The Tax Evasion Social Multiplier: Evidence from Italy*

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Abstract

We investigate the role of externalities in tax evasion arising from congestion of the auditing resources available to local tax authorities. The empirical analysis employs a novel method due to Graham (2008) that mitigates most of the problems that surround the identification of endogenous social effects [Manski (1993)]. Identification exploits the information contained in the variance of concealed income at different levels of aggregation, and an identifying restriction suggested by the different technologies to audit taxable income and tax credits in Italy. We use a unique dataset containing audits of about 80,000 small businesses and professionals in Italy. We find a social multiplier in the range 2.5 - 3, meaning that the equilibrium response to a shock that induces an exogenous variation in mean concealed income is up to three times the initial average response.


Keywords: social interactions, neighborhood effects, social multiplier, tax evasion, tax compliance, crime, excess variance

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Theft – whether from the state, from a fellow citizen or from a looted Jewish store – was so widespread that in the eyes of many people it ceased to be a crime.

–Tony Judt, *Postwar*.1

1 Introduction

Tax evasion is an endemic phenomenon in advanced economic systems. The benchmark model in economics traces back to Allingham and Sandmo (1972) and Yitzhaky (1974): taxpayers maximize expected utility, and rewards and punishments associated with truthful and untruthful income reports provide extrinsic motivations for behavior. A shortcoming of this model is that it predicts too much tax evasion relative to what is observed in advanced economies. An extended version of this approach, exemplified by Gordon (1989), obviates this difficulty by introducing the idea of “tax morale”, i.e. an intrinsic motivation inducing people to abide by their tax obligations.2

Even in this different guise, the benchmark model cannot explain the commonly observed large differences in tax compliance between entities with comparable monetary incentives and arguably similar values or intrinsic motivations.3 As first argued by Glaeser, Sacerdote and Scheinkman (1996), while this is true for crime in general, the large variance of criminal behavior across time and space is not at odds with possible homogeneity in fundamentals if there are local complementarities in the decision to break the law.

An important source of such complementarities in the context of tax evasion is suggested directly by economic models of crime [Sah (1991)]: the perceived probability of apprehension and punishment decreases if more people evade taxes and only a fixed numbers of individuals can be audited. In fact this is an implication

\footnote{1Judt (2005), p. 37.}
\footnote{2An exhaustive summary of theoretical models and evidence is provided by Andreoni et al. (1998) and Slemrod (2007).}
\footnote{3The World Values Survey asks whether it is justifiable to cheat on taxes or not, on a scale ranging from 1 (never justifiable) to 10 (always justifiable). The means in 1999 were 2.3 in the US and Spain, 2.4 in the UK, Portugal, and Italy, 2.6 in Switzerland (Swiss figures refer to 1996). Yet, according to estimates of the size of the shadow economy (an imperfect indicator of tax evasion, although one that is available for most countries) reported by Schneider (2005), the percentage of GDP that went unreported, on average, between 2002 and 2003 was about 9% in the US and Switzerland, 12% in the UK, 22% in Spain and Portugal, and 27% in Italy. In Italy the picture varies greatly from region to region and even within regions, which are fairly homogeneous units. Pisani and Polito (2006) estimate that the ratio between concealed and reported income from productive activities across Italian regions, on average between 1998 and 2002, ranges from 13% in Lombardy and 22% in Emilia Romagna and Veneto, to 66% in Sicily and 94% in Calabria. The variance across provinces within regions is noticeable. Just to mention the extremes, in Lombardy the ratio ranges from 5% in the province of Milan to 34% in the neighboring province of Lodi. In Calabria, it ranges from 53% (Reggio Calabria) to 184% (Vibo Valentia).}
of models where the taxpayers and the tax authority interact strategically [e.g. Sánchez and Sobel (1993)], but one that has not been brought to data yet. The goal of this paper is to identify externalities generated by congestion of the auditing resources in a simple structural model of tax evasion. We employ a standard theoretical framework in which taxpayers and local tax authorities interact strategically in choosing income reports and audit policies, respectively. We focus on the short run and take the amount of local auditing resources as given. As a consequence the perceived individual probability of being audited decreases with the extent of tax evasion in the jurisdiction, a form of neighborhood effects [Durlauf (2004)]. Thus, in our model externalities arise within groups (“neighborhoods”) whose boundaries are exogenously defined by the audit system.

There are of course other examples of neighborhood effects in tax evasion. First, tax cheating is an activity that requires the development of particular skills, and people may learn via social interactions how to conceal their income. Second, cheating on taxes may clash with social norms whose strength decreases with the extent of tax evasion itself [Myles and Naylor (1996)]. Third, business informality can be transmitted across the production chain if value added taxes based on the credit-liability system are in use [de Paula and Scheinkman (2008)]. However, our model is capable of embracing these additional mechanisms, because they all hinge upon a decreasing relationship between the extent of tax evasion and the probability of apprehension.

More precisely, our model generates an “endogenous social effect,” [Manski (1993); Brock and Durlauf (2001a)] which in turn implies a social multiplier [Glaeser, Sacerdote, and Scheinkman (2003)]: small exogenous changes in fundamentals may cause relatively large aggregate responses – because these changes affect individual behavior directly via private incentives and indirectly via the behavior of others – and so “excess variance” relative to fundamentals.

We use a unique collection of about 80,000 tax audits of self-employed individuals (small businesses and professionals) performed in Italy at the beginning of the 1990s, which have now become final. The audits were performed by local branches of the fiscal authority, who can rely on a given amount of resources and have jurisdiction over a precise geographic area. We show that, somewhat surprisingly, this sample is not

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4This same mechanism is an ingredient of recent theoretical work on the optimal design of tax schemes in the presence of equilibria characterized by high tax evasion [Bassetto and Phelan (2008)].

5Interestingly, this last possibility is consistent with the low levels of unreported GDP in the US and Switzerland (see footnote 3), where value-added taxes are not in use and in use at a very low rate, respectively.
Identification exploits a novel method due to Graham (2008), based on a comparison of variances of individual behavior at different levels of aggregation within distinct subpopulations. The method develops the framework of Glaeser, Sacerdote and Scheinkman (1996), which first related excess variance and endogenous social effects. The inferential problem is how to detect excess variance in practice in the presence of individual- and group-level unobservables. For instance, in our application we cannot control for the efficiency or corruptibility of local tax officials, nor for individual tax morale. Similarly, in the context of their application to general crime, Glaeser, Sacerdote and Scheinkman (1996) “admit that there is considerable uncertainty as to how much of the cross-city variance is actually explained by urban characteristics” (p. 509), and note: “In this paper we face an identification problem that can be solved either with an assumption about the functional form of unobserved heterogeneity or by assuming a ‘reasonable’ level of that heterogeneity. Unfortunately, the reader may not find either assumption plausible, in which case our empirical work must be seen as identifying the magnitude of some combination of social interactions and unobserved heterogeneity.” (p. 517). The method developed by Graham (2008) is able to considerably mitigate this fundamental identification problem in linear models of social interactions. In this class of models, as we illustrate below, comparing the conditional variance of individual behavior within groups with the analogous conditional variance between groups (i.e. at different levels of aggregation) allows one to distill the portion of cross-group variation that is due to neighborhood effects only.\(^6\) This requires at least one of the conditioning variables not to affect certain components of the covariance matrix of individual behavior – a restriction admitting a convenient economic interpretation. In our application, such a restriction is provided by the particular nature of tax credits in Italy. At the time our data were collected these could be regarded as sufficient statistic for the number of family dependents. As such, the tax authority could costlessly audit the tax credits claimed by a taxpayer by cross-checking information in the register. In contrast, auditing taxable income is a much more involving process. Such an asymmetry is reflected in our data: few taxpayers correctly reported taxable income, while tax credits are correctly reported by virtually all taxpayers. We argue that this makes the distribution of tax credits in any jurisdiction uninformative with respect to tax evasion. Such uninfor-

\[^6\]This property is not confined to variances: Manacorda (2006) uses the ratio between effects at different levels of aggregation (within- and between-families in this case) to identify interdependencies within families in children’s decision to enter the labor market.
mativeness provides an identifying restriction. Specifically, we show that we can use the dispersion of the distribution of tax credits within tax jurisdictions as an instrument for the within-jurisdiction variance of concealed income. We emphasize that this is an assumption about the variance and not the mean: it is plausible that variation in family composition (as reflected by the dispersion of tax credits) across jurisdictions is correlated with unobservables that could be associated with tax compliance. However, such an association needs not be present – and in fact is not as we show later in the paper – at the level of variances.

We find significant positive externalities that imply a social multiplier in the range 2.5 - 3: the equilibrium aggregate response to an exogenous shock that affects average concealed income is up to three times the initial response.

To our knowledge, the only empirical work that attempts the measurement of social effects in tax evasion is Fortin, Lacroix and Villeval (2007). This experimental study is complementary to ours both in the kind of data the authors use as well as in the estimation method. The main advantage of their approach is that it allows genuinely random audit probabilities. On the other hand, in addition to problems of external validity, lab experiments on tax evasion can be regarded as akin to surveys, as experimental subjects are effectively asked to report to the experimenter how much they evade. The problem, as shown in previous research, is that in surveys about tax compliance there is virtually no correlation between reported and actual tax evasion [Ellfers et al. (1987)]. This is consistent with the zero or even negative effect Fortin, Lacroix and Villeval find.

The paper develops as follows. Section 2 describes our dataset. Section 3 presents the model. The econometric framework is presented in Section 4 along with the identification strategy. In Section 5 we report and discuss our results. Section 6 concludes. All derivations are relegated in an Appendix.

2 Data

Before illustrating the model and the identification strategy, we describe the data we use. This will facilitate the illustration of our modelling strategy and our identifying assumption.

We gained access to the universe of ordinary tax audits of self-employed individuals in Italy, essentially small individual businesses (including farmers) and professionals. The 78,000 odd audits in our dataset
were performed by the Italian *Guardia di Finanza* (Tax Police, a special force dependent on the Ministry of the Economy and Finance that is in charge of tax audits in Italy) at the beginning of the 1990s, and are now final after all possible appeals. Taxpayers in our data belong to identifiable local tax jurisdictions within administrative regions. Jurisdictions are defined by the sphere of competence of local branches of the Ministry of the Economy and Finance, which are responsible for tax management and audits. There are about 400 tax jurisdictions in Italy: each of them comprises one or more municipalities, or a portion of a large city.

In addition to such geographical information and the sector of economic activity, we observe reported income for the purposes of personal income tax, as well as the amount found by the audit. We take the latter to be an accurate estimate of actual income, and we define concealed income as the difference between actual and reported income.\(^7\)

Tables 1-3 report summary statistics. In our data concealed income is about 46% of total taxable income, with considerable variability across regions and sectors of economic activity: non-compliance is more intense in Southern Italy and in agriculture, handicraft, and trade. Consistently with our introductory remarks, the distribution of concealed income across individuals and tax jurisdictions exhibits large dispersion even after conditioning on regions, which are reasonably homogeneous units.\(^8\) This information is summarized in Figures 1-3. Figures 1 and 2 show the conditional distribution of concealed income, relative to total taxable income, at the level of individuals and jurisdictions, respectively. Figure 3 overlaps such conditional distributions and compares them with the unconditional (national) one.

We also observe tax deductions (allowances) and tax credits, both the amount reported and the amount found by the audit. Under the Italian tax code, tax allowances typically include deductions for compulsory contributions to pension funds and charitable deductions. On the other hand, tax credits are granted for dependents, and in most cases can be regarded as a sufficient statistic for family composition.\(^9\) Moreover,
and quite crucially for our identification strategy, there are *de facto* distinct audit processes for taxable income (including allowances) and tax credits. Since the main component of the latter consists of a credit for dependents, the tax authority can easily verify whether a taxpayer has claimed the correct tax credit or not. Such checks are performed routinely and essentially at no cost by simply cross-checking information reported by the taxpayer with family information in the register. This is a fundamental difference with respect to audits on taxable income, which are instead performed only on a small fraction of cases, as reported in the last column of Table 1.\(^{10}\) Consistently with this fact, we observe that while in our dataset only 1 out of 4 taxpayers correctly report taxable income and 1 out of 3 correctly report tax allowances, as many as 90% correctly report tax credits.

A potential problem with our data is that tax audits are not random: tax authorities audit taxpayers on the basis of precise criteria, and one expects tax cheaters to be over-sampled. These criteria were not made public in Italy at the time our data refer to, so one may wonder whether we can draw any credible inference.\(^{11}\)

We claim that our sample is not too far from a random sample of the relevant population, and we offer two pieces of evidence. First, Schneider (2005) estimates that at the end of the 1990s the size of the shadow economy in Italy was about 23% of GDP. We want to compare this figure with the analogue in our data, i.e. the ratio between concealed and gross income. However, the two figures are not fully comparable because we work with a sample of self-employed individuals. Ideally, we would like to know what is the percentage of the shadow economy in the self-employment sector. This is very hard to assess, but we can adjust Schneider’s estimate to get closer to the number in question. We must first exclude the fraction of GDP that originates in the public sector, because by definition this portion cannot originate in the informal sector. In the early 1990s, this was about 28% in Italy (20% from consumption expenditure by general government, and 8% from collective consumption). Next, Schneider’s definition of the shadow economy explicitly excludes illegal activities. However, according to recent estimates, large criminal organizations in Italy\(^{12}\) may account for an astonishing 15% of GDP. At least part of this ends up in official statistics, because in Italy they are corrected for the black economy. A conservative estimate of the residual part is 2%. By correspondingly reducing the measure of GDP in Schneider’s ratio, we make the numerator and denominator

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\(^{10}\)We are grateful to Giorgio Zanella for directing our attention to such an important institutional difference.

\(^{11}\)If selection criteria were known, we could use sampling weights to base our estimates on consistent estimators of population moments. See Pfefferman (1993) and references therein.

\(^{12}\)Mafia in Sicily, Camorra in Campania, ‘Ndrangheta in Calabria, and Sacra Corona Unita in Apulia.
consistent. Therefore, even without further adjustments\textsuperscript{13}, for comparison with our sample Schneider’s figure is $0.23/(1-.28-.2) \approx 33\%$. In our data, the corresponding measure is $37\%$.\textsuperscript{14} We believe that the difference between these two numbers should be much larger if tax cheaters were considerably over-represented in our sample.

Second, and consistently with this belief, in our data the distribution of economic activities and the distribution of tax audits across these same sectors are statistically indistinguishable, in all regions. We performed Kolmogorov-Smirnov tests by region: in no case we can reject the hypothesis that the two distributions are equal. The smallest $p$-value is 0.83, with the majority above 0.99.

These two pieces of evidence suggest that our sample, somewhat surprisingly, is not far from a random sample of taxpayers. Our interpretation is that this reflects the inherent difficulties tax authorities face in detecting tax cheaters. That is, although auditing activities are designed to select individuals who are \textit{ex-ante} more likely to evade taxes – a key implication of theoretical models of optimal enforcement – the outcome is \textit{ex-post} not too far from random sampling.

3 The Model

Our theoretical framework broadly follows Sánchez and Sobel (1993). Consider a population of $N$ taxpayers, indexed by $i = 1, ..., N$. Consistently with our data, we take these to be individual producers distributed across $G$ groups, indexed by $g = 1, ..., G$. For the moment, we assume this distribution is exogenous. Each group has fixed size, denoted $N_g$, with boundaries exogenously defined by the jurisdiction of local tax authorities, who are in charge of tax enforcement. The auditing resources available to tax authority $g$ amount to $A_g$, an amount we treat as exogenous at this stage A taxpayer $i$ whose taxable income is $y_i$ (i.e. gross income minus tax allowances) reports an amount $y_i^R$ to the local tax authority, facing a probability-of-audit schedule $p_i^g(y_i^R)$, and pays a tax at an exogenous flat rate $t$ on reported income. Taxable income is private information and the tax rate is exogenously determined. The tax authority can audit income at a cost of $\alpha > 0$ per audit. When an individual is audited, the tax authority observes taxable income. If $y_i^R \geq y_i$, i.e. at least

\textsuperscript{13}The fraction of employment and corporate income that originates in the informal sector is likely to be considerably smaller than the corresponding fraction of self-employment income [Slemrod (2007)].

\textsuperscript{14}This is smaller than the 46\% odd reported in Table 1 because we are considering gross income: the residual part is therefore accounted for by tax allowances.
the due tax was paid, nothing happens. If \( y^R_i < y_i \), i.e. income was partly concealed, the residual due tax must be paid and a fine is imposed on the unpaid tax, at an exogenous rate \( f > 0 \). Furthermore, the taxpayer deducts a tax credit from the due tax. We denote by \( c_i \) the total tax credit taxpayer \( i \) is entitled to, and by \( c^R_i \) the amount he claims, facing a probability-of-audit schedule \( q_{ig} \left( c^R_i \right) \) and a fine rate \( \phi > 0 \). Consistently with our data, we treat the two audit policies as separate, and we assume that only income audits are costly.\(^{15}\)

The taxpayer’s problem is to choose how much taxable income and tax credits to report, given the audit policies, so as to minimize the expected payment. The tax authority’s problem is to choose the audit policies, given its budget and the optimal behavior of the taxpayer, so as to maximize expected revenue within its jurisdiction. The expectation is taken with respect to the marginal distributions of income and tax credits, denoted \( F \) and \( H \), conditional on individual and jurisdiction-specific information, denoted \( \chi_i \) and \( \Upsilon_g \) respectively. We can write the taxpayer’s problem as follows:\(^{16}\)

\[
\min_{y^R_i,c^R_i} : \quad (1 - p_{ig}) ty^R_i + tp_{ig} (y_i - f (y_i - y^R_i)) \\
- (1 - q_{ig}) c^R_i - q_{ig} (c_i - \phi (c^R_i - c_i))
\]

subject to : \( p_{ig} = p^*_{ig} (y^R_i) \); \( q_{ig} = q^*_{ig} (c^R_i) \),

where \( p^*_{ig} (y^R_i) \) and \( q^*_{ig} (c^R_i) \) are the optimal audit probabilities chosen by the tax authority, i.e. solve:

\[
\max_{\{p_{ig}\}_{i=1}^{N_g},\{q_{ig}\}_{i=1}^{N_g}} : \quad \sum_{i=1}^{N_g} \left[ (1 - p_{ig}) ty^R_i + tp_{ig} \int_0^\infty \left( y + f \left( y - y^R_i \right) \right) dF \left( y | \chi_i, \Upsilon_g \right) \\
- (1 - q_{ig}) c^R_i - q_{ig} \int_0^\infty \left( c - \phi \left( c^R_i - c \right) \right) dH \left( c | \chi_i, \Upsilon_g \right) \right] \\
\text{subject to : } \quad \sum_{i=1}^{N_g} a p_{ig} \leq A_g.
\]

In other words, the tax authority cannot exceed its budget in terms of the expected number of audits.\(^{15}\)

\(^{15}\)In reality, auditing income implies auditing tax credits. Given our assumptions, nothing changes if we write the probability of an audit to tax credits as \( p_{ig} + q_{ig} \).

\(^{16}\)We implicitly condition on truthful or under-reporting, because over-reporting is strictly dominated, for instance, by truthful reporting.
Since the cost of auditing tax credits is zero, it is optimal to set $q_{ig} = 1$ and we can establish the following:

**Proposition 1** Taxpayers truthfully report tax credits, i.e. $c_i^R = c_i$.

Both the proposition and its premise replicate what we observe in our data, as illustrated in the previous Section. Proposition 1 has an important corollary. Although we don’t model this explicitly, misreporting tax credits may signal the tax authority that a taxpayer is evading taxes, which would optimally lead to a larger probability of an audit. But if tax credits are correctly reported, they become uninformative, after conditioning on individual-level information, and do not affect audit probabilities:

**Corollary.** Conditional on individual characteristics, tax credits do not affect audit probabilities.

At an interior optimum, the first-order necessary conditions for a minimum in the taxpayer’s problem balance the expected marginal cost and benefit of under-reporting income and over-reporting tax credits, respectively, net of the marginal effect on the probability of an audit:

$$f p_{ig}^*(y_i^R) = (1 - p_{ig}^*(y_i^R)) + \frac{\partial p_{ig}^*(y_i^R)}{\partial y_i^R} (1 + f) (y_i - y_i^R).$$  \hspace{1cm} (1)

As for the optimal $p_{ig}$, the tax authority can always elicit a truthful report by setting $p_{ig} = (1 + f)^{-1}$. However, in any interesting situation the budget constraint is binding. In this case, Sánchez and Sobel (1993) show that under mild regularity conditions the tax authority optimally divides taxpayers into at most three income classes. Our model differs from theirs because we allow for rich individual and group-level heterogeneity – this is needed for the empirical analysis. However, the nature of the solution is the same because of the additive form of the tax authority’s objective. In this case, income classes are conditional on individual and group information. Let us illustrate in the case of three income classes.

If taxpayer $i$ in jurisdiction $g$ reports income below a threshold $y_{ig}$, he is audited with probability $(1 + f)^{-1}$; if he reports above a threshold $y_{ig}$ he is not audited; if he reports an amount in between he is audited with some constant probability $\pi_{ig}$, as shown in Figure 1.

The thresholds are of course endogenous and depend on the model parameters. Notice that as some marginal taxpayers, i.e. taxpayers on the right of the two thresholds, reduce reported income the probability of being audited decreases.

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17The second order condition for a minimum is satisfied as long as $\partial p_{ig}/\partial y_i^R \leq 0$, and $\partial q_{ig}/\partial c_i^R \leq 0$.

18For any taxpayer, this is the value of $p_{ig}$ that balances expected income under truthful reporting and expected income under tax evasion.
that they are audited increases. Since the resource constraint is binding, the probability that some other taxpayers in the jurisdiction are audited must decrease. This is best seen if we rewrite the budget constraint of the tax authority at the optimum as follows:

$$\sum_{i=1}^{N_g} \frac{1}{1 + f} \mathbb{I} \left[ y_i^R < y_{ig} \right] + \pi_{ig} \mathbb{I} \left[ y_{ig} < y_i^R < y_{ig} \right] \leq \frac{A_g}{\alpha},$$

(2)

where \( \mathbb{I} [\cdot] \) is the indicator function, equal to one if the proposition inside the brackets is true and zero otherwise. Therefore, for the single taxpayer, \( p_{ig} \) must be increasing in the amount reported by other taxpayers, a vector we denoted by \( y_{R_i} \). This is the fundamental externality we want to stress.

Figure 2 illustrates: suppose that everybody except taxpayer \( i \) in jurisdiction \( g \) conceals more, and auditing resources are given. Then the tax authority becomes more lenient towards taxpayer \( i \).

More generally, the presence of such externality allows the optimal audit probability for any taxpayer \( i \) to be written implicitly as follows:

$$p_{ig}^* = p \left( y_i^R, y_{R_i}, \chi_i, \chi_{-i}, t, f, \Upsilon_g, A_g \right),$$

(3)

where variables indexed by \(-i\) contain information about all individuals in jurisdiction \( g \) except \( i \). This expression clarifies the sense in which a model à la Sánchez and Sobel (1993) generates neighborhood effects in tax evasion.

We do not assume that taxpayers know this probability function: tax authorities do not disclose their exact auditing criteria. As in the model of Sah (1991), we simply assume they behave optimally on the basis of a perceived, or inferred, probability of audit. We allow taxpayers to behave like econometricians and use a best linear predictor:

$$\hat{p}_{ig}^* = b_0 + b_1 y_i^R + b_2 y_g^R + b_3 \chi_i + b_4 Z_g,$$

(4)

where \( y_g^R \) is average reported income in jurisdiction \( g \), \( Z_g \equiv (\chi_g, \Upsilon_g, A_g) \) collects group-level variables, \( \chi_g \) is the group-level average of \( \chi_i \), and the coefficients are generated by a hypothetical linear projection. The use of group-level means is a convenient simplification and reflects the idea that by observing what is happening in the jurisdiction, or in communicating with other taxpayers, an individual can inexpensively
form an expectation about local conditions.\footnote{Casual conversations with self-employed individuals suggest that they have a fairly accurate perception of how much tax evasion there is in their area.}

The model imposes restrictions on the sign of $b_1$ and $b_2$: the first is negative, and the second is positive. Figure 3 shows this linear prediction function and its shift following an increase in average concealed income.

We next illustrate two implications of the model when a central authority allocates resources to local jurisdictions and taxpayers are allowed to sort across them. These implications will be later exploited to impose an identifying restriction. First, the optimal audit policy defines a value function for the local tax authority, the maximum expected revenue it raises:

$$
R(A_g) = \sum_{i=1}^{N_g} \left( (1 - p_{ig}^*(A_g)) t y_i^R \right) + t p_{ig}^*(A_g) \int_0^\infty \left( y + f(y - y_i^R) \right) dF(y|\chi_i, \Upsilon_g) - E(c|\chi_i, \Upsilon_g),
$$

(5)

where the last terms in brackets follows from setting $q_{ig} = 1$ in the tax authority’s objective. A revenue-maximizing central authority will allocate resources solving the following problem:

$$
\max_{\{A_g\}} \sum_{g=1}^G R(A_g), \quad \text{s.t.} \quad \sum_{g=1}^G A_g \leq A,
$$

(6)

where $A$ denotes the aggregate amount of resources allocated to tax enforcement. Then, by inspecting (5) and (6), we can establish the following.

**Proposition 2** The optimal allocation of auditing resources is independent of the distribution of tax credits in any given jurisdiction.

This is a direct implication of Proposition 1 and its Corollary: the objective function of the central authority, equation (5), depends on audit probabilities, which are independent of tax credits, and at the same time it is separable, in all jurisdictions, in the average tax credit and the local auditing budget.

Second, we allow individuals to sort across jurisdictions for the purpose of minimizing their expected payment. Since the tax rate is constant, this is equivalent to minimizing the probability of being audited. That is, ignoring mobility costs:
Then, Proposition 1 and its Corollary imply the following:

**Proposition 3** Sorting of taxpayers across jurisdictions is independent of the distribution of tax credits in any given jurisdiction.

Propositions 2 and 3, in turn, imply that the remaining contextual variables, i.e. $\chi_g$ and $\Upsilon_g$, are also independent of the distribution of tax credits. This establishes the following:

**Proposition 4** Matching between contextual variables $\chi_g$, $\Upsilon_g$, and $A_g$ and taxpayer characteristics is independent of the distribution of tax credits in any given jurisdiction.

As we show in the Appendix, substituting the predicted probability of an audit — equation (4) — into the taxpayer’s first-order condition (1), the latter boils down to the linear-in-means behavioral equation of Manski (1993):

$$e_i = \beta X_i + \delta Y_g + Je_g,$$

where $\beta$, $\delta$, and $J$ are coefficients, $X_i \equiv (1, y_i, \chi_i)$, $Y_g \equiv (y_g, \chi_g, \Upsilon_g, A_g)$, and $e_g$ and $y_g$ are group-averages of the corresponding individual-level variables. Equation (8) is a reaction-function, and $J$ is the main parameter of interest, because it measures externalities across taxpayers. In Manski’s (1993) terminology it captures endogenous social effects, since $e_g$ is endogenously determined. This is the parameter we want to estimate. As per derivation, all of the parameters in (8) admit a structural interpretation. In particular, $J \equiv -b_2/2b_1 > 0$, reflecting the distinct and opposite effects of individual ($b_1$) and average ($b_2$) reported income on the perceived probability of an audit. This probability decreases with average concealed income, and so it is optimal to conceal less income if others report more. The individual effect, in turn, dampens the social incentive to report more, because the more an individual reports the lower the perceived probability of an audit. The last step needed to solve the model is to explicitly compute average concealed income in jurisdiction $g$. Averaging $e_i$ within a jurisdiction and solving for $e_g$ yields:
where $X_g$ is the jurisdiction-level mean of $X_i$, and $\gamma \equiv (1 - J)^{-1}$. The model has a unique Nash equilibrium characterized by the following individual level of concealed income, a reduced form obtained by replacing (9) into (8):

$$e_i = \beta X_i + (\gamma - 1) \beta X_g + \gamma \delta Y_g.$$ (10)

Parameter $\gamma$ is the social multiplier, i.e. the ratio between the average cumulative response and the initial individual response following an exogenous shock. Notice that stability requires $J \leq 1$, otherwise concealed income would explode following a tiny shock. Therefore, $\gamma \geq 1$. If $\gamma = 1$, i.e. $J = 0$, there are no externalities, and the average characteristics of others ($X_g$) are irrelevant in equilibrium.

### 4 Identification

The inferential problem is to estimate endogenous social effects, i.e. $J$ in equation (8), and the implied social multiplier, $\gamma$. The reason why identification is a concern is well known since the seminal contribution of Manski (1993), and can be illustrated as follows. Equations (8) and (9) form a system of simultaneous equations. Identification of the first equation, and so of the endogenous social effect requires an exclusion restriction in the form of a variable that affects average but not individual concealed income. This is the essence of the ‘reflection problem’ [Manski (1993)], i.e. the problem of separating the effect of mutual influences ($\gamma$) from the effect of common influences (contextual and correlated effects $\delta$ and $\beta$) in the reduced form, equation (10). Since the same contextual controls, $Y_g$, necessarily appear in both equations, such a restriction must consist of an individual effect whose average is not a contextual effect, i.e. a variable in $X_g$ that is not in $Y_g$ [Brock and Durlauf (2001b)]. Even if such a restriction were available, a number of unobservable individual- and group-level effects would work as confounding factors.

The method devised by Graham (2008) allows us to considerably mitigate these problems. Consider two vectors $W_{1g}$ and $W_{2g}$ containing observable jurisdiction-level information, and rewrite the equilibrium
equation (10) in error-component form, assuming that everything else – except individual concealed income and group composition – is unobservable. That is, define $\alpha_g \equiv dY_g$ as group-level heterogeneity, $\varepsilon_i \equiv cX_i$ as individual-level heterogeneity, and $\varepsilon_g \equiv cX_g$ as the group-level average of the latter. Then equation (10) and (9) become:

\begin{align*}
e_i &= \gamma \alpha_g + \varepsilon_i + (\gamma - 1) \varepsilon_g, \quad (11) \\
e_g &= \gamma (\alpha_g + \varepsilon_g). \quad (12)
\end{align*}

Equation (11) is slightly different from Graham’s behavioral equation, where group-level heterogeneity is not amplified by the social multiplier $\gamma$. This lack of amplification excludes the possibility that shocks to contextual variables may trigger a chain of feedbacks between group and individual behavior, which is instead the case in our framework.\(^{20}\) The reason is that equation (11) is derived from an economic model in which altering institutional variables leads to changes in behavior that are subsequently propagated by social interactions. If one posits a behavioral equation in which interactions occur directly via average individual characteristics (rather than indirectly in the reduced form, as in our model), then contextual variables that are not captured by average individual characteristics cannot generate multiplier effects. Correspondingly, Graham (2008) recognizes that in his framework “$\gamma$ may be a composite function of multiple ‘structural’ parameters. In Manski (1993) it depends on the strength of what he terms ‘exogenous’ and ‘endogenous’ social effects.” (p. 646, fn. 7).

Thanks to the explicit modeling of the mechanism generating externalities, the social multiplier we identify has a sharper interpretation, i.e. it is a known function of structural parameters that reflects endogenous social effects only. Therefore it is the exact measure of the equilibrium effect of exogenous shocks that alter individual behavior.

Let us denote by $\sigma^2_\varepsilon (W_{1g}, W_{2g})$ the conditional variance of individual heterogeneity, by $\sigma_{\varepsilon \varepsilon} (W_{1g}, W_{2g})$ the corresponding conditional covariance across individuals, by $\sigma^2_\alpha (W_{1g}, W_{2g})$ the conditional variance of

\(^{20}\)For instance, an exogenous change in the quality of tax officials in a certain jurisdiction may trigger a change in the perception of the audit probabilities and a sequence of feedbacks between individuals thus leading to a different equilibrium level of tax evasion.
group-level heterogeneity, and by $\sigma_{\alpha \varepsilon}(W_{1g}, W_{2g})$ its conditional covariance with individual heterogeneity.

Notice that $\sigma_{\varepsilon \varepsilon}$ is a measure of the degree of sorting of taxpayers across jurisdictions: it should be zero if they were randomly assigned. Similarly, $\sigma_{\alpha}^2$ reflects the variance of the unobserved characteristics of the tax authority (such as the efficiency of tax officials and the resources they can rely on), as well as other institutional, cultural, or market characteristics common to all taxpayers in a jurisdiction. Finally, $\sigma_{\alpha \varepsilon}$ reflects the degree of matching between such characteristics and taxpayers. This covariance is non-zero when, for instance, resources are allocated to tax authorities on the basis of taxpayer characteristics, or when taxpayers sort across jurisdictions on the basis of the efficiency of the tax authority.

Suppose that $\sigma_{\varepsilon \varepsilon}(W_{1g}, W_{2g})$, $\sigma_{\alpha}^2(W_{1g}, W_{2g})$, and $\sigma_{\alpha \varepsilon}(W_{1g}, W_{2g})$ are all independent of $W_{1g}$. We show in the Appendix, following Graham (2008) closely, that after conditioning on $W_{1g}$ and $W_{2g}$, the within-group variance of concealed income in jurisdiction $g$, denoted $V_{g}^w$, and the corresponding between-group variance, denoted $V_{g}^b$, can be written as follows:

$$V_{g}^w = \mathbb{E} \left[ \frac{\sigma_{\varepsilon}^2(W_{1g}, W_{2g}) - \sigma_{\varepsilon \varepsilon}(W_{2g})}{N_g} | W_{1g}, W_{2g} \right]$$

$$V_{g}^b = \gamma^2 \left( \sigma_{\alpha}^2(W_{2g}) + 2\sigma_{\alpha \varepsilon}(W_{2g}) + \sigma_{\varepsilon \varepsilon}(W_{2g}) + V_{g}^w \right)$$

These conditional variances exhibit an important pattern. First, the within-group variance is independent of social interactions and group-level heterogeneity. The reason is intuitive: if there are differences in individual tax evasion in a jurisdiction where tax officials are corrupt or externalities are at work, such variability cannot be ascribed to corruption or social effects, but only to differences between individuals and to covariance between individual characteristics generated by the process of sorting. Second, the between-group variance depends on group heterogeneity and is amplified by social effects, when these are present (i.e. $\gamma > 1$). This is also intuitive: for instance, part of the variability of tax evasion between two groups, one where tax officials are corrupt and one where they are not, must depend on corruption, as well as on the fact that dishonest taxpayers may tend to locate where tax officials are more easily corruptible. However, since the level of tax evasion in a group depends on social interactions, which alters contextual differences, so must be for the cross-group variation. In other words, the presence of social interactions generates a wedge
between the variance of illegal behavior at different levels of aggregation. In a linear-in-means model the size of the wedge is proportional to $\gamma^2$, a fact that offers a lever for identification.

Referring to equation (14), if we assume that the portion of the between-group variance that is independent of the within-group variance can be written as a linear function of $W_{2g}$, i.e.

$$\gamma^2 \left( \sigma^2_\alpha (W_{2g}) + 2\sigma_{\alpha \varepsilon} (W_{2g}) + \sigma_{\varepsilon \varepsilon} (W_{2g}) \right) = \theta W_{2g},$$

(15)

and if we rewrite conditional variances as conditional expectations of the appropriate statistics $G^w_g$ and $G^b_g$ (see Appendix), i.e.

$$V^w_g \equiv \mathbb{E} \left( G^w_g | W_{1g}, W_{2g} \right),$$

(16)

$$V^b_g \equiv \mathbb{E} \left( G^b_g | W_{1g}, W_{2g} \right),$$

(17)

then (14) becomes:

$$\mathbb{E} \left( G^w_g | W_{1g}, W_{2g} \right) = \theta W_{2g} + \gamma^2 \mathbb{E} \left( G^b_g | W_{1g}, W_{2g} \right).$$

(18)

This equation generates a conditional moment restriction:

$$\mathbb{E} [G^b_g - \theta W_{2g} - G^w_g | W_{1g}, W_{2g}] = 0,$$

(19)

which in turn implies the following unconditional moment restriction:

$$\mathbb{E} \left[ \begin{pmatrix} W_{1g} \\ W_{2g} \end{pmatrix} (G^b_g - \theta W_{2g} - \gamma^2 G^w_g) \right] = 0.$$

(20)

This equation provides the basis for estimating $\gamma^2$ by GMM, with $W_{1g}$ as an instrument. Effectively, the latter restricts the covariance matrix of cross-group tax evasion. Therefore, as well illustrated by Durlauf and Tanaka (2008), such a covariance restriction parallels the exclusion restriction needed to identify social
interactions in a regression framework based on model (8). The important advantage is that this identification strategy is robust to arbitrary individual and group-level unobservables.

Propositions 2, 3, and 4 suggest looking for a restriction in the distribution of tax credits. Since the individual tax credit, and so its mean, in our data can be regarded as a sufficient statistic for family composition and the latter may be part of salient individual characteristics (i.e. variables denoted by $\chi_i$ in the theoretical model), we use the dispersion of the distribution as an instrument: while it does not affect allocation of resources, sorting, and matching because of its uninformativeness to the tax authority, our maintained assumption is that it affects the within-group conditional variance of concealed income because it is a measure of individual heterogeneity.\(^{21}\) Under this assumption, the dispersion of the jurisdiction-level distribution of tax credits, $W_{1g}$, affects the between-group variance of concealed income only through the within-group variance, as equations (13) and (14) illustrate.

In our data the correlation between individual tax evasion and the dispersion of local tax credits is extremely low (0.007), thus corroborating the idea that the latter is uninformative with respect to tax evasion. On the other hand, as we report in the next Section, standard diagnostics (tests for rank condition and weak instruments) indicate that our instrument performs quite well.\(^{22}\)

Feasibility requires an estimate of $G_{b}^{h}$: we use the predicted value from the regression of $e_i$ on a constant, $W_{1g}$ and $W_{2g}$. After identifying $\gamma^2$, and assuming $J > 0$ in line with the theory, we use the delta method to recover the social multiplier $\gamma$, the externality coefficient $J$, and their standard errors.

\section{Results}

The model suggests including in $W_{2g}$ information that may affect sorting, matching, and the allocation of auditing resources. The following information seems appropriate given the limitations of our dataset. One, an indicator for whether the jurisdiction is large or small relative to the regional median. Two, an indicator

\footnote{This is a standard rank condition:}

\[E\left[G_{g}^{w} \mid W_{1g}, W_{2g}\right] \neq E\left[G_{g}^{w} \mid W_{1g}', W_{2g}\right] \text{ for } W_{1g} \neq W_{1g}'.\]

\footnote{Notice that by writing the unconditional moment restriction as in (20) we are not addressing the issue of the optimal set of instruments, so our estimates will be consistent but not necessarily efficient.}
of whether the local share of aggregate taxable income that is reported is above or below the regional share. For instance, more efficient tax officials may be assigned to larger jurisdictions, as well as to jurisdictions that do not generate enough revenue relative to the region. Three, a South dummy: Table 1 indicates that concealed income is larger in Southern than in Northern Italy. It is quite intuitive that this additional variable may at least affect matching and allocation of resources. Four, the shares of individuals self-employed in agriculture and handicraft: Table 2 indicates that concealed income is a very large portion of taxable income in these two sectors, so the expected return on auditing resources increases with such shares.

Our results are reported in Table 4 below. Column 1 contains a baseline specification, without regional and sector controls. Column 2 adds regional information, so it is more reliable although significance levels are slightly lower. The first-stage $F$-statistic and the coefficient on the instrument at first stage indicate that neither identification nor weak identification are concerns.

<table>
<thead>
<tr>
<th>Table 4. Results.</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$</td>
<td>0.68</td>
<td>0.61</td>
</tr>
<tr>
<td>(robust s.e.)</td>
<td>(0.09)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.13</td>
<td>2.60</td>
</tr>
<tr>
<td>(robust s.e.)</td>
<td>(0.91)</td>
<td>(0.97)</td>
</tr>
</tbody>
</table>

**1st stage:**

- $F$-statistic: 11.4 13.9
- Coefficient on instrument: 18.2 20.5
  - (robust s.e.): (5.4) (5.5)

**Reduced form:**

- Regional and Sector controls: NO YES
- Jurisdictions: 412 412
- Individuals: 78,863 78,863

These estimates imply a strong amplifying role of social effects: an exogenous shock altering concealed income independently across individuals produces an equilibrium variation that is up to three times the initial
response. Our model suggests this is a short run effect due to congestion of local auditing resources. This strong effect reflects the importance of reported income in the jurisdiction with respect to the perceived probability of an audit: recall the structural expression for the social interactions coefficient, \( J = -b_2/2b_1 \), where \( b_1 \) and \( b_2 \) are the effects of own and average report, respectively, on the perceived probability of an audit. Our estimates imply that, in absolute value, \( b_2 \) is 20-30% larger than \( b_1 \). This is suggestive of the importance of externalities: for a dishonest taxpayer, ceteris paribus, doing business in a jurisdiction where other taxpayers tend to behave honestly is perceived to be more risky than concealing income.

6 Concluding Remarks

Despite social externalities constituting a plausible explanation for the high variation of tax compliance, empirical research on tax evasion has largely ignored this possible determinant of individual behavior. This lack of empirical research is largely due to the fact that social effects are extremely difficult to identify. In this paper we have exploited a method recently devised by Graham (2008) that exploits conditional variances at different levels of aggregation. This “variance method”, which allows to overcome some of the most worrying aspects of identification in a traditional regression framework, still requires a restrictions on the covariance matrix of concealed income that parallels the restriction needed to identify social interactions based on the traditional “regression method” [Durlauf and Tanaka (2008)]. Such restriction arises naturally in our application thanks to an institutional property of the tax audit system in Italy: contrary to income reports, claims for tax credits are routinely and costlessly audited, so that tax credits are correctly reported and their distribution does not affect the probability of an audit.

The method we use requires linearity, which arises naturally in our model from the risk-neutrality of individual entrepreneurs and their attempt to optimally forecast audit probabilities. This also allows us to provide a simple structural interpretation of our estimates: we find a social multiplier of 2.5 - 3, implying among other things that the government can reduce tax evasion at fraction of the cost needed to directly induce single taxpayers to report honestly. A related implication is that social effects generated by the short-run resource constraint of local tax authorities make honest behavior of other taxpayers more risky – in terms of the perceived probability of an audit – than own dishonest behavior.
It may sound odd to refer to such interdependency as social interactions. However, ‘social’ refers to the particular nature of these externalities, which affect only individuals belonging to the same group, regardless of whether the forces that generate the externalities are genuinely ‘social’ or more ‘technological’, as in our model. In either case the meaning and the policy implications of the multiplier effects we identify are unaffected.

From a methodological viewpoint, we regard our work as an example of how the social interactions literature can make concrete progress in an empirical direction.

7 Appendix

7.1 Derivation of equation (8)

Substitute first $\partial \tilde{R}^i / \partial y^R_i = b_1$, from equation (4), into (1). Then the latter becomes:

$$f p_{ig} = (1 - p_{ig}) + b_1 (1 + f) e_i,$$

or, equivalently, collecting terms and dividing both sides by $(1 + f)$:

$$p_{ig} = (1 + f)^{-1} + b_1 e_i.$$

Next, substitute (4) into this expression. From the definition of concealed income, we can also substitute $y^R_i \equiv y_i - e_i$ and $y^R_g \equiv y_g - e_g$ and obtain:

$$b_0 + b_1 y_i - b_1 e_i + b_2 y_g - b_2 e_g + b_3 t_i + b_4 \chi_i + b_5 Z_g = (1 + f)^{-1} + b_1 e_i.$$ 

Solving for $e_i$ yields:

$$e_i = \frac{b_0 - (1 + f)^{-1}}{2b_1} + \frac{b_4}{2b_1} \chi_i + \frac{b_3}{2b_1} t_i + \frac{1}{2} y_i + \frac{b_5}{2b_1} Z_g + \frac{b_2}{2b_1} y_g - \frac{b_2}{2b_1} e_g.$$

Defining $k \equiv \frac{b_0 - (1 + f)^{-1}}{2b_1}$, $\beta \equiv \left( \frac{b_4}{2b_1}, \frac{b_3}{2b_1}, \frac{1}{2} \right)$, $\delta \equiv \left( \frac{b_5}{2b_1}, \frac{b_2}{2b_1} \right)$, $J \equiv -\frac{b_2}{2b_1}$, and $Y_g \equiv (Z_g, y_g,)$ this equation is the same as (8).
7.2 Derivation of conditional moments (16) and (17)

For each group $g$ that comprises $N_g$ individuals in the population in question, we have a random sample of size $n_g \leq N_g$. Denote with $e_g^*$ the sample mean of concealed income in group $g$, and manipulate the data in terms of within-group ($w$) and between-group ($b$) deviations from the respective means, with the cross-group mean conditioned on observable group-level information, that is $W_{1g}$ and $W_{2g}$:

$$G^w_g \equiv \frac{1}{N_g n_g - 1} \sum_{i=1}^{n_g} (e_i - e_g^*)^2, \quad (21)$$

$$G^b_g \equiv (e_g^* - \mathbb{E} (e_i|W_{1g}, W_{2g}))^2 - \left( \frac{1}{n_g} - \frac{1}{N_g} \right) N_g G^w_g. \quad (22)$$

This means that $G^w_g$ is simply the within-group sample variance of tax evasion, normalized by population size, while $G^b_g$ is the square deviation of group average evasion in the sample from the conditional population mean, minus a correction term to account for the discrepancy between sample and population means. The role of this correction term is made clear below. The purpose of these statistics is to derive an estimator based on the variance of concealed income at different levels of aggregation. Notice that after using individual-level data to construct $G^w_g$ and $G^b_g$, the analysis uses only these transformations and $(W_{1g}, W_{2g})$, i.e. aggregate group-level data: jurisdictions, not individual taxpayers, are the units of observation.

The normalized conditional within-group variance of concealed income, within the jurisdictions defined by $(W_{1g}, W_{2g})$, can be computed by taking the conditional expectation of (21), as expressed in equation (13). In the following we will use equation (11):

$$e_i = \gamma \alpha_g + \varepsilon_i + (\gamma - 1) \varepsilon_g. \quad (23)$$

Assume, without loss of generality, that $\mathbb{E} (\varepsilon_i|N_g, n_g, W_{2g}) = 0.23$ Then, the conditional mean of concealed income is:

$$\mathbb{E} (e_i|W_{1g}, W_{2g}) = \gamma \mathbb{E} (\alpha_g|W_{1g}, W_{2g}). \quad (24)$$

---

23 Notice that this is an assumption about the theoretical mean. The sample mean, as assumed above, is $\varepsilon_g = \frac{1}{n_g} \sum_i \varepsilon_i$. 

22
Taking the conditional expectations of $G_{g}^{w}$ yields:

\[
\mathbb{E} \left[ G_{g}^{w} | N_{g}, n_{g}, W_{1g}, W_{2g} \right] \\
= \mathbb{E} \left[ \frac{1}{N_{g}} \frac{1}{n_{g} - 1} \sum_{i=1}^{n_{g}} (e_{i} - e_{g})^{2} | N_{g}, n_{g}, W_{1g}, W_{2g} \right] \\
= \mathbb{E} \left[ \frac{1}{N_{g}} \frac{1}{n_{g} - 1} \sum_{i=1}^{n_{g}} (\gamma \alpha_{g} + \varepsilon_{i} + \gamma - 1) \varepsilon_{g} - \gamma \alpha_{g} - \varepsilon_{g} - (\gamma - 1) \varepsilon_{g})^{2} | N_{g}, n_{g}, W_{1g}, W_{2g} \right] \\
= \mathbb{E} \left[ \frac{1}{N_{g}} \frac{1}{n_{g} - 1} \sum_{i=1}^{n_{g}} (\varepsilon_{i} - \varepsilon_{g})^{2} | N_{g}, n_{g}, W_{1g}, W_{2g} \right] \\
= \mathbb{E} \left[ \frac{1}{N_{g}} \frac{1}{n_{g} - 1} \sum_{i=1}^{n_{g}} (\varepsilon_{i}^{2} + \varepsilon_{g}^{2} - 2\varepsilon_{i}\varepsilon_{g}) | N_{g}, n_{g}, W_{1g}, W_{2g} \right] \\
= \frac{n_{g}}{N_{g} n_{g} - 1} \mathbb{E} \left( \varepsilon_{i}^{2} | N_{g}, n_{g}, W_{1g}, W_{2g} \right) + \frac{n_{g}}{N_{g} n_{g} - 1} \mathbb{E} \left( \left( \frac{1}{n_{g}} \sum_{i=1}^{n_{g}} \varepsilon_{i} \right)^{2} | N_{g}, n_{g}, W_{1g}, W_{2g} \right) \\
- 2 \frac{n_{g}}{N_{g} n_{g} - 1} \mathbb{E} (\varepsilon_{i} \varepsilon_{g} | N_{g}, n_{g}, W_{1g}, W_{2g}) \\
= \frac{n_{g}}{N_{g} n_{g} - 1} \sigma_{\varepsilon}^{2} + \frac{1}{N_{g} (n_{g} - 1)} \frac{1}{n_{g}} \mathbb{E} \left( \left( \sum_{i=1}^{n_{g}} \varepsilon_{i} \right)^{2} | N_{g}, n_{g}, W_{1g}, W_{2g} \right) \\
- 2 \frac{n_{g}}{N_{g} (n_{g} - 1)} \mathbb{E} \left( \varepsilon_{i} \sum_{j=1}^{n_{g}} \varepsilon_{j} | N_{g}, n_{g}, W_{1g}, W_{2g} \right) \\
= \frac{n_{g}}{N_{g} n_{g} - 1} \sigma_{\varepsilon}^{2} + \frac{1}{N_{g} (n_{g} - 1)} \frac{1}{n_{g}} \left( n_{g} \sigma_{\varepsilon}^{2} + n_{g} (n_{g} - 1) \sigma_{\varepsilon \varepsilon} \right) \\
- 2 \frac{n_{g}}{N_{g} (n_{g} - 1)} \left( \sigma_{\varepsilon}^{2} + (n_{g} - 1) \sigma_{\varepsilon \varepsilon} \right) \\
= \frac{n_{g}}{N_{g} n_{g} - 1} \sigma_{\varepsilon}^{2} - \frac{1}{N_{g} (n_{g} - 1)} \sigma_{\varepsilon}^{2} - \frac{1}{N_{g}} \sigma_{\varepsilon \varepsilon} = \frac{\sigma_{\varepsilon}^{2} - \sigma_{\varepsilon \varepsilon}}{N_{g}},
\]

where we abbreviate $\sigma_{\varepsilon}^{2} (W_{2g})$ and $\sigma_{\varepsilon \varepsilon} (W_{2g})$ to $\sigma_{\varepsilon}^{2}$ and $\sigma_{\varepsilon \varepsilon}$. By the law of iterated expectations $\mathbb{E} \left[ G_{g}^{w} | W_{2g} \right] = \mathbb{E} \left[ \mathbb{E} \left[ G_{g}^{w} | N_{g}, n_{g}, W_{2g} \right] | W_{2g} \right]$. That is:

\[
\mathbb{E} \left[ G_{g}^{w} | W_{1g}, W_{2g} \right] = \mathbb{E} \left[ \frac{\sigma_{\varepsilon}^{2} - \sigma_{\varepsilon \varepsilon}}{N_{g}} | W_{1g}, W_{2g} \right],
\]

23
which is equation (13). Similarly, the between-jurisdiction conditional variance can be computed by taking
the conditional expectation of (22), as expressed in equation (14):

\[
\mathbb{E} [G_g^B | N_g, n_g, W_{2g}] \\
= \mathbb{E} [(e_g - E(e_i|\nu, \omega))^2 | N_g, n_g, W_{2g}] - \mathbb{E} \left[ \left( \frac{1}{n_g} - \frac{1}{N_g} \right) N_g G_g^W | N_g, n_g, W_{2g} \right] \\
= \mathbb{V}(e_g | N_g, n_g, W_{2g}) - \mathbb{E} \left[ \left( \frac{1}{n_g} - \frac{1}{N_g} \right) (\sigma_e^2 - \sigma_{\varepsilon e}) | N_g, n_g, W_{2g} \right],
\]

(23)

where \( \mathbb{V} \) denotes variance. Denote with \( \varepsilon_g^s \) the sample mean of individual characteristics in group \( g \), as opposed to population mean \( \varepsilon_g \). Then we can write the first term in this equation, i.e. the between-group conditional variance of concealed income, as:

\[
\mathbb{V}(e_g | N_g, n_g, W_{2g}) \\
= \mathbb{V} \left( \frac{1}{n_g} \sum_{i=1}^{n_g} (\gamma \alpha_g + \varepsilon_i + (\gamma - 1) \varepsilon_g) | N_g, n_g, W_{2g} \right) \\
= \mathbb{V} (\gamma \alpha_g + \varepsilon_g^s + (\gamma - 1) \varepsilon_g | N_g, n_g, W_{2g}) \\
= \gamma^2 \mathbb{V}(\alpha_g | N_g, n_g, W_{2g}) + \mathbb{V}(\varepsilon_g^s | N_g, n_g, W_{2g}) + (\gamma - 1)^2 \mathbb{V}(\varepsilon_g | N_g, n_g, W_{2g}) \\
+ 2\gamma \mathbb{C}(\alpha_g, \varepsilon_g^s | N_g, n_g, W_{2g}) + 2\gamma (\gamma - 1) \mathbb{C}(\alpha_g, \varepsilon_g | N_g, n_g, W_{2g}) \\
+ 2(\gamma - 1) \mathbb{C}(\varepsilon_g^s, \varepsilon_g | N_g, n_g, W_{2g}) \\
= \gamma^2 \sigma_\alpha^2 + \mathbb{V} \left( \frac{1}{n_g} \sum_{i=1}^{n_g} \varepsilon_i | N_g, n_g, W_{2g} \right) + (\gamma - 1)^2 \mathbb{V} \left( \frac{1}{N_g} \sum_{i=1}^{N_g} \varepsilon_i | N_g, n_g, W_{2g} \right) \\
+ 2\gamma \sigma_{\alpha \varepsilon} + 2\gamma (\gamma - 1) \sigma_{\alpha \varepsilon} + 2(\gamma - 1) \mathbb{C} \left( \frac{1}{n_g} \sum_{i=1}^{n_g} \varepsilon_i, \frac{1}{N_g} \sum_{i=1}^{N_g} \varepsilon_i | N_g, n_g, W_{2g} \right) \\
= \gamma^2 \sigma_\alpha^2 + \frac{1}{n_g} \sigma_\varepsilon^2 + \frac{n_g - 1}{n_g} \sigma_{\varepsilon \varepsilon} + (\gamma - 1)^2 \frac{1}{N_g} \sigma_\varepsilon^2 + (\gamma - 1)^2 \frac{N_g - 1}{N_g} \sigma_{\varepsilon \varepsilon} + \\
+ 2\gamma \sigma_{\alpha \varepsilon} + 2\gamma (\gamma - 1) \sigma_{\alpha \varepsilon} + 2(\gamma - 1) \left( \frac{1}{N_g} \sigma_\varepsilon^2 + \frac{N_g - 1}{N_g} \sigma_{\varepsilon \varepsilon} \right) \\
= \gamma^2 \sigma_\alpha^2 + 2\gamma^2 \sigma_\alpha^2 + ((\gamma - 1)^2 + 2(\gamma - 1) + 1) \sigma_{\varepsilon \varepsilon} \\
+ \frac{\sigma_\varepsilon^2 - \sigma_{\varepsilon \varepsilon}}{n_g} + ((\gamma - 1)^2 + 2(\gamma - 1)) \frac{\sigma_\varepsilon^2 - \sigma_{\varepsilon \varepsilon}}{N_g}
\]
\[
\begin{align*}
\gamma^2 \sigma_a^2 & + 2\gamma^2 \sigma_{\alpha\varepsilon} + \gamma^2 \sigma_{\varepsilon\varepsilon} + \frac{\sigma_{\varepsilon}^2 - \sigma_{\varepsilon\varepsilon}}{n_g} + \left( \frac{1}{n_g} - \frac{1}{N_g} \right) \left( \sigma_{\varepsilon}^2 - \sigma_{\varepsilon\varepsilon} \right),
\end{align*}
\]

where we have again, for brevity, omitted the argument of the conditional variances and covariances. Now replace (24) into (23):

\[
\begin{align*}
\mathbb{E} \left[ G^B_g | N_g, n_g, W_{2g} \right] &= \gamma^2 \left( \sigma_a^2 + 2\sigma_{\alpha\varepsilon} + \sigma_{\varepsilon\varepsilon} + \frac{\sigma_{\varepsilon}^2 - \sigma_{\varepsilon\varepsilon}}{N_g} \right) + \left( \frac{1}{n_g} - \frac{1}{N_g} \right) \left( \sigma_{\varepsilon}^2 - \sigma_{\varepsilon\varepsilon} \right) \\
&- \mathbb{E} \left[ \left( \frac{1}{n_g} - \frac{1}{N_g} \right) \left( \sigma_{\varepsilon}^2 - \sigma_{\varepsilon\varepsilon} \right) | N_g, n_g, W_{2g} \right].
\end{align*}
\]

Finally, take the expectation of this expression, conditional on \( W_{2g} \):

\[
\begin{align*}
\mathbb{E} \left[ \mathbb{E} \left[ G^B_g | N_g, n_g, W_{2g} \right] | W_{2g} \right] &= \gamma^2 \left( \sigma_a^2 + 2\sigma_{\alpha\varepsilon} + \sigma_{\varepsilon\varepsilon} + \frac{\sigma_{\varepsilon}^2 - \sigma_{\varepsilon\varepsilon}}{N_g} \right) + \gamma^2 \mathbb{E} \left[ \frac{\sigma_{\varepsilon}^2 - \sigma_{\varepsilon\varepsilon}}{N_g} | W_{2g} \right] \\
&+ \mathbb{E} \left[ \left( \frac{1}{n_g} - \frac{1}{N_g} \right) \left( \sigma_{\varepsilon}^2 - \sigma_{\varepsilon\varepsilon} \right) | W_{2g} \right] \\
&- \mathbb{E} \left[ \left( \frac{1}{n_g} - \frac{1}{N_g} \right) \left( \sigma_{\varepsilon}^2 - \sigma_{\varepsilon\varepsilon} \right) | N_g, n_g, W_{2g} \right] | W_{2g} \right].
\end{align*}
\]

Applying the law of iterated expectations, the LHS reduces to \( \mathbb{E} \left[ G^B_g | W_{1g}, W_{2g} \right] \), and the last two terms on the RHS cancel out. Therefore this equation reduces to equation (14).

### 7.3 Some useful properties of the covariance

Three basic properties of the covariance are used in the previous derivations. We summarize them here for convenience. First, for any sequence of \( n \) random variables \( X_i \) with common variance \( \sigma_X^2 \) and covariance
\( \sigma_{XX} : \)

\[
V \left( \frac{1}{n} \sum_{i=1}^{n} X_i \right) = \frac{1}{n^2} \left( n\sigma_X^2 + 2 \sum_{i<j}^{n} \text{cov} \left( X_i, X_j \right) \right) \\
= \frac{1}{n^2} \left( n\sigma_X^2 + 2 \frac{n(n-1)}{2} \sigma_{XX} \right) \\
= \frac{1}{n} \sigma_X^2 + \frac{(n-1)}{n} \sigma_{XX}.
\]

Second, for the same sequence and another random variable \( Y \) whose covariance with any random variable in the series is \( \sigma_{YX} \),

\[
C \left( Y, \frac{1}{n} \sum_{i=1}^{n} X_i \right) = \frac{1}{n} \sum_{i=1}^{n} \text{cov} \left( Y, X_i \right) = \sigma_{YX}.
\]

Third, if we extend the sequence to \( N > n \), then:

\[
C \left( \frac{1}{n} \sum_{i=1}^{n} X_i, \frac{1}{N} \sum_{i=1}^{N} X_i \right) = \frac{1}{nN} \left( \sum_{i=1}^{n} \text{cov} \left( X_i, X_i \right) + \sum_{i<j}^{N} \text{cov} \left( X_i, X_j \right) \right) \\
= \frac{1}{nN} \left( n\sigma_X^2 + (nN-n) \sigma_{XX} \right) \\
= \frac{1}{N} \sigma_X^2 + \frac{N-1}{N} \sigma_{XX}.
\]
References


Table 1. Concealed income and tax audits.

<table>
<thead>
<tr>
<th>Region</th>
<th>Evasion</th>
<th>Jurisdictions</th>
<th>Audits</th>
<th>Population</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aosta Valley</td>
<td>36.8%</td>
<td>2</td>
<td>267</td>
<td>6,852</td>
<td>3.9%</td>
</tr>
<tr>
<td>Piedmont</td>
<td>37.6%</td>
<td>38</td>
<td>6,887</td>
<td>237,068</td>
<td>2.9%</td>
</tr>
<tr>
<td>Lombardy</td>
<td>32.5%</td>
<td>60</td>
<td>12,634</td>
<td>520,765</td>
<td>2.4%</td>
</tr>
<tr>
<td>Friuli-Venezia Giulia</td>
<td>56.3%</td>
<td>10</td>
<td>1,407</td>
<td>65,208</td>
<td>2.2%</td>
</tr>
<tr>
<td>Trentino-Südtirol</td>
<td>45.0%</td>
<td>12</td>
<td>1,067</td>
<td>50,697</td>
<td>2.1%</td>
</tr>
<tr>
<td>Veneto</td>
<td>50.4%</td>
<td>31</td>
<td>4,840</td>
<td>259,584</td>
<td>1.9%</td>
</tr>
<tr>
<td>Liguria</td>
<td>45.6%</td>
<td>10</td>
<td>3,661</td>
<td>95,602</td>
<td>3.8%</td>
</tr>
<tr>
<td>Emilia-Romagna</td>
<td>37.5%</td>
<td>24</td>
<td>6,016</td>
<td>248,353</td>
<td>2.4%</td>
</tr>
<tr>
<td>Tuscany</td>
<td>40.7%</td>
<td>34</td>
<td>5468</td>
<td>215,758</td>
<td>2.5%</td>
</tr>
<tr>
<td>Marche</td>
<td>46.5%</td>
<td>14</td>
<td>2,122</td>
<td>88,906</td>
<td>2.4%</td>
</tr>
<tr>
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<td>1,257</td>
<td>43,581</td>
<td>2.9%</td>
</tr>
<tr>
<td>Lazio</td>
<td>52.1%</td>
<td>14</td>
<td>5,729</td>
<td>273,343</td>
<td>2.1%</td>
</tr>
<tr>
<td>Abruzzo</td>
<td>44.6%</td>
<td>13</td>
<td>2,413</td>
<td>66,495</td>
<td>3.6%</td>
</tr>
<tr>
<td>Molise</td>
<td>67.1%</td>
<td>4</td>
<td>569</td>
<td>15,194</td>
<td>3.7%</td>
</tr>
<tr>
<td>Campania</td>
<td>55.5%</td>
<td>27</td>
<td>6,720</td>
<td>228,824</td>
<td>2.9%</td>
</tr>
<tr>
<td>Basilicata</td>
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<td>1,056</td>
<td>27,335</td>
<td>3.9%</td>
</tr>
<tr>
<td>Apulia</td>
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<td>16</td>
<td>4,968</td>
<td>195,460</td>
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</tr>
<tr>
<td>Calabria</td>
<td>66.1%</td>
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<td>84,175</td>
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</tr>
<tr>
<td>Sicily</td>
<td>54.4%</td>
<td>40</td>
<td>7,434</td>
<td>217,394</td>
<td>3.4%</td>
</tr>
<tr>
<td>Sardinia</td>
<td>56.1%</td>
<td>11</td>
<td>1,500</td>
<td>73,717</td>
<td>2.0%</td>
</tr>
<tr>
<td><strong>Italy</strong></td>
<td><strong>46.4%</strong></td>
<td><strong>412</strong></td>
<td><strong>78,863</strong></td>
<td><strong>3,014,311</strong></td>
<td><strong>2.6%</strong></td>
</tr>
</tbody>
</table>

Note to Table 1. Column 1: concealed income as % of taxable income. Column 2: number of tax jurisdictions. Column 3: total number of audits performed. Column 4: number of self-employed individuals. Column 5: total number of audits as % of number of self-employed individuals.
Table 2. Concealed income as % of taxable income, by sector.

<table>
<thead>
<tr>
<th>Sector of Activity</th>
<th>Evasion</th>
<th>Audits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>68.9%</td>
<td>380</td>
</tr>
<tr>
<td>Handicraft</td>
<td>65.6%</td>
<td>13,496</td>
</tr>
<tr>
<td>Trade</td>
<td>60.5%</td>
<td>29,571</td>
</tr>
<tr>
<td>Services</td>
<td>24.8%</td>
<td>18,537</td>
</tr>
<tr>
<td>Missing</td>
<td>37.4%</td>
<td>16,906</td>
</tr>
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</table>

Table 3. Small and large tax jurisdictions.
<table>
<thead>
<tr>
<th>Region</th>
<th>Jurisdictions</th>
<th>Audits</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#</td>
<td>Small</td>
<td>Large</td>
<td>Small</td>
</tr>
<tr>
<td>Aosta Valley</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>67</td>
</tr>
<tr>
<td>Piedmont</td>
<td>38</td>
<td>29</td>
<td>9</td>
<td>3,380</td>
</tr>
<tr>
<td>Lombardy</td>
<td>60</td>
<td>52</td>
<td>8</td>
<td>6,108</td>
</tr>
<tr>
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<td>10</td>
<td>7</td>
<td>3</td>
<td>483</td>
</tr>
<tr>
<td>Trentino-Alto Adige/Südtirol</td>
<td>12</td>
<td>8</td>
<td>4</td>
<td>440</td>
</tr>
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<td>Veneto</td>
<td>31</td>
<td>25</td>
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<td>2,239</td>
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<td>Liguria</td>
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<td>3</td>
<td>1,675</td>
</tr>
<tr>
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<td>5</td>
<td>2,876</td>
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<td>Tuscany</td>
<td>34</td>
<td>28</td>
<td>6</td>
<td>2,705</td>
</tr>
<tr>
<td>Marche</td>
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<td>10</td>
<td>4</td>
<td>1,020</td>
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<td>2,441</td>
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<td>4</td>
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<tr>
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<td>193</td>
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<tr>
<td>Campania</td>
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<td>4</td>
<td>472</td>
</tr>
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<td>5</td>
<td>2,291</td>
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<tr>
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<td>21</td>
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<td>1,115</td>
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<tr>
<td>Sicily</td>
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<td>33</td>
<td>7</td>
<td>3,572</td>
</tr>
<tr>
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<td>8</td>
<td>3</td>
<td>696</td>
</tr>
<tr>
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<td>412</td>
<td>316</td>
<td>96</td>
<td>36,478</td>
</tr>
</tbody>
</table>

Note to Table 3. The table breaks down the number of jurisdictions and audits into small and large jurisdictions. A small jurisdiction is defined as a jurisdiction whose size in terms of the auditable self-employed is below the regional median.
Figure 1. Distribution (kernel density estimates) of concealed income across individuals, as % of total taxable income
Figure 2. Distribution (kernel density estimates) of concealed income across jurisdictions, as % of total taxable income

Figure 3. Distribution (kernel density estimates) of concealed income across individuals and jurisdictions, as % of total taxable income
Figure 1. Optimal audit probability.
Figure 2. Effect of increased tax evasion

Figure 3. Optimal and optimally predicted audit probability