The Evolution of Cooperation by Image Scoring in a Lattice-Structured Population

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Abstract

In this paper we extend the model of Nowak and Sigmund by considering a two-dimensional lattice structure of the population. We find that all the individuals in a population adopt the same cooperative strategy at steady state. However, this stable strategy varies with the instance of initial distribution of strategy value (following uniform distribution). We explain this result by making comparison between any pair of strategies. In addition, we also discuss the impact of the interaction number per generation on cooperation and find that cooperation can emerge more easily in a lattice-structured population than in a perfectly mixed population, just like the case in direct reciprocity.

Key words: evolution of cooperation, reciprocity, image score, spatially structured

1 Introduction

Cooperation is ubiquitous in ecological and social system. "How can natural selection favor cooperative acts within groups, given that selfish individuals can gain by cheating?" Recently, cooperation between unrelated individuals has been investigated mainly from two sides. One is direct reciprocity. The seminal work was achieved by Trivers, in which he introduced the idea of reciprocal altruism: a donor may help a recipient if the recipient is likely to return the favour [1]. Axelrod and Hamilton extended Trivers' argument using an evolutionary game-theoretic model of repeated prisoner's dilemma [2]. Following

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this line, a lot of works have been achieved [3–5]. The effects of spatial structure in interaction and reproduction on the evolution of cooperation have also been discussed widely. Axelrod studied the competition between TFT strategy and AllD strategy in a population structured by a two-dimensional lattice [6]. Nowak & May studied a Prisoner's Dilemma game on a similar lattice structured population [7]. Watts studied Generalized TFT strategy in a population structured by a small-world network [8]. The common conclusion is that spatial structure promotes cooperation, since cooperators can form tight clusters and thus benefit from more frequent mutual cooperation to resist invasion of defectors [9,10].

The other is indirect reciprocity, which was presented by Alexander to understand the phenomena that one doesn't expect a return from the recipient, but from someone else. Two types of models were proposed to extend Alexander's argument, by Boyd & Richerson and by Nowak and Sigmund, respectively. The model of Boyd & Richerson was developed on the basis of a loop-structured population [11]. Such a structure ensures that a cooperative action can be returned to the original donor. As a result, cooperation only emerges in small populations. In contrast, Nowak and Sigmund proposed the model by introducing the mechanism of image scoring [12,13]. In their model, a potential donor can help a recipient at a cost to increase his own image score, which will give him more chances to become the recipient of an altruistic act later. On the contrary, he can also refuse to offer help, which will decrease his image score. This means that he will be refused more probably when he needs help in the future. The model of Nowak and Sigmund can explain the emergence of cooperation in large-scale populations well, and thus is more successful. In this line, many papers have been published [14–17].

However, so far all the work that investigated the mechanism of image scoring was done in a perfectly mixed population. The effect of spatial structure has not been considered. As we mentioned above, in the case of direct reciprocity the effect of spatial structure can promote cooperation obviously. Thus it is necessary to take the effect of spatial structure into account in the model which adopts the mechanism of image scoring. In this paper, we investigate the evolution of cooperation by image scoring in a population structured by a two-dimensional lattice using computer simulations. We find that all the individuals in a population adopt the same cooperative strategy at steady state. However, this stable strategy varies with the instance of initial distribution of strategy value (following uniform distribution). We explain this result by making comparison between any pair of strategies. In addition, we also discuss the impact of the interaction number per generation on cooperation and find that cooperation can emerge more easily in a lattice-structured population than in a perfectly mixed population, just like the case in direct reciprocity. This paper is organized as follows. In the next section we extend the model of Nowak and Sigmund by taking a two-dimensional lattice structure of the population into account. In Section 3 we perform computer simulations and explain the results obtained through simulations. We finish with some conclusions in Section 4.

2 Two-Dimensional Lattice Model

Consider a population which is structured by a $n \times n$ two-dimensional lattice. Each individual in the population occupies a lattice site, and can only interact with its eight immediate neighbors. Each individual has an image score (s) and a strategy value (k). In each interaction, an individual is chosen randomly as the donor, and one of its immediate neighbors is chosen at random as the recipient. The donor makes a decision whether to help the recipient. If the image score of the recipient is more than or equal to the strategy value of the donor, the donor will help the recipient. This means the donor pays a cost (c) and the recipient obtains a benefit (b). Meanwhile the donor's image score increases one unit. On the contrary, if the image score of the recipient is less than the strategy value of the donor, the donor will refuse to offer help. Both of them have no payoff, and the donor's image score decreases one unit. In our model one generation includes m interactions. At the beginning of each generation, all the individuals have image score 0 (assuming that image scores of parents are not inherited by children). And at the end of each generation, each individual has a payoff from m interactions in this generation. In next generation, a lattice site will be occupied by the offspring of the individual with maximal payoff in nine individuals nearby (eight immediate neighbors and the individual occupying this site). If the individual occupying this site and one of its immediate neighbors have the same maximal payoff, then the offspring of the individual occupying this site will occupy this site. The image scores range from -5 to $+5$, and the strategy values from -5 to $+6$. If strategy value of an individual is 0 or less, then the individual tends to cooperate. So we call the strategy with non-positive value cooperative strategy. On the contrary, if strategy value of an individual is more than 0, then the individual tends to defect. So we call the strategy with positive value defective strategy. In the paper, at the beginning of each simulation, we distribute strategy value evenly among the population and fix the parameters $b = 1.0$, $c = 0.1$ and $n = 35$.

3 Simulations and results

We perform computer simulations as the number of interactions per generation is set to 1500. It is found that after many generations, all the individuals will adopt the same cooperative strategy, i.e., the stable strategy value is 0 or less. This is in accord with the results of the model of Nowak and Sigmund.

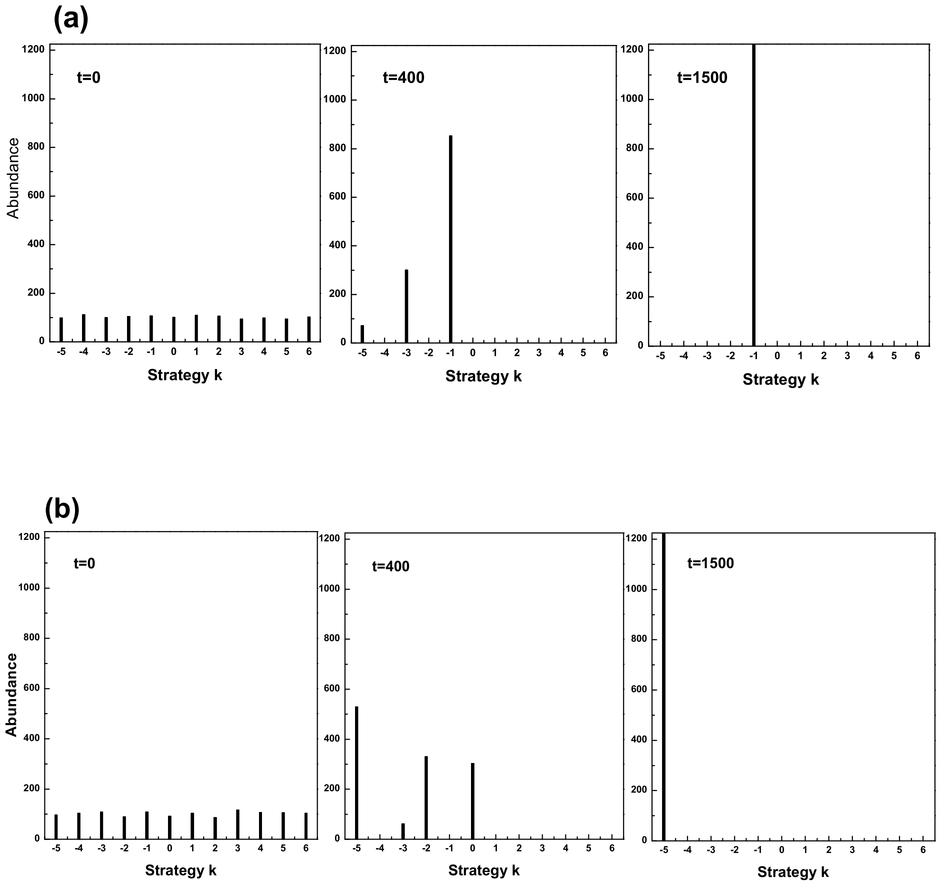

Fig. 1. Evolution of strategies. It can be seen that under different instances of initial distribution of strategy value, the stable strategies are different. (a) The stable strategy value is -1. (b) The stable strategy value is -5.

However, when we change the instance of initial distribution of strategy value to another one, the stable strategy varies. It is different from the results of the model of Nowak and Sigmund. In their model, the stable strategy doesn't vary with the instance of initial distribution of strategy value, as long as the number of interactions per generation is large enough [16]. Thus we change the number of interactions per generation to 8500, which is large enough, but the stable strategy still varies. Fig. 1 shows the results of computer simulations under two different instances of initial distribution of strategy value. In Fig. 1(a), the stable strategy value is -1, while the stable one changes to -5 in Fig. 1(b).

To explain this result, we make comparison between any pair of strategies in terms of their merits. We choose a pair of strategies from twelve strategies in the model. At the beginning, each individual adopts one of these two strategies at equal probability. Let individuals interact continually according to the

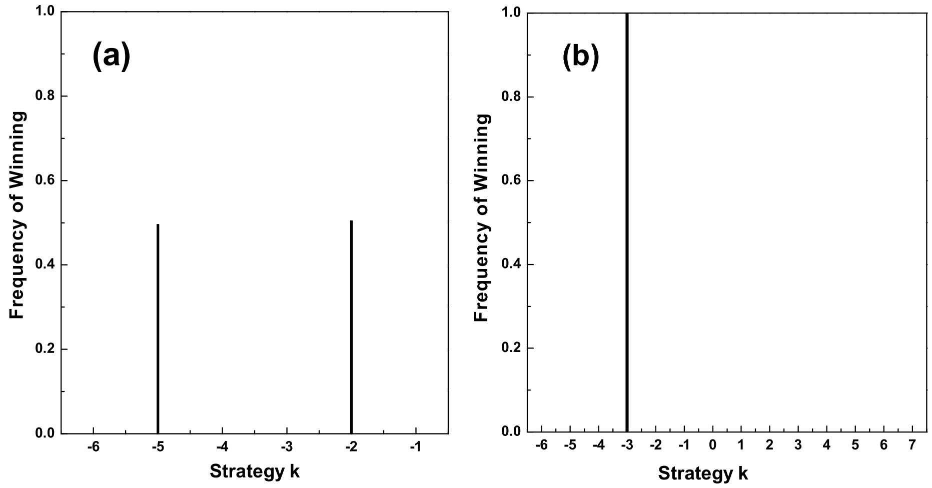

Fig. 2. Comparison between strategies. (a) Comparison between a pair of cooperative strategies (-5 and -2). Two strategies have almost the same frequencies of winning and are as good. (b) Comparison between a cooperative strategy (-3) and a defective strategy (4). Obviously, the cooperative strategy is better than the defective strategy.

rule illuminated in the previous section. Eventually at the steady state, the strategy which is adopted by more individuals wins. Repeat this process many times, and if an strategy wins more times, then it is better than its competitor. We find that there are no difference between any pair of defective strategies. This can be understood as follows. When two defective strategies are compared, image score of any individual is equal to zero and less than strategy value of any individual at the beginning. So the population doesn't evolve at all. Obviously, defective strategies are as good. Similarly, cooperative strategies are also equally good. When two cooperative strategies are compared, the strategy which dominates slightly due to the bias of initial configuration will win. However, after repeating many times, each strategy's frequency of winning is almost the same. So there is also no difference between any cooperative strategies. In contrast, any cooperative strategy is better than any defective strategy. When a cooperative strategy and a defective one are chosen to compare, the defective strategy always becomes extinct. Fig. 2 shows the result of comparison between any pair of strategies. Each result is obtained by repeating competition 200 times. The same frequencies of winning prove that two cooperative strategies are as good, see Fig. 2(a); while the large difference of frequencies of winning demonstrates that cooperative strategy is better than defective strategy clearly, see Fig. 2(b).

Applying these results, we can explain well why the stable strategy values are different under different instances of initial distribution of strategy value. In

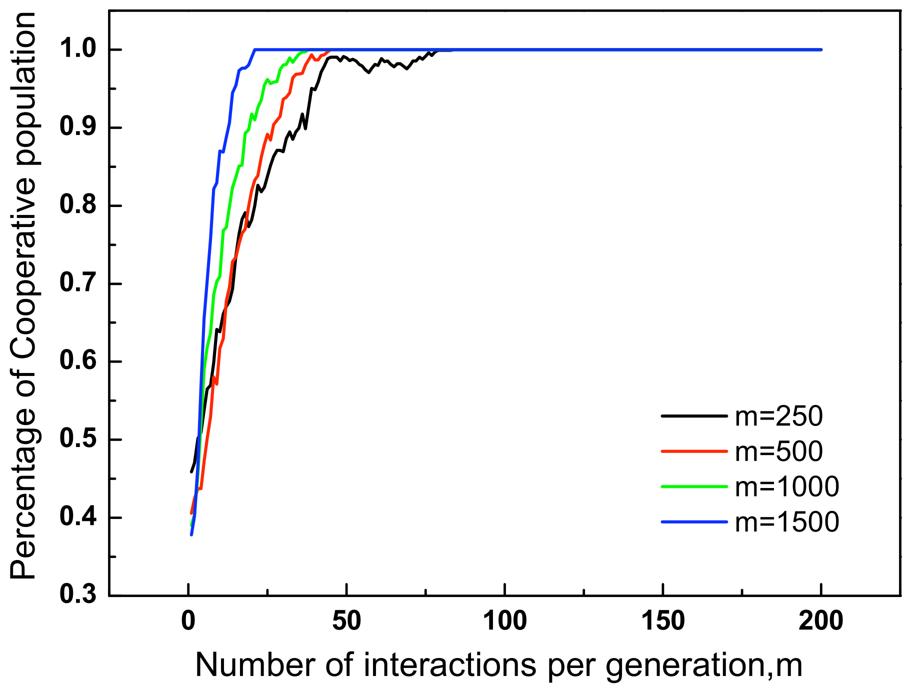

Fig. 3. Evolution of the percentage of cooperative population for different interaction number per generation. It can be seen that eventually all the percentages of cooperative population reach 100%.

the simulations, since cooperative strategies are better than defective ones, defective strategies are eliminated soon afterward. The strategy which dominates slightly in all cooperative strategies under an instance of initial distribution of strategy value will be fixed in a population eventually. Under different instances of initial distribution, cooperative strategy which dominates is different. Thus the stable strategy is different.

We also investigate the impact of the number of interactions per generation on collective cooperative behavior. It is seen that at steady state the percentage of cooperative population always keeps 100% however we change the number of interactions per generation. This is different from the result of the model of Nowak and Sigmund which has been investigated by Cheng and Ouyang in Ref. [16]. They found that the percentage of cooperative population decreases as the number of interactions per generation decreases. However, in our model, even if the number of interactions per generation is reduced to one, cooperation can be still established. Thus it can be concluded that spatial structure promotes cooperation by image scoring. Fig. 3 shows the evolution of the percentage of cooperative population. It can be seen that at steady state the percentages of cooperative population all reach 100% when the numbers of interactions per generation are set to 250, 500, 1000 and 1500, respectively.

4 Conclusions and discussion

In this paper, we extend the model of Nowak and Sigmund by introducing a two-dimensional lattice structure of the population. On the basis of this modified model, we investigate the evolution of cooperation by image scoring. We find that all the individuals in a population adopt the same cooperative strategy at the steady state. However, this stable strategy varies with the instance of initial distribution of strategy value. We explain this result by making comparison between any pair of strategies. Moreover, we also discuss the impact of the interaction number per generation on cooperation and find that cooperation can emerge more easily in a lattice-structured population than in a perfectly mixed population, just like the case in direct reciprocity. Recently some work also suggests some shortcomings of the model of Nowak and Sigmund, such as genetic drift and evolutionary instability [14,15], which may also be referred to in our further studies.

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