1.23]</td>
<td>0.96</td>
<td>295</td>
<td>China 2002</td>
</tr>
<tr>
<td>GDP</td>
<td>1.26</td>
<td>[1.09,1.46]</td>
<td>0.64</td>
<td>196</td>
<td>EU 1999–2003</td>
</tr>
<tr>
<td>GDP</td>
<td>1.13</td>
<td>[1.03,1.23]</td>
<td>0.94</td>
<td>37</td>
<td>Germany 2003</td>
</tr>
<tr>
<td>Total electrical</td>
<td>1.07</td>
<td>[1.03,1.11]</td>
<td>0.88</td>
<td>392</td>
<td>Germany 2002</td>
</tr>
<tr>
<td>Total electrical</td>
<td>1.23</td>
<td>[1.18,1.29]</td>
<td>0.76</td>
<td>93</td>
<td>U.S. 2002–2003</td>
</tr>
<tr>
<td>Serious crimes</td>
<td>1.16</td>
<td>[1.11,1.18]</td>
<td>0.89</td>
<td>287</td>
<td>U.S. 2003</td>
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<tr>
<td>Total housing</td>
<td>1.00</td>
<td>[0.99,1.01]</td>
<td>0.99</td>
<td>316</td>
<td>U.S. 1990</td>
</tr>
<tr>
<td>Total employment</td>
<td>1.01</td>
<td>[0.99,1.02]</td>
<td>0.98</td>
<td>331</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Household electrical</td>
<td>1.00</td>
<td>[0.94,1.06]</td>
<td>0.88</td>
<td>377</td>
<td>Germany 2002</td>
</tr>
<tr>
<td>Household electrical</td>
<td>1.05</td>
<td>[0.89,1.22]</td>
<td>0.91</td>
<td>295</td>
<td>China 2002</td>
</tr>
<tr>
<td>Household water</td>
<td>1.01</td>
<td>[0.89,1.11]</td>
<td>0.96</td>
<td>295</td>
<td>China 2002</td>
</tr>
<tr>
<td>consumption</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gasoline stations</td>
<td>0.77</td>
<td>[0.74,0.81]</td>
<td>0.93</td>
<td>318</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Gasoline sales</td>
<td>0.79</td>
<td>[0.73,0.80]</td>
<td>0.94</td>
<td>318</td>
<td>U.S. 2001</td>
</tr>
<tr>
<td>Length of electrical</td>
<td>0.87</td>
<td>[0.82,0.92]</td>
<td>0.75</td>
<td>380</td>
<td>Germany 2002</td>
</tr>
<tr>
<td>cables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Road surface</td>
<td>0.83</td>
<td>[0.74,0.92]</td>
<td>0.87</td>
<td>29</td>
<td>Germany 2002</td>
</tr>
</tbody>
</table>

Data sources are shown in SI Text. CI, confidence interval; Adj-$R^2$, adjusted $R^2$; GDP, gross domestic product.

See supplementary online materials for further details and data sources.
Increasing returns in cities

\[ \beta = 1.12 \quad R^2 = 0.97 \]

wealth creation/year
Employment patterns

Births vs. Metropolitan Population

\[ y = 0.0507x^{1.029} \]

\[ R^2 = 0.9765 \]

\[ \beta = 1.029 \]
Deaths vs. Population

\[ y = 0.0133x^{0.9642} \]

\[ R^2 = 0.9597 \]

\[ \beta = 0.964 \]
Car Thefts and Urban Population
Italy 2001

\[ y = 1.8386x - 17.206 \]

\[ R^2 = 0.8651 \]
Material Infrastructure
optimized global design for economies of scale

<table>
<thead>
<tr>
<th>Y</th>
<th>$\beta$</th>
<th>95% CI</th>
<th>adj.- $R^2$</th>
<th>observations</th>
<th>Country/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoline Stations</td>
<td>0.77</td>
<td>[0.74, 0.81]</td>
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<td>318</td>
<td>USA/2001</td>
</tr>
<tr>
<td>Gasoline Sales</td>
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<td>[0.73, 0.80]</td>
<td>0.94</td>
<td>318</td>
<td>USA/2002</td>
</tr>
<tr>
<td>Length of electrical cables</td>
<td>0.88</td>
<td>[0.82, 0.94]</td>
<td>0.82</td>
<td>387</td>
<td>Germany/2001</td>
</tr>
<tr>
<td>Road surface</td>
<td>0.83</td>
<td>[0.74, 0.92]</td>
<td>0.87</td>
<td>29</td>
<td>Germany/2002</td>
</tr>
</tbody>
</table>

Note that although there are economies of scale in cables the network is still delivering energy at a superlinear rate:

Social rates drive energy consumption rates, not the opposite.
**Basic Individual needs**

proportionality to population

<table>
<thead>
<tr>
<th>Y</th>
<th>$\beta$</th>
<th>95% CI</th>
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<th>observations</th>
<th>Country/year</th>
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<tbody>
<tr>
<td>Total establishments</td>
<td>0.98</td>
<td>[0.95, 1.02]</td>
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<td>331</td>
<td>USA/2001</td>
</tr>
<tr>
<td>Total employment</td>
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<td>[0.99, 1.02]</td>
<td>0.98</td>
<td>331</td>
<td>USA/2001</td>
</tr>
<tr>
<td>Total Household electrical consumption</td>
<td>1.00</td>
<td>[0.94, 1.06]</td>
<td>0.70</td>
<td>387</td>
<td>Germany/2001</td>
</tr>
<tr>
<td>Total Household electrical consumption</td>
<td>1.05</td>
<td>[0.89, 1.22]</td>
<td>0.91</td>
<td>295</td>
<td>China/2002</td>
</tr>
<tr>
<td>Total Household water consumption</td>
<td>1.01</td>
<td>[0.89, 1.11]</td>
<td>0.96</td>
<td>295</td>
<td>China/2002</td>
</tr>
</tbody>
</table>

Also true for the scaling of **number of housing units**
The urban economic miracle across time, space, level of development or economic system

<table>
<thead>
<tr>
<th>Y</th>
<th>$\beta$</th>
<th>95% CI</th>
<th>adj.- $R^2$</th>
<th>observations</th>
<th>Country/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Wages/yr</td>
<td>1.12</td>
<td>[1.09, 1.13]</td>
<td>0.96</td>
<td>361</td>
<td>USA/2002</td>
</tr>
<tr>
<td>GDP/yr</td>
<td>1.15</td>
<td>[1.06, 1.23]</td>
<td>0.96</td>
<td>295</td>
<td>China/2002</td>
</tr>
<tr>
<td>GDP/yr</td>
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<td>[1.03, 1.23]</td>
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<td>37</td>
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<td>[1.03, 1.46]</td>
<td>0.64</td>
<td>196</td>
<td>EU/2003</td>
</tr>
</tbody>
</table>
Innovation as the engine

<table>
<thead>
<tr>
<th>Y</th>
<th>$\beta$</th>
<th>95% CI</th>
<th>adj.- $R^2$</th>
<th>observations</th>
<th>Country/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Patents/yr</td>
<td>1.27</td>
<td>[1.25, 1.29]</td>
<td>0.72</td>
<td>331</td>
<td>USA/2001</td>
</tr>
<tr>
<td>Inventors/yr</td>
<td>1.25</td>
<td>[1.22, 1.27]</td>
<td>0.76</td>
<td>331</td>
<td>USA/2001</td>
</tr>
<tr>
<td>Private R&amp;D employment</td>
<td>1.34</td>
<td>[1.29, 1.39]</td>
<td>0.92</td>
<td>266</td>
<td>USA/2002</td>
</tr>
<tr>
<td>“Supercreative”</td>
<td>1.15</td>
<td>[1.11, 1.18]</td>
<td>0.89</td>
<td>287</td>
<td>USA/2003</td>
</tr>
<tr>
<td>R&amp;D employment</td>
<td>1.26</td>
<td>[1.18, 1.43]</td>
<td>0.93</td>
<td>295</td>
<td>China/2002</td>
</tr>
</tbody>
</table>
### Social Side Effects

<table>
<thead>
<tr>
<th>Y</th>
<th>$\beta$</th>
<th>95% CI</th>
<th>adj.- $R^2$</th>
<th>observations</th>
<th>Country/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total elect. consumption</td>
<td>1.09</td>
<td>[1.03, 1.15]</td>
<td>0.72</td>
<td>387</td>
<td>Germany/2001</td>
</tr>
<tr>
<td>New AIDS cases</td>
<td>1.23</td>
<td>[1.18, 1.29]</td>
<td>0.76</td>
<td>93</td>
<td>USA/2002</td>
</tr>
<tr>
<td>Serious Crime</td>
<td>1.16</td>
<td>[1.11, 1.18]</td>
<td>0.89</td>
<td>287</td>
<td>USA/2003</td>
</tr>
<tr>
<td>Walking Speed</td>
<td>0.09</td>
<td>[0.07, 0.11]</td>
<td>0.79</td>
<td>21</td>
<td>Several/1979</td>
</tr>
</tbody>
</table>

Disease transmission is a social contact process:

\[
\frac{dT}{dt} = \beta_c SI \quad \text{Standard Incidence}
\]
TAXONOMY OF EXPONENTS FALL INTO THREE “UNIVERSAL” CLASSES

i) $b \sim 0.8 < 1$  
INFRASTRUCTURE (BIOLOGICAL)
SUB-LINEAR  $\rightarrow$  ECONOMIES OF SCALE  
DRIVEN BY EFFICIENCY

ii) $b = 1$
LINEAR  $\rightarrow$  NON-INNOVATIVE

iii) $b \sim 1.15 > 1$  
SOCIO-ECONOMIC
SUPER-LINEAR  $\rightarrow$  INNOVATIVE DRIVEN BY WEALTH CREATION
Pace of biological life vs. Pace of social life

Heart Rate vs. Body Size  
Walking Speed vs. Population Size
Urban exponents and the dynamics of growth

<table>
<thead>
<tr>
<th>Scaling Exponent</th>
<th>Driving Force</th>
<th>Organization</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta &lt; 1 )</td>
<td>Optimization, Efficiency</td>
<td>Biological</td>
<td>Sigmoidal long-term population limit</td>
</tr>
<tr>
<td>( \beta &gt; 1 )</td>
<td>Creation of Information, Wealth and Resources</td>
<td>Sociological</td>
<td>Boom / Collapse finite-time singularity/unbounded growth accelerating growth rates / discontinuities</td>
</tr>
<tr>
<td>( \beta = 1 )</td>
<td>Individual Maintenance</td>
<td>Individual</td>
<td>Exponential</td>
</tr>
</tbody>
</table>
2003 Patenting Rankings

Cornwallis, OR (1)
San Jose (4)
Boston (107)
Phoenix (155)
Denver (206)
New York (272)
Abilene (359)
Table 1. Hunter-gatherer group sizes and frequencies.

<table>
<thead>
<tr>
<th>Organizational level</th>
<th>Horton order</th>
<th>Sample size</th>
<th>ln mean</th>
<th>s.d.</th>
<th>Geometric mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group size (g)</td>
<td>ω</td>
<td>n</td>
<td>(ln g)</td>
<td>s_g</td>
<td>g</td>
</tr>
<tr>
<td>Individual</td>
<td>1</td>
<td>—</td>
<td>0</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td>Family</td>
<td>2</td>
<td>114</td>
<td>1.50</td>
<td>0.23</td>
<td>4.48</td>
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<tr>
<td>Dispersed group</td>
<td>3</td>
<td>227</td>
<td>2.75</td>
<td>0.46</td>
<td>15.60</td>
</tr>
<tr>
<td>Aggregated group</td>
<td>4</td>
<td>297</td>
<td>3.98</td>
<td>0.71</td>
<td>53.66</td>
</tr>
<tr>
<td>Periodic aggregation</td>
<td>5</td>
<td>213</td>
<td>5.11</td>
<td>0.66</td>
<td>165.32</td>
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<tr>
<td>Population size</td>
<td>6</td>
<td>339</td>
<td>6.73</td>
<td>1.25</td>
<td>839.19</td>
</tr>
<tr>
<td>Frequency, N(g)</td>
<td>ω</td>
<td>n</td>
<td>(ln N(g))</td>
<td>s_{N(g)}</td>
<td>N̄(g)</td>
</tr>
</tbody>
</table>

Graphs:

(a) Equation: $Y = -1.28x + 7.78$, $r^2 = 0.99$

(b) Equation: $Y = -1.19x + 7.38$, $r^2 = 0.99$
Growth Equation

Total incoming rate \approx \textbf{Maintenance} + \textbf{Growth}

(Resources, Products, … “Energy” or “Dollar” equivalent) + (Repair, Replacement, Sustenance, …) + \textbf{Growth}

\[ R \approx NR_0 + E_0 \frac{dN}{dt} \]

Resources, etc. needed to maintain individual

Energy/resources, etc. needed to create new individual
\[
\frac{dN}{dt} = \left( \frac{R_1}{E_0} \right) \left[ N^\beta - \left( \frac{R_0}{R_1} \right) N \right]
\]

**SOLUTION:**

\[
N^{1-\beta} = \frac{R_1}{R_0} + \left[ N^{1-\beta}(0) - \frac{R_1}{R_0} \right] e^{-\frac{R_0}{E_0}(1-\beta)t}
\]

**CHARACTER OF SOLUTION SENSITIVE TO** \( \beta >, =, < 1 \)
\( b < 1 \)  
SIGMOIDAL BOUNDED GROWTH
$b>1$ : Finite time Boom and Collapse
$b > 1$ UNBOUNDED GROWTH UP TO FINITE TIME SINGULARITY
$N(t_c) = \infty!$

UNBOUNDED GROWTH UP TO FINITE TIME SINGULARITY
TO MAINTAIN CONTINUOUS GROWTH, MUST HAVE:

i) $b > 1$ AND

ii) CONTINUOUS MAJOR INNOVATIONS OR PARADIGM SHIFTS AT AN ACCELERATING RATE

iii) TIME BETWEEN INNOVATIONS DECREASES SYSTEMATICALLY WITH GROWTH:

$$t_c \sim N^{1-b} \sim N^{-0.15} \sim 1/t$$
Population growth for New York City
1790 - 2003
Successive cycles of superlinear innovation reset the singularity and postpone instability and subsequent collapse. The relative population growth rate of New York City over time reveals periods of accelerated (super-exponential) growth. Successive shorter periods of super exponential growth appear, separated by brief periods of deceleration. (Inset) $t_c$ for each of these periods vs. population at the onset of the cycle. Observations are well fit with $\beta = 1.09$ (green line).
Social Corporate Urban

- NON-LINEAR SCALING LAWS
- THREE UNIVERSAL CLASSES
- SUPER-LINEAR EXPONENTS (> 1)
- WEALTH CREATION INNOVATION (∼ N^{0.15})
- PACE OF LIFE INCREASES WITH SIZE:
  - TIMES ∼ N^{-0.15}
  - RATES ∼ N^{0.15}
- UNBOUNDED SUPER-EXPONENTIAL GROWTH
- FINITE TIME SINGULARITY
- ACCELERATING CYCLES OF INNOVATION
- SUSTAINABLE?
- GOVERNED BY NETWORKS (FRACTALS?)
Singularity is technological change so rapid and so profound that it represents a rupture in the fabric of human history.
Singularity is near

Registered genetic pairs (75% in last 2 yrs)

Years to reach 10 million customers (US)

Time
Singularity is near

The ever accelerating progress of technology....gives the appearance of approaching some essential singularity in the history of the race beyond which human affairs, as we know them, could not continue.

John von Neumann (1903 - 1957)
PER CAPITA POWER CONSUMPTION AS A FUNCTION OF PER CAPITA GDP 1980-2000

OUTLIERS REMOVED

Woodruff et al. unpublished
Human ecology: reproductive rate in modern nations

- Biological metabolic rate ($B$) is 100 watts.
- Per capita rate of total energy use, including fossil fuels, varies from 300 watts in developing nations to 11,000 watts in developed nations.
- Predicted fecundity rate ($F$)
  
  \[ F \propto M^{-1/4} \text{ and } B \propto M^{3/4}, \]

  so \[ F \propto B^{-1/3} \]
Reproductive rates of human females

Bottom line:
• The per capita energy use of a woman in the U.S., western Europe, or Japan is equivalent to that of a 30,000 kg primate
• Metabolic theory predicts the reproductive rate of this “Queen Kong”: one offspring per 15 years
• Why? It costs $250,000 and the equivalent quantity of energy to rear one child, not including college education
Reproductive rates of mammals, primates, and humans

Moses and Brown 2002

Fecundity \( \propto B^{-1/3} \)

- Wild mammals
- Wild primates
- Modern human nations

Annual fertility rate (births per female per yr)

Per capita power consumption or metabolic rate (W)
Escaping the singularity with $\beta > 1$: cycles of successive growth & innovation

$$t_{\text{crit}} \approx \frac{E_0}{(\beta - 1)R_a} N^{1-\beta}(0) \approx 50 \frac{T}{n^{\beta-1}} \text{ years}.$$  

$t_{\text{crit}}$ shortens with population size $N$. 

[Graphs and charts illustrating the relationship between population size and growth rate over time.]
Combined truck traffic through the Laredo area

![Graph showing the relationship between reproductive rate (births per thousand per year) and per capita power consumption (watts). The graph includes data points and a trend line with an exponent of -0.31.]

Moses and Brown 2002
Energy, Urbanization and Sustainability; Lessons from Biology

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